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Visco-instability of shear viscoelastic collisional dusty plasma systems

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In this paper, the stability of Newtonian and non-Newtonian viscoelastic collisional shear-velocity dusty plasmas is studied, using the framework of a generalized hydrodynamic (GH) model. Motivated by Banerjee *et al.*'s work (Banerjee *et al.*, *New J. Phys.*, vol. 12 (12), 2010, p. 123031), employing linear perturbation theory as well as the local approximation method in the inhomogeneous direction, the dispersion relations of the Fourier modes are obtained for Newtonian and non-Newtonian dusty plasma systems in the presence of a dust–neutral friction term. The analysis of the obtained dispersion relation in the non-Newtonian case shows that the inhomogeneous viscosity force depending on the velocity shear profile can be the genesis of a free energy source which leads the shear system to be unstable. Study of the dust–neutral friction effect on the instability of the considered systems using numerical analysis of the dispersion relation in the Newtonian case demonstrates that the maximum growth rate decreases considerably by increasing the collision frequency in the hydrodynamic regime, while this reduction can be neglected in the kinetic regime. Results show a more significant stabilization role of the dust–neutral friction term in the non-Newtonian cases, through decreasing the maximum growth rate at any fixed wavenumber and construction of the instable wavenumber region. The results of the present investigation will greatly contribute to study of the time evolution of viscoelastic laboratory environments with externally applied shear; where in these experiments the dust–neutral friction process can play a considerable role.

Key words: dusty plasmas, plasma instabilities, strongly coupled plasmas

1. Introduction

The study of dust plasma systems has recently received a great deal of interest in terms of understanding the many puzzling phenomena in the laboratory and the field of astrophysics, particularly in shear flow structures including planetary nebulas, interstellar matter, interplanetary medium, comets, planetary rings, stars, the tokomak edge plasma and so on (Whipple 1981; Whipple, Northrop & Mendis 1985; Horanyi, Houpis & Mendis 1988; Goertz 1989; Angelis 1992; Tsytovich & Havnes 1993; Mendis & Rosenberg 1994; Thomas *et al.* 1994; Ussenov *et al.* 2014). Over recent years, the dust particle motions of shear outflows have been a subject of considerable

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attention as they can lead to an interesting novel phenomenology (Pesceli, Rasmussen & Thomsen 1984; Ikezi 1986; Rosenberg & Kalman 1997; Gruzinov 2008; Jana, Banerjee & Chakrabarti 2015). These systems can be categorized into different classes defined by the Coulomb coupling parameter $\Gamma = z_d^2 e^2/k_B T_d a$ where a is the average interparticle distance related to dust number density n_d as $a \approx n_d^{-1/3}$, and z_d, T_d and k_B are, respectively, the charge on the dust particle, the temperature of dust component and the Boltzmann constant (Rosenberg & Kalman 1997; Nunomura *et al.* 1999; Mishra, Kaw & Sen 2000). In this category, when the potential energy of the constituent particles is much smaller than their kinetic energy ($\Gamma \ll 1$) the dust plasma system is called weakly coupled, while it is strongly coupled when the potential energy of the constituent particles is much greater than their kinetic energy ($\Gamma \gg 1$). In this classification, most of the space and laboratory dust plasmas lie in the strongly coupled plasma category due to their low dust temperature and high dust charge. The members of this class itself can be divided in two sub-groups by defining the Γ_c as a critical coupled value over which the system becomes crystalline: (i) the group of $\Gamma > \Gamma_c$ where the elasticity effect of the considered system dominates over the viscosity effect so that the viscosity effect can be neglected and (ii) the group of $\mathbf{1} \ll \Gamma < \Gamma_c$ in which both the viscosity and elasticity features of the system are equally important, known as a viscoelastic fluid. In studying the latter case, using the generalized hydrodynamic (GH) model, Banerjee and his coworkers recently showed that non-Newtonian (in which the viscosity coefficient depends on the velocity shear rate) collisionless shear flow systems suffer the instability of due to the viscosity gradient. On the other hand, in a low-frequency dusty plasma system it has been shown that dust–neutral collisions can play an important role in the stability of the system (Rosenberg & Kalman 1997). As far as we understand, the previous works on the stability analysis of strongly coupled dusty plasma systems were limited, up to now, to the Newtonian and collisionless regimes (Pesceli *et al.* 1984). The goal of this paper is to study the stability of the collisional viscoelastic dusty plasma system in Newtonian and non-Newtonian regimes. This paper attempts to evaluate the effects of dust–neutral collision (called 'collision' in abbreviation, hereafter) on the stability of the viscoelastic dusty plasma systems with constant and velocity-dependent viscosity coefficients. Accordingly, recalling the generalized hydrodynamic equation coupled with Maxwell's equations, the time evolution of the shear collisional dusty plasma system in the strongly coupled regime, $1 \ll \Gamma$, can be investigated by deriving the dispersion relation of excited modes in the system. For this purpose, we consider a shear dusty plasma system consisting of positive ions, electrons and negative dust grains whose equilibrium space-dependent velocity profile, $\boldsymbol{v}_{0y}(x)$, can be obtained from the GH equation. In the following, employing the linear velocity profile (at equilibrium state) as $\boldsymbol{v}_{0y}(x) = v_0 + v_0 \alpha x$ the effects of collisions on the instability growth rate of the considered system is investigated by deriving the corresponding dispersion relations for the Newtonian and non-Newtonian fluids, where the derivations are based on those of Angelis (1992) but with the addition of the dust–gas friction term (Banerjee *et al.* 2010). Numerical analysis of the obtained dispersion relations shows that the collision term affects the stability of these fluids in different ways. In this regard, our study of the Newtonian shear dusty plasma system in the hydrodynamic limit shows that the maximum normalized growth rate of the system decreases by increasing the collision frequency while the effect of reducing the collision term on the growth rate of instability in kinetic limit is not considerable. This investigation of the non-Newtonian regime of a strongly coupled dusty plasma system shows that the stabilization role of the collision term for these systems is more significant through constriction of the unstable wavenumber region and a substantial decrease in the maximum growth rate at any arbitrary wavenumber.

The remaining parts of this paper are organized as follow: in the next section, a mathematical model for the equilibrium state of a Newtonian shear viscoelastic dusty plasma system is formulated, employing the linear perturbation theory as well as the local approximation method the linear dispersion relation for the general velocity profile is yielded. The analyses of the obtained dispersion relation for the linear velocity profile in the hydrodynamic ($\omega\tau_m \ll 1$) and kinetic regimes ($\omega\tau_m \gg 1$) are detailed in § 3. In the final section, the collision effect on the instability of a non-Newtonian collisional viscose dusty plasma system due to the viscosity gradient is discussed.

2. Newtonian viscoelastic collisional shear dusty plasma system

2.1. *Equilibrium state and dispersion relation*

Consider a Newtonian strongly coupled dusty plasma system consisting of warm electrons, ions and negative massive dust grains satisfying the charge-neutrality condition at the equilibrium state. To study the time evolution of the system in the low-frequency regime ($\omega \ll kv_{Te(i)}$ where $v_{Te(i)}$ represents the thermal velocities of electrons and ions), the electrons and ions are generally found in a weakly coupled state due to their high temperature and low electric charges, therefore they can gain enough time to obey the Boltzmann distribution. The negative massive dust grains in this system must be described by the GH model, where in this investigation it is assumed that the coupling parameter of dust grains satisfies the condition $1 \ll \varGamma < \varGamma_c$ in which both viscosity and elasticity features of the system are equally important. Assuming the dust grains flow in the y direction with an x-dependent velocity profile ($v_0(x)\boldsymbol{e}_y$) in a background of fixed neutral particles, the equilibrium state of these particles can be founded from following GH equation (Frenkel 1946; Rosenberg & Kalman 1997; Kaw 2001; Shukla & Mamun 2001; El-Awady & Djebli 2012), as in equation (1) of Angelis (1992) but taking into account the neutral–dust friction term,

$$\left[1+\tau_m\left(\frac{\partial}{\partial t}\right)\right]\left[\rho_d\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla}\right)\boldsymbol{v}+\boldsymbol{\nabla}p_d+q_dn_d\boldsymbol{E}+m_dn_d\nu\boldsymbol{v}\right]=\frac{\partial\sigma_{ij}}{\partial x_j}, \tag{2.1}$$

in which

$$\sigma_{ij}=\eta\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+\left(\xi-\frac{2}{3}\eta\right)\delta_{ij}(\boldsymbol{\nabla}\cdot\boldsymbol{v}). \tag{2.2}$$

In this equation the dust–neutral collision (called ‘collision’ in abbreviation, hereafter) effect is indicated by the ν parameter which is the collision frequency. The symbols m_d, n_d, p_d, τ_m and $\boldsymbol{E}$ stand for the mass of dust species, the dust particles number density, the dust particles pressure, the viscoelastic relaxation time and the electric field, respectively. Note that, the viscoelastic relaxation time demonstrated by τ_m in (2.1) constitutes a bridge between solid and liquid properties of the plasma system discussed

In this step, considering incompressibility and uniformity constraints on the number density of the dust grains (to compare with previous works) the mass conservation equation of dust grains at the equilibrium state is

$$\frac{\partial\rho_d}{\partial t}+\boldsymbol{\nabla}\cdot(\rho\boldsymbol{v})=0, \tag{2.3}$$

where $\rho_d=m_dn_d$.

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To find the equilibrium velocity profile of the dust grains, we must examine the steady state form of (2.1)

$$\left[\rho_d\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}+\nabla p_d+q_d n_d \boldsymbol{E}+m_d n_d \nu \boldsymbol{v}\right]=\frac{\partial \sigma_{ij}}{\partial x_j}. \tag{2.4}$$

After some algebra and with the considered assumptions, equation (2.4) can be reduced to the following form

$$m_d n_d \nu \boldsymbol{v}_{0y}(x)=\eta\frac{\mathrm{d}^2 v_{0y}(x)}{\mathrm{d}x^2}. \tag{2.5}$$

This equation leads to the dust grain velocity profile at the equilibrium state as

$$\boldsymbol{v}_{0y}(x)=v_0\exp\left(-\alpha\,|x|\right)\boldsymbol{e}_{\boldsymbol{y}}, \tag{2.6}$$

where

$$\alpha=\sqrt{\frac{m_d n_{0d}\nu_{dn}}{\eta}}. \tag{2.7}$$

Now, to find the time evaluation of the considered Newtonian shear dusty plasma flow system, we have employed linear perturbation theory for the under study two-dimensional incompressible plasma system in the x–y plane. After some straightforward algebra, one obtains the x- and y-components of the perturbed GH equation as

$$\begin{aligned}&\left[1+\tau_m\left(\frac{\partial}{\partial t}\right)\right]\left[\rho_d\left(\frac{\partial}{\partial t}+v_{1x}(x,y,t)\frac{\partial}{\partial x}+(v_{0y}(x)+v_{1_y}(x,y,t))\frac{\partial}{\partial y}\right)\right.\\&\left. v_{1x}(x,y,t)+\rho_d\nu v_{1x}(x,y,t)\right]=\eta\left(\frac{\partial^2 v_{1x}(x,y,t)}{\partial y^2}+\frac{\partial^2 v_{1y}(x,y,t)}{\partial y\partial x}\right),\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}&\left[1+\tau_m\left(\frac{\partial}{\partial t}\right)\right]\left[\rho_d\left(\frac{\partial}{\partial t}+v_{1x}\,(x,y,t)\,\frac{\partial}{\partial x}+(v_{0y}\,(x)+v_{1_y}(x,y,t))\frac{\partial}{\partial y}\right)\right.\\&\left.\times(v_{0y}(x)+v_{1_y}(x,y,t))+\rho_d\nu(v_{0y}(x)+v_{1_y}(x,y,t))\right]\\&=\eta\left(\frac{\partial^2 v_{1y}(x,y,t)}{\partial x^2}+\frac{\partial^2 v_{1x}(x,y,t)}{\partial x\partial y}+\frac{\partial^2 v_{0y}(x)}{\partial x^2}\right),\end{aligned} \tag{2.9}$$

where $v_1(x, y, t)$ is the velocity perturbed around the equilibrium state. Using the Fourier transport in the y-direction, the perturbed velocity can be considered as $v_1(x, y, t)=v_1(x)\exp(\mathrm{i}k_y y-\mathrm{i}\omega t)$, where k_y is the wave vector in the y direction and ω is the frequency of the corresponding perturbation mode. Inserting the perturbed velocity into (2.8) and (2.9), they reduce to the following eigenvalue equations

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$$
\begin{aligned}
&[1-\mathrm{i}\omega\tau_m]\left[\rho_d\left(-\mathrm{i}\omega+v_{1x}(x)\frac{\partial}{\partial x}+(v_{0y}(x)+v_{1y}(x))\mathrm{i}k_y\right)v_{1x}(x)+\rho_d\nu v_{1x}(x)\right]\\
&\quad=\eta\left(-k_y^2v_{1x}(x)+\mathrm{i}k_y\frac{\partial v_{1y}(x)}{\partial x}\right).
\end{aligned}
\tag{2.10}
$$

and

$$
\begin{aligned}
&[1-\mathrm{i}\omega\tau_m]\left[\rho_d\left(-\mathrm{i}\omega+v_{1x}(x)\frac{\partial}{\partial x}+(v_{0y}(x)+v_{1y}(x))\mathrm{i}k_y\right)(v_{0y}(x)+v_{1y}(x))\right.\\
&\quad\left.+\rho_d\nu(v_{0y}(x)+v_{1y}(x))\right]=\eta\left(\frac{\partial^2 v_{1y}(x)}{\partial x^2}+\mathrm{i}k_y\frac{\partial v_{1x}(x)}{\partial x}+\frac{\partial^2 v_{0y}(x)}{\partial x^2}\right).
\end{aligned}
\tag{2.11}
$$

As the last step, we impose the local approximation method for $k_x\lambda_{\mathrm{scale}}\gg 1$, where the inhomogeneity of the system is defined by $\lambda_{\mathrm{scale}}=v_0/v_0'$ (v_0' is the derivation of v_0 relative to x). We then explain the perturbed velocity in the X-direction as a harmonic function in the form of $v_1(x)\sim\exp(\mathrm{i}k_xx)$ in which k_x is the wave vector in the x-direction. Substituting this relation into (2.10) and (2.11), one obtains the simplified form of x- and y-component of the GH equation as follows

$$
\begin{aligned}
&v_{1x}(x)\left[-\mathrm{i}\omega\rho_d+\mathrm{i}k_y\rho_dv_{0y}(x)+\rho_d\nu-\rho_d\tau_m\omega^2+\omega k_y\rho_d\tau_mv_{0y}(x)-\mathrm{i}\rho_d\tau_m\nu\omega+\eta k_y^2\right]\\
&\quad+\eta k_yk_xv_{1y}(x)=0,
\end{aligned}
\tag{2.12}
$$

and

$$
\begin{aligned}
&v_{1y}(x)\left[-\mathrm{i}\omega\rho_d+\mathrm{i}k_y\rho_dv_{0y}(x)+\rho_d\nu-\rho_d\tau_m\omega^2+\omega k_y\rho_d\tau_mv_{0y}(x)-i\rho_d\tau_m\nu\omega+\eta k_x^2\right]\\
&\quad+v_{1x}(x)\left[\rho_d\frac{\partial v_{0y}(x)}{\partial x}-\mathrm{i}\omega\rho_d\tau_m\frac{\partial v_{0y}(x)}{\partial x}+\eta k_yk_x\right]=0.
\end{aligned}
\tag{2.13}
$$

Eliminating $v_{1y}(x)$ in these equations and assuming $\omega\gg k_yv_0$, it is straightforward to find the dispersion relation:

$$
\begin{aligned}
&\left[-\mathrm{i}\gamma+\bar{\nu}-\tau\gamma^2-\mathrm{i}\tau\bar{\nu}\gamma+\eta^*K_y^2\right]\left[-\mathrm{i}\gamma+\bar{\nu}-\tau\gamma^2-\mathrm{i}\tau\bar{\nu}\gamma+\eta^*K_x^2\right]-\eta^*K_yK_x\\
&\quad\times\left[\frac{\partial V_{0y}(X)}{\partial X}-\frac{\partial V_{0y}(X)}{\partial X}\mathrm{i}\tau\gamma+\eta^*K_yK_x\right]=0.
\end{aligned}
\tag{2.14}
$$

The normalized parameters employed in this equation are defined as

$$
\left.
\begin{aligned}
&\gamma=\frac{\omega}{\omega_{pd}},\quad K_{x(y)}=k_{x(y)}\lambda_{\mathrm{scale}},\quad \tau=\tau_m\omega_{pd},\quad \bar{\nu}=\frac{\nu}{\omega_{pd}},\\
&\eta^*=\frac{\eta}{\rho_d\omega_{pd}\lambda_{\mathrm{scale}}^2},\quad V_0=\frac{v_{0y}(x)}{\omega_{pd}\lambda_{\mathrm{scale}}},\quad X=\frac{x}{\lambda_{\mathrm{scale}}},
\end{aligned}
\right\}
\tag{2.15}
$$

in which $\omega_{pd}=\sqrt{4\pi(z_de)^2n_{0d}/m_d}$ is the dust plasma frequency.

Before starting the numerical analysis of the obtained dispersion relation in detail in the following section, the effect of the dust–neutral friction on the stability of a special experimental collisional dusty plasma system is presented. For this purpose, solving the obtained dispersion relation, the maximum growth rate of the instability for the typical experimental parameters $\eta=2.1\times10^{-8}$ kg (s m)$^{-1}$, $\tau_m=3$, $T=2.5\times10^{-4}$ K, and dust–neutral collision frequency $\nu=2.4$ s^{-1} (Feng, Goree & Liu 2012) has been illustrated versus the normalized wavenumbers K_x and K_y

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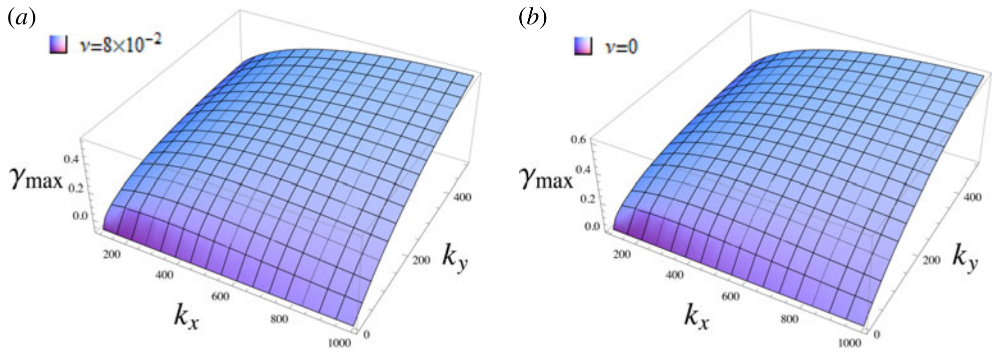

FIGURE 1. Three-dimensional graph of the normalized growth rate plotted in terms of the normalized K_x and K_y for typical experimental parameters $\eta^* = 2.1 \times 10^{-8}$ and $\tau = 3$; (*a*) for experimental collisional shear dusty plasma system ($\bar{\nu} = 8 \times 10^{-2}$) and (*b*) for collisionless system ($\bar{\nu} = 0$).

in figure 1(*a*), and is compared with the corresponding collisionless case shown in figure 1(*b*). These panels (figure 1) show that the maximum growth rate of the instability for both of the considered shear collisional and collisionless viscoelastic dusty plasma systems increases with their normalized wavenumbers. Comparing the collisional and collisionless cases demonstrates that in this experimental system the dust–neutral friction effect plays a significant stabilizing role on the evolution of the systems.

2.2. *Numerical analysis of the obtained dispersion relation*

In this section, we investigate the collision term effect on the instability of the Newtonian shear dusty plasma system in two distinct regimes, namely, the weakly coupled or hydrodynamic regime ($\omega\tau_m \ll 1$) and the strongly coupled or kinetic regime ($\omega\tau_m \gg 1$). In the following numerical analysis, the equilibrium velocity profile (2.6) of the dust grains has been considered in a linear form $\boldsymbol{v}_{0y}(x) = v_0 + v_{0\alpha}x$ (where $v_{0\alpha} = \alpha v_0$), using the Taylor approximation for $\alpha \ll 1$.

(A) In the hydrodynamic regime, $\omega\tau_m \ll 1$, the dimensionless dispersion relation (2.14) reduces to

$$\left[-\mathrm{i}\gamma + \bar{\nu} + \eta^* K_y^2\right]\left[-\mathrm{i}\gamma + \bar{\nu} + \eta^* K_x^2\right] - \eta^* K_y K_x \left[V_0\alpha + \eta^* K_y K_x\right] = 0. \tag{2.16}$$

Under this limitation, the medium behaves as a liquid-like system and the effect of dust–neutral collisions on the instability of the system can be found by numerical analysis of the dispersion relation (2.16). Figure 2(*a*) illustrates the normalized growth rate versus K_x at the fixed value of $K_y = 10$ for given parameters satisfying the considered assumptions, where the dispersion relation is derived with $\alpha = 1$, $v_0 = 0.2$, $\eta^* = 1.4 \times 10^{-8}$, $\tau = 2.08 \times 10^{-8}$, and three values of the normalized collisional frequency $\bar{\nu} = 0$ (figure 2*a* – red), 3.4×10^{-4} (figure 2*a* – green) and 8.4×10^{-4} (figure 2*a* – blue). As seen in these curves, the normalized instability growth rate decreases considerably on increasing the collision frequency for any arbitrary K_x value and it is obvious that the optimum values of the instability growth rate occur in the collisionless cases. The normalized growth rate as a function of K_y is plotted in figure 2(*b*) for a fixed value of $K_x = 100$ where the other parameters are the same as for figure 2(*a*). As indicated in this figure, the behaviour of the

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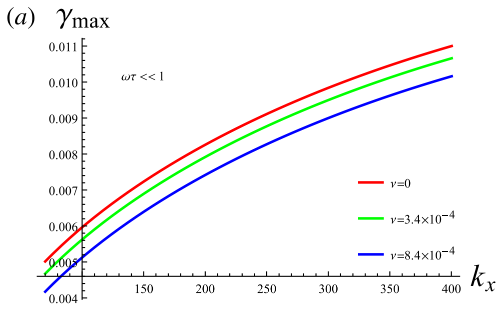

FIGURE 2. Variations of the growth rate for different collision frequencies in the case of $v_{0\alpha} = 2.6$, $\eta^* = 1.4 \times 10^{-8}$, $\tau = 2.08 \times 10^{-8}$, $\bar{\nu} = 0$, 3.4×10^{-4} and 8.4×10^{-4}; (*a*) for a fixed value of $K_y = 10$ versus K_x and (*b*) for a fixed value of $K_x = 10^2$ versus K_y.

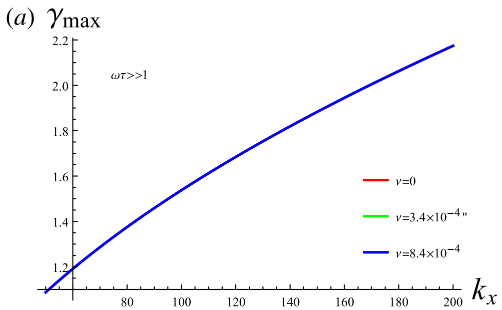

FIGURE 3. Graph of maximum growth rate in the kinetic regime ($\omega\tau \gg 1$) for various collision frequencies in terms of the normalized K_x and K_y for the case of in which $v_{0\alpha} = 2.6$, $\eta^* = 1.4 \times 10^{-8}$, $\tau = 5 \times 10^{-2}$, $\bar{\nu} = 0$, 3.4×10^{-4} and 8.4×10^{-4}; (*a*) versus K_x for $K_y = 10$ and (*b*) versus K_y for $K_x = 100$.

normalized growth rate is similar to figure 2(*a*) for the same given parameters. As an important result, the collision term has a stabilizing effect on the instability of the Newtonian viscoelastic shear dusty plasma in the hydrodynamic regime.

(B) In the kinetic limit, $\omega\tau_m \gg 1$, the dispersion relation (DR) can be obtained from the general DR (2.14) as

$$[-\tau\gamma^2 - \mathrm{i}\tau\bar{\nu}\gamma + \eta^* K_y^2][-\tau\gamma^2 - \mathrm{i}\tau\bar{\nu}\gamma + \eta^* K_x^2] - \eta^* K_y K_x[-\mathrm{i}\tau\gamma V_0\alpha + \eta^* K_y K_x] = 0. \tag{2.17}$$

In this limit, to show the collision effect on the growth of the instability, the normalized growth rate versus K_x and K_y are depicted in figure 3(*a*,*b*) for $\tau \cong 5 \times 10^{-2}$ and the other parameters as the same as were used in the hydrodynamic limit analysis, figure 2. As seen in these figures, all curves for different values of the collision frequency coincide with each others. Thus, although in this limit the instability is stronger than in the hydrodynamic limit, the collision term does not have a relatively large role ($(\Delta\gamma_{\max}/\gamma_{\max}) \cong 0$, where $\Delta\gamma_{\max}$ illustrates the role of the collision term in the instability growth rate) in the instability of the system.

It should be emphasized that the existence of instability in each class of shear viscoelastic collisional dusty plasma is a consequence of the inhomogeneity property

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of the viscous force in the system, where the shear velocities supply the required free energy of the instability which can be affected by collisions in the system.

3. Non-Newtonian viscoelastic collisional dusty plasma system

3.1. *Equilibrium state and dispersion relation*

The strong dependency of the viscosity parameter on the physical structure of the system is an interesting property observed in complex dusty plasmas because of the large amount of charge on each dust particle (Steinberg *et al.* 2008). For the under-investigated viscoelastic collisional dusty plasma system with shear-velocity structure, the viscosity coefficient can depend on the velocity shear rate, in which case the fluid is known as a non-Newtonian fluid. In this case, the viscosity coefficient can be decreased (increased) by increasing the velocity shear rate just beyond some critical point called the shear thinning (or thickening) region. Recently, Ivlev and coworkers have experimentally demonstrated the shear thinning property of dusty plasmas and proposed a power-law model for the dependence of the viscosity coefficient function on the velocity shear rate (Ivlev *et al.* 2007). Motivated with these results, in the following analysis of the described non-Newtonian system, the functional form of $\eta(s_0)$ in the shear thinning region is considered as (Pesceli *et al.* 1984; Ivlev *et al.* 2007)

$$\eta\,(s_0) = \overline{\eta_0}\left(\frac{s_0}{s_c}\right)^{-2\delta/(1+\delta)}, \tag{3.1}$$

where δ is a positive exponent, s_c is of the order of unity and $\overline{\eta_0}$ is a constant with the dimensions of the viscosity coefficient and s_0 is the equilibrium value of the shear parameter defined as $s_0 = \mathrm{d}v_{0y}/\mathrm{d}x$.

Now, recalling the GH equation in the kinetic limit ($\omega\tau_m \gg 1$) for the mentioned equilibrium velocity ($\boldsymbol{v}_{0y}(x) = v_{0y}(x)\boldsymbol{e}_y$) in § 2, the equilibrium state of the viscous non-Newtonian collisional dusty plasma system is described by (Pesceli *et al.* 1984)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\eta(s_0)\frac{\mathrm{d}v_{0y}}{\mathrm{d}x}\right] = 0. \tag{3.2}$$

Substituting $\eta(s_0)$ into equilibrium equation (3.2), $\mathrm{d}v_{0y}/\mathrm{d}x$ is found to be a constant, hence, as with the Newtonian case investigated in the previous section (§ 2), the equilibrium velocity profile can be written as $v_{0y}(x) = v_0 + v_{0\alpha}x$ where $v_{0\alpha}$ is a constant velocity shear rate having the dimensions of frequency (Pesceli *et al.* 1984).

Similar to Banerjee *et al.*'s work (Banerjee *et al.* 2010), for the incompressible uniform shear dusty plasma system, the differential equation for v_{1x} can be obtained from the general hydrodynamic equation by defining $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ as

$$\begin{aligned}&\left(\frac{\partial}{\partial t}\right)\left[\frac{\partial}{\partial t} + v_{0y}(x)\frac{\partial}{\partial x}\right]\nabla_\perp^2 v_{1x}(x, y, t) + \frac{\nu}{\tau}\nabla_\perp^2 v_{1x}(x, y, t)\\ &\quad = \left(\frac{\eta_0}{\rho\tau}\right)\nabla_\perp^4 v_{1x}(x, y, t) + \left(\frac{\eta_0' v_0'}{\rho\tau}\right)\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right)^2 v_{1x}(x, y, t),\end{aligned} \tag{3.3}$$

where use is made of linear perturbation theory for a small perturbation around the equilibrium flow.

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At the next step, using Fourier mode analysis by considering $v_{1x}(x, y, t) = v_1(x)\exp(\mathrm{i}k_y y - \mathrm{i}\omega t)$ where k_y is the wave vector in the y-direction and ω is the frequency mode, the differential equation (3.3) is reduced to

$$\frac{\omega_s^2}{k_y^2}\left(1+\frac{\eta_0' v_0'}{\rho\tau}\right)\frac{\mathrm{d}^4 v_{1(x)}}{\mathrm{d}x^4} + \left[\omega^2 - \omega k_y v_{0y}(x) - \frac{\nu}{\tau} - 2\omega_s^2\left(1-\frac{\eta_0' v_0'}{\rho\tau}\right)\right]\frac{\mathrm{d}^2 v_{1(x)}}{\mathrm{d}x^2}$$
$$+\left[k_y^2\omega_s^2\left(1+\frac{\eta_0' v_0'}{\rho\tau}\right) - k_y^2\left(\omega^2 - \omega k_y v_{0y}(x) - \frac{\nu}{\tau}\right)\right] v_1(x) = 0, \tag{3.4}$$

in which $\omega_s^2 = k_y^2\eta_0/\rho\tau$. This differential equation can be analysed by employing the local approximation method, $k_x\lambda_{scale} \gg 1$, where the perturbation wavelength is assumed to be much smaller than the inhomogeneity scale length along the x-direction $\lambda_{scale} = v_0/v_0'$. In this approximation, by considering $v_1(x) \sim \exp(\mathrm{i}k_x x)$, where k_x is the wave vector in the x-direction, and substituting it into (3.4) the dispersion relation in the limit of $\omega \gg k_y v_0$ will be obtained, after some algebra, as

$$\omega^2 = \frac{\omega_s^2\left(k_x^2+k_y^2\right)}{k_y^2}\left(1+\frac{\eta_0' v_0'}{\eta_0}\left(\frac{k_x^2-k_y^2}{k_x^2+k_y^2}\right)^2\right) + \frac{\nu}{\tau}, \tag{3.5}$$

in which η_0' is defined by $\mathrm{d}\eta_0/\mathrm{d}v_0'$. Regarding the negative value of $\eta_0' v_0'/\eta_0$ (see (3.2)), it is clearly evident from (3.5) that the system will be unstable when $(\eta_0' v_0'/\eta_0)((k_x^2-k_y^2)/(k_x^2+k_y^2))^2 > 1 + (k_y^2/\omega_s^2(k_x^2+k_y^2))(\nu/\tau)$. As compared with the dispersion relation obtained in Angelis (1992), the role dust–neutral friction term appears in the last term of relation (3.5).

3.2. *Numerical analysis of the obtained dispersion relation*

In this section, the effect of the neutral–dust friction term on the instability growth rate of a non-Newtonian viscoelastic shear dusty plasma in the kinetic limit is investigated numerically. Before this however, for a fixed value of the collision frequency, the maximum normalized growth rate as a function of K_x and β is depicted in the three-dimensional graph, figure 4, for the chosen parameter values $K_y = 1$, $\eta^* = 0.1 \times 10^{-9}$, $\tau = 2.08 \times 10^{-8}$ and $\bar{\nu} = 3.4 \times 10^{-8}$, where β is defined from (3.2) as $\beta = \eta_0' v_0'/\eta_0 = -(2\delta/(1+\delta))$. As seen in this figure, on increasing the absolute value of β, the normalized maximum growth rate will also be increased for any fixed value of K_x. Moreover, the instability growth rate s enhanced with K_x for any fixed value of β.

As the last step, to study the dust–neutral friction effect on the instability, the instability growth rate versus K_x is illustrated in figure 5 for the frictionless case studied in Angelis (1992) and three frictional cases with different collision frequency values. In this figure, $\beta = -1.5$ and the other parameter values are the same as those in figure 4. As seen in this figure, the collision effect has a more significant role in the stabilization of the system, even compared to the hydrodynamic limit of the Newtonian case. In this sense, not only does the maximum normalized growth rate decrease substantially by increasing the collision frequency, but also the cutoff wavenumber shifts the instability region to high values. Thus, considering the neutral–dust friction in this paper compared to the previous study (Banerjee *et al.* 2010), the current work shows the significant role of this process on the theoretical stability analysis of shear dusty plasmas, especially experiments with laboratory shear dusty plasmas.

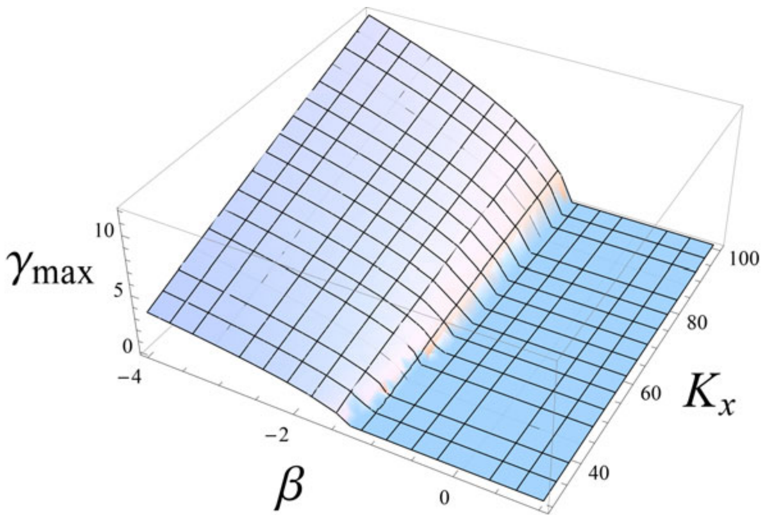

FIGURE 4. Three-dimensional graph of the normalized growth rate for a non-Newtonian viscoelastic collisional dusty plasma plotted in terms of β for the case of for $K_y = 1$, $\eta^* = 0.1 \times 10^{-9}$, $\tau = 2.08 \times 10^{-8}$ and $\bar{\nu} = 3.4 \times 10^{-8}$.

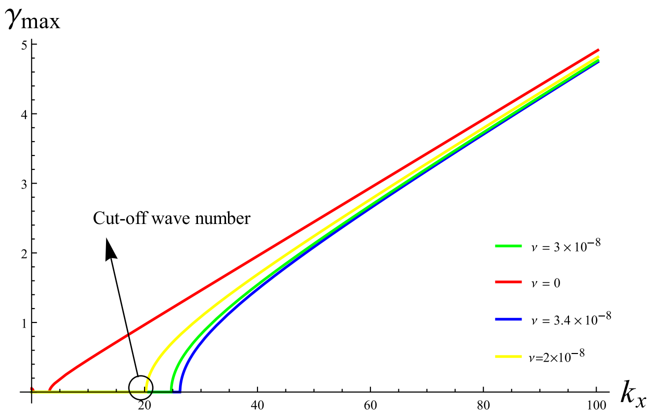

FIGURE 5. Graph of maximum growth rate in the kinetic regime ($\omega\tau \gg 1$) for collisionless (red line) and collisional (blue line $\bar{\nu} = 3.4 \times 10^{-8}$, green line $\bar{\nu} = 3 \times 10^{-8}$, yellow line $\bar{\nu} = 2 \times 10^{-8}$ and red line $\bar{\nu} = 0$) cases for $K_y = 1$ and $\beta = -1.5$ versus K_x.

4. Conclusion

The physics of the shear dusty plasma is of great interest due to applications in the vast range of plasma environments ranging from astrophysical objects to laboratory experiments. Over recent years, it has been widely recognized that, in these systems, the highly charged dust component is in a strongly coupled state. Moreover, recently, an interesting property of the velocity shear rate dependence of the viscosity parameter in this system has been observed which can lead to remarkable new features in these non-Newtonian systems, for example in deriving the novel instability due to coupling between velocity fluctuations and equilibrium fluids, called viscosity-gradient-driven instability.

In this paper, we have studied the instability of Newtonian and non-Newtonian shear viscoelastic dusty plasma systems in the presence of a considerable neutral–dust collision term. For this purpose, considering the electrons and ions to have Boltzmann distributions, we have used the generalized hydrodynamic formalism to find the equilibrium states and the dispersion relation of the Fourier modes in the systems, employing linear perturbation theory as well as the local approximation method. Investigations into the effects of the collision term on the instability growth rate of the modes in the Newtonian cases show that, although this term can be neglected in the kinetic regime, it has a considerable role in decreasing the instability in the hydrodynamic regime. Moreover, results demonstrated the more significant stabilization role of the neutral–dust collisions in the non-Newtonian system in the kinetic regime compared to the corresponding Newtonian cases, through decreasing the maximum growth rate at any fixed wavenumber and the construction of an unstable wavenumber region. The present instability analysis shows good agreement with previous works where collisional stabilization of the streaming instabilities has been proposed as a trigger for condensation of the dust into a solid state in the experiments.

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