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Idempotency in Optimality Theory[1]

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An *idempotent* phonological grammar maps phonotactically licit forms faithfully to themselves. This paper establishes tight sufficient conditions for idempotency in (classical) Optimality Theory. Building on Tesar (2013), these conditions are derived in two steps. First, idempotency is shown to follow from a general formal condition on the faithfulness constraints. Second, this condition is shown to hold for a variety of faithfulness constraints which naturally arise within McCarthy & Prince's (1995) Correspondence Theory of faithfulness. This formal analysis provides an exhaustive toolkit for modeling *chain shifts*, which have proven recalcitrant to a constraint-based treatment.

This paper contributes to a research program in constraint-based phonology which aims at distilling analytically the implications of constraint theories for formal typological properties (Prince 2007). For instance, Moreton (2004b) develops constraint conditions for the property of *eventual idempotency* and Tesar (2013) develops constraint conditions for the property of *output-drivenness*. This paper focuses on a third formal property which is intermediate between those two, namely *idempotency*. Building on Tesar's analysis of output-drivenness, the paper develops tight sufficient conditions for idempotency within classical *Optimality Theory* (OT; Prince & Smolensky 2004, Moreton 2004b). Magri (to appear) extends the theory of idempotency developed in this paper to the case of *Harmonic Grammar* (Legendre, Miyata & Smolensky 1990a, b; Smolensky & Legendre 2006) and discusses the relationship between idempotency and Tesar's output-drivenness.

A formal theory of idempotency is relevant both for phonological theory and for modeling the acquisition of phonology. In fact, idempotency is related to opacity: a grammar fails at idempotency provided it displays a *chain shift*, which corresponds to counter-feeding ordering in a rule-based phonological framework. An understanding of the conditions that ensure idempotency thus yields a toolkit for modeling chain shifts, which have proven recalcitrant to constraint-based analyses. Furthermore, various models of the early acquisition of phonotactics

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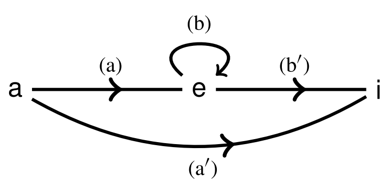
(Gnanadesikan 2004, Hayes 2004, Prince & Tesar 2004) assume that the learner posits a fully faithful underlying form for each training phonotactically licit surface form. These models thus effectively assume that the typology explored by the learner consists of idempotent grammars. Magri (2015) explores the implications of the analysis of idempotency developed in this paper for this class of models of the acquisition of phonotactics.

1. Overview

Within classical generative phonology, a phonological grammar maps underlying to surface forms (Kenstowicz & Kisseberth 1977, Heinz 2011). Suppose that there are no representational differences between underlying and surface forms: any given representation can be construed as both an underlying and a surface form. In this case, a phonological grammar is called *idempotent* provided that any form that is phonotactically licit (as a surface form) is faithfully mapped (as an underlying form) to itself (as a surface form). Section 2 formalizes this notion of idempotency within a representational framework where underlying and surface forms are strings of segments related by correspondence relations (McCarthy & Prince 1995).

Within OT, formal grammatical properties follow from properties of the candidate and constraint sets. Which conditions on these two sets suffice to guarantee that the grammars in the corresponding typology are all idempotent? The answer to this question developed in Section 3 can be informally previewed as follows. (It was anticipated in Moreton & Smolensky 2002: Section 3; Prince 2007 and Buccola 2013. Furthermore, it is analogous to the analysis in Tesar 2013: Section 3.2.) Suppose that an OT grammar maps the underlying form /a/ to the surface form [e], as represented by the arrow (1a). This means that [e] is phonotactically licit. Idempotency then requires the underlying form /e/ to be faithfully mapped to [e], as represented by the loop (1b).

(1)



We reason by contradiction. Thus, we make the *contradictory assumption* that idempotency fails and that /e/ is instead mapped to something else; say it is raised further to [i] for concreteness, as represented by the arrow (1b′). In order to establish idempotency, we want to derive the *contradictory conclusion* that /a/ is also mapped to [i], as represented by the long arrow (1a′), against the hypothesis that /a/ be mapped to [e].

Assume that every constraint C in the constraint set satisfies the implication (2). The contradictory assumption that /e/ is raised to [i] rather than faithfully

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mapped to [e] intuitively means that high ranked constraints fail at penalizing the contradictory mapping (/e/, [i]) in (1b′) with respect to the idempotent mapping (/e/, [e]) in (1b), thus satisfying the antecedent of (2).

(2) **If:** constraint C does not prefer the idempotent mapping (/e/, [e]) to the contradictory mapping (/e/, [i])

namely: $C(\text{/e/, [i]}) \leqslant C(\text{/e/, [e]})$

Then: constraint C does not prefer the actual mapping (/a/, [e]) to the contradictory mapping (/a/, [i])

namely: $C(\text{/a/, [i]}) \leqslant C(\text{/a/, [e]})$

The implication (2) thus ensures that high ranked constraints also fail at penalizing the contradictory mapping (/a/, [i]) in (1a′) with respect to the actual mapping (/a/, [e]) in (1a). In conclusion, we intuitively expect this implication (2) to provide a sufficient condition for the contradictory assumption to entail the contradictory conclusion, thus guaranteeing the idempotent mapping of /e/ to [e].

The mappings (/e/, [e]) and (/e/, [i]) compared in the antecedent of (2) feature the underlying form /e/. The mappings (/a/, [e]) and (/a/, [i]) compared in the consequent only differ because they feature the underlying form /a/. The implication (2) thus trivially holds for the markedness constraints, because they are insensitive to the underlying forms, so that antecedent and consequent coincide. The implication (2) is thus a condition on the faithfulness constraints. For a faithfulness constraint, the number of violations assigned to the identity mapping (/e/, [e]) is zero. The implication (2) therefore becomes (3), where I have replaced ‘C’ with ‘F’, to highlight the fact that the implication only needs to be checked for the faithfulness constraints.

(3) **If:** $F(\text{/e/, [i]}) = 0$

Then: $F(\text{/a/, [i]}) \leqslant F(\text{/a/, [e]})$

This sufficient condition for idempotency (3) is referred to as the *faithfulness idempotency condition* (FIC).

The problem of establishing OT idempotency is thus reduced to the problem of determining which faithfulness constraints satisfy the FIC. The latter problem is taken on by Sections 4–6, for a variety of faithfulness constraints that naturally arise within McCarthy & Prince’s (1995) Correspondence Theory. To start, Section 4 looks at the three basic faithfulness constraints, MAX, DEP, and IDENT. MAX is shown to satisfy the FIC under no additional assumptions, while DEP and IDENT require no correspondence relation in the candidate set to break any underlying segment. This edge of MAX over DEP and IDENT can be intuitively explained as follows. The left- and right-hand sides of the inequality in the consequent of the FIC (3) look at two mappings that have the same underlying form (in this specific case, the underlying vowel /a/) but two different surface forms (in this case, the two surface vowels [i] and [e]). While MAX only ‘counts’ over underlying segments, DEP and IDENT are also sensitive to the surface

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segments and thus need some additional assumption (in the form of the no-breaking condition) to guarantee the 'commensurability' of the two candidates being compared.

Section 5 extends the analysis of the FIC to restricted variants of these basic constraints, such as faithfulness constraints that penalize consonant deletion but not vowel deletion, that penalize de-nasalization but not nasalization, or that penalize obstruent devoicing only before a sonorant. Section 6 concludes the analysis of the FIC by looking at a variety of other constraints, such as INTEGRITY, UNIFORMITY, featural $\text{DEP}_{[\pm\varphi]}$ and $\text{MAX}_{[\pm\varphi]}$, CONTIGUITY, ALIGNMENT, and LINEARITY, as well as constraint conjunction and disjunction.

Section 7 articulates the phonological implications of the theory of idempotency. A grammar fails at idempotency provided that it enforces a *chain shift* (Łubowicz 2011) such as the mapping $\mathsf{a} \rightarrow \mathsf{e} \rightarrow \mathsf{i}$ in (1). From this perspective, the result obtained in Section 3 can be reinterpreted as follows: any analysis of chain shifts within (classical) OT requires at least one faithfulness constraint that fails at the FIC (in fact, if all faithfulness constraints in the constraint set did satisfy the FIC, every grammar in the corresponding typology would be idempotent, and would thus display no chain shifts). This result is intuitive: in order to get the chain shift $\mathsf{a} \rightarrow \mathsf{e} \rightarrow \mathsf{i}$, the top ranked relevant faithfulness constraint cannot penalize the mapping $\mathsf{e} \rightarrow \mathsf{i}$ relative to $\mathsf{e} \rightarrow \mathsf{e}$, thus satisfying the antecedent of the FIC (3); and cannot penalize the mapping $\mathsf{a} \rightarrow \mathsf{e}$ relative to $\mathsf{a} \rightarrow \mathsf{i}$, thus failing the consequent of the FIC (3). Section 7 systematizes various approaches to chain shifts in the OT literature by showing that they differ in how they choose the culprit faithfulness constraint from the list of non-FIC abiding constraints compiled in Sections 4–6. The final Section 8 summarizes the results obtained in this paper and sketches the results presented in the two companion papers Magri (to appear) and Magri (2015). The presentation is kept informal throughout the paper, with proofs relegated to an appendix.

2. IDEMPOTENCY

This section introduces the notion of idempotent phonological grammar within a representational framework which is a segmental version of McCarthy & Prince's (1995) Correspondence Theory.

2.1 *Representational framework*

Consider a finite set of *segments* (for instance, the segments in the IPA table, or some subset thereof), denoted by $\mathsf{a}, \mathsf{b}, \mathsf{c}, \ldots$. Strings obtained through segment concatenation are denoted by $\mathbf{a}, \mathbf{b}, \mathbf{c}, \ldots$. The notation $\mathbf{a} = \mathsf{a}_1 \cdots \mathsf{a}_\ell$ says that the string $\mathbf{a}$ is the concatenation of the segments $\mathsf{a}_1, \ldots, \mathsf{a}_\ell$ and thus has *length* ℓ. This paper assumes the representational framework (4). Underlying and surface forms are strings of segments. Phonological candidates establish a correspondence between the segments of these underlying and surface strings.

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(4) The *candidate set* consists of triplets $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ of an *underlying* segment string $\mathbf{a}$ and a *surface* segment string $\mathbf{b}$ together with a *correspondence relation* $\rho_{\mathbf{a},\mathbf{b}}$ between the segments of $\mathbf{a}$ and $\mathbf{b}$.[2]

Correspondence relations will be denoted by thin lines. To illustrate, (5) represents the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose underlying string $\mathbf{a}$ is /bnɪk/, whose surface string $\mathbf{b}$ is [blɪk], whose correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ maps underlying to surface segments respecting their position in the strings.

(5)
$$\begin{array}{l}\mathbf{a} = \mathrm{b\ n\ \imath\ k} \\ \qquad\ \ |\ \ |\ \ |\ \ | \\ \mathbf{b} = \mathrm{b\ l\ \imath\ k}\end{array}$$

The representational assumption (4) is consistent with additional restrictions on the candidate set (Blaho, Bye & Krämer 2007). This flexibility will be exploited in this paper, which will explore the implications of various restrictions on the correspondence relations that can figure in the candidate set.

2.2 *Identity candidates*

As anticipated in Section 1, idempotency is about phonotactically licit forms being mapped to themselves. It thus requires the distinction between underlying and surface forms to be blurred. This is achieved through axiom (6). It can be interpreted as a candidacy *reflexivity axiom*, as it requires each (surface) string to be in correspondence with itself. This axiom will play a crucial role in the definition of idempotency in the next subsection.

(6) If the candidate set contains a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ with a surface form $\mathbf{b}$, it also contains the *identity* candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, where $\mathbb{I}_{\mathbf{b},\mathbf{b}}$ is the identity correspondence relation among the segments of $\mathbf{b}$.

By (6), any surface form can be construed as an underlying form (of the corresponding identity candidate). In other words, the set of surface forms is a subset of the set of underlying forms. This is a slightly weaker condition than Moreton's (2004b) *homogeneity* condition, which requires the sets of underlying and surface forms to coincide. Both reflexivity and homogeneity hold when underlying and surface representations are constructed out of the same 'building blocks'. Moreton claims that 'most phonological representations are in fact present in both [underlying and surface forms]', so that reflexivity and homogeneity hold for 'much of

[2] Correspondence relations might want to distinguish between multiple occurrences of the same segment in a string. Thus, correspondence relations cannot be defined simply as relations between the two *sets* of underlying and surface segments. To keep the presentation straightforward, this paper will follow common practice and ignore these subtleties.

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the core business of phonology'. Yet, the reflexivity axiom (6) obviously does not hold in full generality. To illustrate, suppose that the candidate set contains the candidate $(\mathbf{a}, \mathbf{b}) =$ (/mabap/, [ma.ba]). The reflexivity axiom (6) requires the candidate set to also contain the candidate $(\mathbf{b}, \mathbf{b}) =$ (/ma.ba/, [ma.ba]). This contravenes the plausible assumption that syllabification is a property of the surface representations and is absent in the underlying representations. In the case of constraint-based phonology, this difficulty can be circumvented by switching from the identity candidates required by (6) to McCarthy's (2002) *fully faithful candidates*, as explained in Section 3.6.

2.3 *Idempotency*

Within the representational framework just defined, a phonological grammar is a map G that takes an underlying form $\mathbf{a}$ and returns a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose underlying string is indeed $\mathbf{a}$.[3] A string $\mathbf{b}$ is called *phonotactically licit* according to a grammar G provided that there exists at least one string $\mathbf{a}$ (with $\mathbf{a}$ possibly identical to $\mathbf{b}$) such that the grammar G maps the underlying form $\mathbf{a}$ to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose surface string is $\mathbf{b}$. A grammar G is *idempotent* provided that it maps any phonotactically licit surface form to itself, as formalized by the implication (7) in the following definition. The antecedent of the implication says that the surface form $\mathbf{b}$ is phonotactically licit relative to the grammar G, because it is the surface realization of some underlying form $\mathbf{a}$. The consequent says that $\mathbf{b}$ is then mapped faithfully to itself. The reflexivity axiom (6) ensures the existence of the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ in the consequent of (7).

Definition. A grammar G is idempotent provided that it satisfies the implication

(7) **If:** $G(\mathbf{a}) = \big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big)$

Then: $G(\mathbf{b}) = \big(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}}\big)$

for any candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in the candidate set. □

To illustrate, suppose that a grammar raises the low vowel /a/ to [e]. The mid vowel [e] is therefore phonotactically licit. In order for that grammar to comply with condition (7) and thus qualify as idempotent, the underlying form /e/ must be mapped faithfully to [e].[4]

[3] For the sake of simplicity, this paper assumes that a grammar maps an underlying form to a *single* candidate. This assumption is not crucial, and the results obtained extend to a framework where grammars map an underlying form to a *set* of candidates, thus modeling phonological variation.

[4] Usually, idempotency is a notion that applies to a function f between a set X and itself and requires the identity $f(f(x)) = f(x)$ for every argument $x \in X$. The connection between this notion and the definition above is straightforward. Given a grammar G, let g be the corresponding string function, namely the function from strings to strings defined by the condition $g(\mathbf{a}) = \mathbf{b}$ provided that $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ for some correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$. The grammar G is idempotent according to the definition above if and only if the corresponding string function g satisfies the condition $g(g(\mathbf{a})) = g(\mathbf{a})$ for any underlying string $\mathbf{a}$.

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2.4 *Chain shifts*

The candidate set might provide many different relations $\rho_{\mathbf{b},\mathbf{b}}$ to put a string $\mathbf{b}$ in correspondence with itself. Some options are illustrated in (8) in the case where $\mathbf{b} = \mathsf{amba}$. Axiom (6) requires one of these correspondence relations provided by the candidate set to be the identity relation $\mathbb{I}_{\mathbf{b},\mathbf{b}}$, illustrated by the left-most candidate in (8). This identity relation $\mathbb{I}_{\mathbf{b},\mathbf{b}}$ is intuitively the *best* way to put the string $\mathbf{b}$ in correspondence with itself. A grammar G is *well-behaved* provided that it abides by this intuition: whenever G maps an underlying string $\mathbf{b}$ to the same surface string $\mathbf{b}$, it does so through the identity correspondence relation $\mathbb{I}_{\mathbf{b},\mathbf{b}}$. In other words, $G(\mathbf{b}) = (\mathbf{b}, \mathbf{b}, \rho_{\mathbf{b},\mathbf{b}})$ is impossible when $\rho_{\mathbf{b},\mathbf{b}} \neq \mathbb{I}_{\mathbf{b},\mathbf{b}}$.

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(8)  b = a m b a          b = a m b a          b = a m b a
         | | | |              |  X  |              \| | |
     b = a m b a          b = a m b a          b = a m b a
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Suppose now that a grammar G fails at the idempotency implication (7) for some candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, as stated in (9): G maps the underlying form $\mathbf{a}$ to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, as required by the antecedent of the idempotency implication; but G fails to map the underlying form $\mathbf{b}$ to the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as required by the consequent.

(9) A grammar G fails at idempotency on a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ iff:

  (a) $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$;

  (b) $G(\mathbf{b}) \neq (\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$.

Condition (9b) means that G maps the underlying form $\mathbf{b}$ to some candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ different from $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$. This means that either the two strings $\mathbf{b}$ and $\mathbf{c}$ differ, or $\mathbf{b}$ and $\mathbf{c}$ coincide but the two correspondence relations $\rho_{\mathbf{b},\mathbf{c}}$ and $\mathbb{I}_{\mathbf{b},\mathbf{b}}$ differ. The latter option is impossible when G is well-behaved. The strings $\mathbf{b}$ and $\mathbf{c}$ must thus differ and condition (9) becomes (10).

(10) A (well-behaved) grammar G fails at idempotency on a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ iff there is a candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ with $\mathbf{b} \neq \mathbf{c}$ such that:

  (a) $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$;

  (b) $G(\mathbf{b}) = (\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$.

Condition (10) says that G maps $\mathbf{a}$ to $\mathbf{b}$ and then in turn maps $\mathbf{b}$ to $\mathbf{c}$. Since $\mathbf{b} \neq \mathbf{c}$, this scheme $\mathbf{a} \rightarrow \mathbf{b} \rightarrow \mathbf{c}$ is called a *chain shift* in the phonological literature (see Łubowicz 2011 for a comprehensive review). In conclusion, a (well-behaved) grammar G fails at idempotency if and only if it enforces chain shifts.

3. Idempotency in Optimality Theory

The notion of idempotency introduced in the preceding section is independent of any specific phonological framework used to define the grammar G. Starting with

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this section, I focus on the specific framework of *Optimality Theory* (OT; Prince & Smolensky 2004) (Magri to appear extends the theory of idempotency to the related framework of *Harmonic Grammar*). Which conditions guarantee that all of the grammars in an OT typology are idempotent? The answer developed in this section has two parts: a condition on the candidate set, in the form of a candidacy *transitivity axiom*, and a condition on the violation profiles of the faithfulness constraints, illustrated in Section 1 with (3).

3.1 *Optimality Theory (OT)*

A *constraint* C is a function that takes a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and returns a *number of violations* $C(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ that is large when the candidate scores poorly from the perspective relevant to that constraint. A constraint C *prefers* a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ to another candidate $(\mathbf{c}, \mathbf{d}, \rho_{\mathbf{c},\mathbf{d}})$ provided that it assigns fewer violations to the former than to the latter, namely $C(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}) < C(\mathbf{c}, \mathbf{d}, \rho_{\mathbf{c},\mathbf{d}})$. A *constraint ranking* is an arbitrary linear order $\gg$ over a set of constraints. A constraint ranking $\gg$ *prefers* a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ to another candidate $(\mathbf{c}, \mathbf{d}, \rho_{\mathbf{c},\mathbf{d}})$ provided that the $\gg$-highest constraint among those which assign a different number of violations to the two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{c}, \mathbf{d}, \rho_{\mathbf{c},\mathbf{d}})$, prefers the former candidate to the latter. Given a ranking $\gg$, the corresponding OT grammar $G_{\gg}$ maps an underlying form $\mathbf{a}$ to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ that is preferred by the ranking $\gg$ to all other candidates $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$ that share that underlying form $\mathbf{a}$.[5]

3.2 *Classical OT*

A *faithfulness* constraint F has the property that it never assigns any violations to any identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as stated in (11a). A *markedness* constraint M has the property that it is blind to underlying forms, so that it assigns the same number of violations to any two candidates $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ sharing the surface form $\mathbf{c}$ (independently of their underlying forms), as stated in (11b).

(11) (a) $F(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}}) = 0$

(b) $M(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}}) = M(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$

Given the candidacy reflexivity axiom (6), no (non-trivial) constraint can be *both* a faithfulness and a markedness constraint. In fact, suppose by contradiction that were the case for some constraint C. Consider an arbitrary candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in the candidate set. The reflexivity axiom thus ensures that the

[5] As noted in footnote 84, I assume throughout the paper that grammars map an underlying form to a single candidate. This condition holds for OT grammars provided that the constraint set is sufficiently rich relative to the candidate set, in the following sense: for any two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$ that share the underlying form $\mathbf{a}$, the constraint set contains a constraint C which assigns them a different number of violations.

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candidate set also contains the corresponding identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$. These two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ share the surface form $\mathbf{b}$. Since C is a markedness constraint, C must assign the same number of violations to those two candidates, as stated in (12a). Since C is also a faithfulness constraint, it does not penalize the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as stated in (12b).

(12) $C(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}) \overset{(a)}{=} C(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}}) \overset{(b)}{=} 0$

In conclusion, C does not penalize any candidate, and it is therefore trivial.[6]

Although no constraint can be *both* a faithfulness and a markedness constraint, it can easily be *neither* (for instance, *comparative markedness constraints* are neither; see McCarthy 2002, 2003a as well as Section 7 below for additional references). To rule out the latter case, I assume throughout the paper that the constraint set only consists of faithfulness and markedness constraints (this is Moreton (2004b) *conservativity* assumption).

(13) Constraint set = faithfulness constraints ∪ markedness constraints

Let me call *classical* the version of OT endowed with the latter restriction (13) on the constraint set.

3.3 *A sufficient condition for chain shifts*

The classical assumption (13) that each constraint is either a faithfulness constraint (11a) or a markedness constraint (11b) ensures that the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ harmonically bounds any candidate $(\mathbf{b}, \mathbf{b}, \rho_{\mathbf{b},\mathbf{b}})$ with $\rho_{\mathbf{b},\mathbf{b}} \neq \mathbb{I}_{\mathbf{b},\mathbf{b}}$. To illustrate, the left-most candidate in (8) outperforms the other candidates listed. In fact, faithfulness constraints cannot prefer $(\mathbf{b}, \mathbf{b}, \rho_{\mathbf{b},\mathbf{b}})$ to $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, by (11a); and markedness constraints do not distinguish between two such candidates, by (11b). In other words, the OT grammar $G_{\gg}$ corresponding to any ranking $\gg$ is well-behaved in the sense of Section 2.4: $G_{\gg}(\mathbf{b}) \neq (\mathbf{b}, \mathbf{b}, \rho_{\mathbf{b},\mathbf{b}})$ whenever $\rho_{\mathbf{b},\mathbf{b}} \neq \mathbb{I}_{\mathbf{b},\mathbf{b}}$. The characterization of non-idempotency in terms of chain shifts in Section 2.4 thus applies to (classical) OT grammars. To distill the implications of that characterization, let me weaken the 'if-and-only-if' statement (10) into the 'if' statement (14). In fact, if the grammar $G_{\gg}$ maps the underlying form $\mathbf{a}$ to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, as stated in (10a), the ranking $\gg$ must in particular prefer the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ to any other loser candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}})$, as stated in (14a). Furthermore, if the grammar $G_{\gg}$ maps the underlying form $\mathbf{b}$ to the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$, as stated in (10a), the ranking $\gg$ must in particular prefer this candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ to the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as stated in (14b).

[6] This conclusion crucially rests on the candidacy reflexivity axiom (6), which intuitively ensures that the candidate set has enough identity candidates. Without this axiom, the assumption that C is a faithfulness constraint would indeed have no bite, as the faithfulness definitional condition (11a) is stated in terms of identity candidates.

(14) If $G_{\gg}$ fails at idempotency on a candidate (a, b, $\rho_{\mathsf{a,b}}$), there exists some candidate (b, c, $\rho_{\mathsf{b,c}}$) with $\mathsf{b} \neq \mathsf{c}$ such that:

- (a) $\gg$ prefers (a, b, $\rho_{\mathsf{a,b}}$) to (a, c, $\rho_{\mathsf{a,c}}$), for any correspondence $\rho_{\mathsf{a,c}}$ in the candidate set (if any);
- (b) $\gg$ prefers (b, c, $\rho_{\mathsf{b,c}}$) to (b, b, $\mathbb{I}_{\mathsf{b,b}}$).

Condition (14b) that the ranking $\gg$ prefers (b, c, $\rho_{\mathsf{b,c}}$) to the identity candidate (b, b, $\mathbb{I}_{\mathsf{b,b}}$) means that the constraint set contains a constraint that prefers (b, c, $\rho_{\mathsf{b,c}}$) to (b, b, $\mathbb{I}_{\mathsf{b,b}}$), such that all of the constraints that are ranked by $\gg$ above it assign the same number of violations to the two candidates. By (11a), this constraint that prefers (b, c, $\rho_{\mathsf{b,c}}$) to (b, b, $\mathbb{I}_{\mathsf{b,b}}$) cannot be a faithfulness constraint and must instead be a markedness constraint M. Condition (14b) can thus be explicitated as (15b) and (15c).

(15) If $G_{\gg}$ fails at idempotency on (a, b, $\rho_{\mathsf{a,b}}$), there exist a candidate (b, c, $\rho_{\mathsf{b,c}}$) with $\mathsf{b} \neq \mathsf{c}$ and a markedness constraint M such that:

- (a) $\gg$ prefers (a, b, $\rho_{\mathsf{a,b}}$) to (a, c, $\rho_{\mathsf{a,c}}$), for any correspondence $\rho_{\mathsf{a,c}}$ in the candidate set (if any);
- (b) M assigns fewer violations to (b, c, $\rho_{\mathsf{b,c}}$) than to (b, b, $\mathbb{I}_{\mathsf{b,b}}$);
- (c) any faithfulness or markedness constraint $\gg$-ranked above M assigns (b, c, $\rho_{\mathsf{b,c}}$) and (b, b, $\mathbb{I}_{\mathsf{b,b}}$) the same number of violations.

Since faithfulness constraints assign no violations to identity candidates by (11a), condition (15c) that any faithfulness constraint ranked above M assigns the same number of violations to (b, c, $\rho_{\mathsf{b,c}}$) and (b, b, $\mathbb{I}_{\mathsf{b,b}}$) means that it assigns no violations to (b, c, $\rho_{\mathsf{b,c}}$). Condition (15c) can thus be made explicit as in (16c) and (16d).

(16) If $G_{\gg}$ fails at idempotency on (a, b, $\rho_{\mathsf{a,b}}$), there exist a candidate (b, c, $\rho_{\mathsf{b,c}}$) with $\mathsf{b} \neq \mathsf{c}$ and a markedness constraint M such that:

- (a) $\gg$ prefers (a, b, $\rho_{\mathsf{a,b}}$) to (a, c, $\rho_{\mathsf{a,c}}$), for any correspondence $\rho_{\mathsf{a,c}}$ in the candidate set (if any);
- (b) M assigns fewer violations to (b, c, $\rho_{\mathsf{b,c}}$) than to (b, b, $\mathbb{I}_{\mathsf{b,b}}$);
- (c) any faithfulness constraint $\gg$-ranked above M assigns no violations to (b, c, $\rho_{\mathsf{b,c}}$);
- (d) any markedness constraint $\gg$-ranked above M assigns the same number of violations to (b, c, $\rho_{\mathsf{b,c}}$) and (b, b, $\mathbb{I}_{\mathsf{b,b}}$).

The designated markedness constraint M prefers (b, c, $\rho_{\mathsf{b,c}}$) to (b, b, $\mathbb{I}_{\mathsf{b,b}}$), by (16b). Furthermore, it is blind to the underlying forms, by (11b). Hence, M also prefers (a, c, $\rho_{\mathsf{a,c}}$) to (a, b, $\rho_{\mathsf{b,b}}$). Assumption (16a) thus requires M to be ranked below some constraint with the opposite preference. The latter constraint cannot be a markedness constraint, because of (16d). It must therefore be a faithfulness constraint. Condition (16a) can thus be made explicit as in (17a).

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(17) If $G_{\gg}$ fails at idempotency on $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, there exist a candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ with $\mathbf{b}{\neq}\mathbf{c}$ and a markedness constraint M such that:

(a) for any correspondence $\rho_{\mathbf{a},\mathbf{c}}$ in the candidate set (if any), there exists a faithfulness constraint $\gg$-ranked above M which assigns fewer violations to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ than to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$;

(b) M assigns fewer violations to $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ than to $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$;

(c) any faithfulness constraint $\gg$-ranked above M assigns no violations to $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$;

(d) any markedness constraint $\gg$-ranked above M assigns the same number of violations to $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ and $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$.

Condition (17) just derived is necessary for idempotency to fail.

3.4 *The faithfulness idempotency condition (FIC)*

I am now ready to tackle the central question of this section: which conditions ensure that the OT grammars corresponding to any ranking of a given constraint set are idempotent? The answer to this question is provided by the following Lemma 1. The assumption made by the lemma is twofold. First, it restricts the candidate set: if it contains two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ that share a string $\mathbf{b}$ as the surface and underlying form respectively, it must also contain a candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$ that puts the underlying string $\mathbf{a}$ of the former candidate in correspondence with the surface string $\mathbf{c}$ of the latter candidate, as in (18).

(18)
$$\begin{tikzcd} \mathbf{a} \arrow[r, "\rho_{\mathbf{a},\mathbf{b}}"] \arrow[rr, bend right, "\rho_{\mathbf{a},\mathbf{c}}"] & \mathbf{b} \arrow[r, "\rho_{\mathbf{b},\mathbf{c}}"] & \mathbf{c} \end{tikzcd}$$

Second, the assumption of the lemma restricts the constraint set: it requires all the faithfulness constraints to satisfy the implication (19), which is referred to as the *faithfulness idempotency condition* (FIC). The specific implication (3) in Section 1 is a concrete example of the FIC.

Lemma 1. *Assume that, for any two candidates $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ and $(\boldsymbol{b}, \boldsymbol{c}, \rho_{\boldsymbol{b},\boldsymbol{c}})$ that share $\boldsymbol{b}$ as the underlying and surface form respectively, the candidate set also contains a candidate $(\boldsymbol{a}, \boldsymbol{c}, \rho_{\boldsymbol{a},\boldsymbol{c}})$ such that the following implication holds for every faithfulness constraint F in the constraint set.*

(19) **If:** $F\big(\boldsymbol{b},\boldsymbol{c}, \rho_{\boldsymbol{b},\boldsymbol{c}}\big) = 0$
Then: $F\big(\boldsymbol{a},\boldsymbol{c}, \rho_{\boldsymbol{a},\boldsymbol{c}}\big) \leqslant F\big(\boldsymbol{a},\boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}}\big)$

Then, the OT grammar corresponding to any ranking of the constraint set is idempotent. □

Lemma 1 follows straightforwardly from the discussion in the preceding subsection: the FIC (19) makes the two conditions (17a) and (17c) incompatible and thus prevents idempotency from failing. In fact, (17a) requires the designated markedness constraint M to be outranked by a faithfulness constraint F which assigns fewer violations to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ than to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{c}})$. This means that the consequent of the FIC (19) fails. The antecedent must therefore fail as well. This means in turn that F assigns some violations to $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$, contradicting (17c).

3.5 *Composition candidates and the FIC$_{comp}$*

Lemma 1 makes no assumptions on the nature of the correspondence relation $\rho_{\mathbf{a},\mathbf{c}}$ depicted in (18) and in particular on its relationship with the two other correspondence relations $\rho_{\mathbf{a},\mathbf{b}}$ and $\rho_{\mathbf{b},\mathbf{c}}$. For instance, $\rho_{\mathbf{a},\mathbf{c}}$ could be the empty relation. This would make the FIC (19) trivial when F is an identity faithfulness constraint (because the quantity on the left-hand side of the inequality in the consequent would be equal to zero) but difficult when F is DEP or MAX (because the quantity on the left-hand side of the inequality would be large in this case). At the opposite extreme, $\rho_{\mathbf{a},\mathbf{c}}$ could be the total relation, which puts any underlying segment in correspondence with any surface segment. This would make the FIC (19) trivial when F is DEP or MAX but difficult when F is an identity faithfulness constraint.

A natural assumption is that $\rho_{\mathbf{a},\mathbf{c}}$ is the *composition* $\rho_{\mathbf{a},\mathbf{c}} = \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ of the two correspondence relations $\rho_{\mathbf{a},\mathbf{b}}$ and $\rho_{\mathbf{b},\mathbf{c}}$.[7] This means that a segment a of the string $\mathbf{a}$ and a segment c of the string $\mathbf{c}$ are in correspondence through $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ if and only if there exists some 'mediating' segment b of the string $\mathbf{b}$ such that a is in correspondence with b through $\rho_{\mathbf{a},\mathbf{b}}$ and furthermore b is in correspondence with c through $\rho_{\mathbf{b},\mathbf{c}}$ (many examples will be provided in Sections 4–6). The existence of this *composition* candidate is guaranteed by (20), which can thus be interpreted as a candidacy *transitivity axiom*, complementing the reflexivity axiom (6).

(20) If the candidate set contains two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ that share $\mathbf{b}$ as the surface and the underlying form, it also contains the *composition candidate* $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ whose correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ is the composition of $\rho_{\mathbf{a},\mathbf{b}}$ and $\rho_{\mathbf{b},\mathbf{c}}$.

The original FIC (19) can now be specialized in terms of this composition candidate as the implication (21), which will be referred to as the FIC$_{\text{comp}}$ to highlight the fact that the left-hand side of the inequality in the consequent features the composition candidate. The FIC$_{\text{comp}}$ entails the original FIC and thus provides a sufficient condition for the idempotency of all of the grammars in an OT typology.

[7] The operation of composition between two relations is usually denoted by '$\circ$'. In the rest of the paper, I write more succinctly $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ instead of $\rho_{\mathbf{a},\mathbf{b}} \circ \rho_{\mathbf{b},\mathbf{c}}$.

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(21) **If:** $F(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}) = 0$
Then: $F(\mathbf{a}, \mathbf{c}, \underbrace{\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}}_{\rho_{\mathbf{a},\mathbf{c}}}) \leqslant F(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$

The $\mathrm{FIC}_{\mathrm{comp}}$ is only a *sufficient* condition for idempotency, not a necessary-and-sufficient characterization of idempotency.[8] Yet, the $\mathrm{FIC}_{\mathrm{comp}}$ is a *tight* sufficient condition: for any faithfulness constraint that fails at the $\mathrm{FIC}_{\mathrm{comp}}$, it is possible to construct a counterexample where idempotency indeed fails, as will be shown in Section 7.

3.6 *Refinements*

The definition of idempotency in Section 2.3 crucially relies on the existence of the identity candidate, as guaranteed by the reflexivity axiom (6). Yet, as discussed in Section 2.2, this reflexivity axiom fails when surface representations are richer than underlying representations. For instance, the identity candidate $(\mathbf{b}, \mathbf{b})$ = (/ma.ba/, [ma.ba]) makes no sense if syllabification is construed as a surface property. Within a constraint-based framework such as OT, this difficulty can be circumvented as follows.[9] Following McCarthy (2002: Section 6.2), $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ is called a *fully faithful candidate* (FFC) relative to a constraint set provided that it violates no faithfulness constraints in that constraint set. Identity candidates $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ are FFCs, because of the definition (11a) of faithfulness constraints. Yet, non-identity candidates can also qualify as FFCs. For instance, the candidate (/maba/, [ma.ba]) is not the identity candidate and yet qualifies as an FFC, under the plausible assumption that syllabification of tautomorphemic sequences is never contrastive and that no faithfulness constraint is therefore sensitive to syllabification. The reasoning presented in this section holds unchanged if idempotency is re-defined as follows: whenever $G(\mathbf{a}) = (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, there exists an FFC $(\boldsymbol{\beta}, \mathbf{b}, \rho_{\boldsymbol{\beta},\mathbf{b}})$ such that $G(\boldsymbol{\beta}) = (\boldsymbol{\beta}, \mathbf{b}, \rho_{\boldsymbol{\beta},\mathbf{b}})$. This definition of idempotency does not require the existence of the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$ and thus dispenses with the problematic reflexivity axiom (6). Instead, it requires the following weaker axiom on the candidate set: for any candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ with a surface string $\mathbf{b}$, the candidate set also contains an FFC $(\boldsymbol{\beta}, \mathbf{b}, \rho_{\boldsymbol{\beta},\mathbf{b}})$ with that same surface form $\mathbf{b}$. This assumption complements McCarthy's (2002) assumption that each *underlying* form $\mathbf{a}$ admits an FFC $(\mathbf{a}, \boldsymbol{\alpha}, \rho_{\mathbf{a},\boldsymbol{\alpha}})$ with that underlying form.

[8] Looseness has loomed at two steps in the derivation of the $\mathrm{FIC}_{\mathrm{comp}}$: first, in the replacement of the if-and-only-if condition (10) with the if-condition (14); second, in the replacement of the original FIC (19) for an arbitrary correspondence relation $\rho_{\mathbf{a},\mathbf{c}}$ with the $\mathrm{FIC}_{\mathrm{comp}}$ (21) for the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$.

[9] I thank an anonymous reviewer for discussion on the content of this subsection.

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4. Establishing the FIC$_{\text{COMP}}$: Basic constraints

The preceding section has established the FIC$_{\text{comp}}$ (21) as a sufficient condition for idempotency in OT. This condition places no restrictions on the markedness constraints and instead only concerns the faithfulness constraints. The theory of idempotency in the rest of this paper thus turns into an investigation of the formal underpinning of theories of faithfulness. Consider the *strictest* faithfulness constraint $F_{\text{strictest}}$, which is violated by every candidate except the identity candidate and thus demands perfect string identity. This constraint $F_{\text{strictest}}$ satisfies the implication (22). In fact, the antecedent of (22) requires the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ to be the identity candidate. This means that the two strings $\mathbf{b}$ and $\mathbf{c}$ are identical and that the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$ is the identity relation, so that the composition $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$ coincides with $\rho_{\mathbf{a},\mathbf{b}}$. The equation in the consequent of (22) thus holds because the two candidates being compared are identical.

(22) **If:** $F_{\text{strictest}}(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}) = 0$
Then: $F_{\text{strictest}}(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}) = F_{\text{strictest}}(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$

The faithfulness constraints adopted in the phonological literature require something weaker than perfect string identity. Correspondingly, the FIC$_{\text{comp}}$ (21) is weaker than (22), because the consequent of the former features an inequality while the consequent of the latter features an identity. Is it the case that what is left of perfect string identity in the definition of the common faithfulness constraints suffices to satisfy the FIC$_{\text{comp}}$ (21)? This section starts to address this question, focusing on the three core constraints in McCarthy & Prince's (1995) Correspondence Theory: Max, Dep, and Ident. A variety of other constraints will be considered in Sections 5 and 6.

4.1 Max

The faithfulness constraint Max assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each *deleted* underlying segment, namely for each segment of the underlying string $\mathbf{a}$ that has no corresponding segments in the surface string $\mathbf{b}$ according to $\rho_{\mathbf{a},\mathbf{b}}$ (McCarthy & Prince 1995, Harris 2011, and references therein). To illustrate, Max assigns two violations to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (23), because of its two underlying deleted segments /s/ and /e/.

(23) $\mathbf{a} =$ s t o e m t
$\mathbf{b} =$ t o n

Let us consider two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ together with their composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$. Does the faithfulness constraint Max satisfy the FIC$_{\text{comp}}$ (21)?

If the antecedent of the implication is false, the implication trivially holds. Thus, let us suppose that the antecedent is true, namely that the candidate

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$(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate MAX. For instance, assume that the strings $\mathbf{b}$ and $\mathbf{c}$ consist of two corresponding consonants each, as represented in (24a).

(24) (a) $\textsc{Max}(\mathbf{b}, \mathbf{c})=0$ (b) $\textsc{Max}(\mathbf{a}, \mathbf{c})=1$ (c) $\textsc{Max}(\mathbf{a}, \mathbf{b})=1$

(a)
$\mathbf{b}$ = s t
$\mathbf{c}$ = s t

(b)
$\mathbf{a}$ = s t r
$\mathbf{c}$ = s t

(c)
$\mathbf{a}$ = s t r
$\mathbf{b}$ = s t

Let us now turn to the inequality in the consequent of the FIC_{comp}. If the left-hand side of the inequality is zero, the inequality trivially holds. Thus, let us suppose that the left-hand side is larger than zero, namely that the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ does violate MAX. For instance, assume that the last of the three consonants of the string $\mathbf{a}$ is deleted in $\mathbf{c}$ according to the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$, as represented in (24b). If the consonant /r/ of $\mathbf{a}$ had a correspondent [s] or [t] in $\mathbf{b}$ according to $\rho_{\mathbf{a},\mathbf{b}}$, then it would also have a correspondent in $\mathbf{c}$ according to $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$, because both segments /s/ and /t/ of $\mathbf{b}$ have a correspondent in $\mathbf{c}$ relative to $\rho_{\mathbf{b},\mathbf{c}}$. Thus, the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ must fail to provide a surface correspondent of /r/ in $\mathbf{b}$, as represented in (24c). This says in turn that the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ which figures in the right-hand side of the FIC_{comp} inequality violates MAX as well, so that the inequality holds in this case.

This reasoning suggests that the FIC_{comp} (21) holds because the assumption that no segment of $\mathbf{b}$ is deleted in $\mathbf{c}$ (the antecedent of the FIC_{comp}) entails that any segment of $\mathbf{a}$ that is deleted in $\mathbf{c}$ (as quantified by the left-hand side of the inequality in the consequent) is also deleted in $\mathbf{b}$ (as quantified by the right-hand side of the inequality). Lemma 2 thus obtained will be refined in Section 5 and proved in Appendix A.1.

Lemma 2 (provisional). *The faithfulness constraint* MAX *satisfies the* $\textit{FIC}_{comp}$ *(21) under no additional assumptions.* □

4.2 DEP

The faithfulness constraint DEP assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each *epenthetic* surface segment, namely for each segment of the surface string $\mathbf{b}$ that has no corresponding segments in the underlying string $\mathbf{a}$ according to $\rho_{\mathbf{a},\mathbf{b}}$ (McCarthy & Prince 1995, Hall 2011, and references therein). To illustrate, DEP assigns two violations to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (25), because of its two epenthetic vowels [ə] and [e] (from Temiar; Itô 1989).

(25) $\mathbf{a}$ = s n g l ɔ g
$\mathbf{b}$ = s ə n e g l ɔ g

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Let us consider two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ together with their composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$. Does the faithfulness constraint DEP satisfy the FIC_{comp} (21)?

We can reason exactly as in the preceding Section 4.1. We assume that the antecedent of the FIC_{comp} holds, namely that the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate DEP, as in (26a). Moreover, we assume that the left-hand side of the inequality in the consequent of the FIC_{comp} is larger than zero, namely that the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ does violate DEP, say because of the surface [ə] with no underlying correspondents in (26b).

(26) (a) DEP($\mathbf{b}, \mathbf{c}$)=0

b = s ə l ɔ g

c = s ə l ɔ g

(b) DEP($\mathbf{a}, \mathbf{c}$)=1

a = s l ɔ g

c = s ə l ɔ g

(c) DEP($\mathbf{a}, \mathbf{b}$)=1

a = s l ɔ g

b = s ə l ɔ g

By definition of the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$, it follows that the vowel [ə] of **b** cannot have a correspondent relative to $\rho_{\mathbf{a},\mathbf{b}}$, as represented in (26c). This says in turn that the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ that figures in the right-hand side of the FIC_{comp} inequality violates DEP as well, so that the inequality holds in this case.

In order to secure the FIC_{comp} for DEP, some additional care is needed, though: the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$ must be prevented from breaking any underlying segments into two or more surface segments, as shown by the counter-example (27).

(27) (a) DEP($\mathbf{b}, \mathbf{c}$)=0

b = s ə l ɔ g

c = s e i l ɔ g

(b) DEP($\mathbf{a}, \mathbf{c}$)=2

a = s l ɔ g

c = s e i l ɔ g

(c) DEP($\mathbf{a}, \mathbf{b}$)=1

a = s l ɔ g

b = s ə l ɔ g

The antecedent of the FIC_{comp} holds, as shown in (27a): the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate DEP, because every segment of **c** has a correspondent, although the two surface vowels [e] and [i] share the underlying correspondent /ə/. The right-hand side of the FIC_{comp} inequality is equal to 1, as shown in (27c): the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ violates DEP once, because it has a unique epenthetic vowel [ə]. The FIC_{comp} inequality fails because its left-hand side is instead equal to 2, as shown in (27b): the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ violates DEP twice, because both [e] and [i] are epenthetic.

These considerations lead to Lemma 3, which will be refined in Section 5 and proved in Appendix A.2.

Lemma 3 (provisional). *The faithfulness constraint* DEP *satisfies the FIC$_{comp}$ (21) provided that no correspondence relation in the candidate set breaks any underlying segment.* □

Lemmas 2 and 3 highlight a difference between MAX and DEP: the former satisfies the FIC$_{\text{comp}}$ without additional assumptions; the latter instead requires the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$ not to break any underlying segments, forbidding scenarios such as (27a). The reason behind this difference can be intuitively appreciated as follows. DEP quantifies over epenthetic *surface* segments, and the two candidates ($\mathbf{a}$, $\mathbf{c}$, $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$) and ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) compared by the inequality in the consequent of the FIC$_{\text{comp}}$ have different surface strings $\mathbf{b}$ and $\mathbf{c}$. In order to make these two strings 'commensurate', the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$ that links them cannot break underlying segments. MAX instead quantifies over deleted *underlying* segments, and the two candidates ($\mathbf{a}$, $\mathbf{c}$, $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$) and ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) compared by the FIC$_{\text{comp}}$ inequality share the underlying form $\mathbf{a}$, so that no additional 'commensurability' assumptions are needed.

4.3 IDENT

A phonological *feature* φ takes a segment a and returns a *feature value*. A feature is called *binary* if it takes only two values; otherwise, it is called *multi-valued*. For instance, the feature [nasal] is binary while the feature [place] could be construed as distinguishing between three major places of articulation, making it multi-valued (de Lacy 2006: Section 2.3.2.1.1). A feature φ is called *total* (relative to the candidate set) provided that there is no underlying or surface string that contains a segment for which the feature φ is undefined. The identity faithfulness constraint IDENT$_{\varphi}$ corresponding to a total feature φ assigns to a candidate ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) one violation for each pair (a, b) of an underlying segment a and a surface segment b that are put in correspondence by $\rho_{\mathbf{a},\mathbf{b}}$ despite the fact that they are assigned different values by the feature φ (McCarthy & Prince 1995). To illustrate, IDENT$_{\text{[nasal]}}$ assigns two violations to the candidate ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) in (28), because of the two corresponding pairs (/n/, [t]) and (/k/, [ŋ]).

(28)
```
a = a n t a ŋ k
    | |/ / / /
b = a t a ŋ ŋ
```

Let us consider two candidates ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) and ($\mathbf{b}$, $\mathbf{c}$, $\rho_{\mathbf{b},\mathbf{c}}$) together with their composition candidate ($\mathbf{a}$, $\mathbf{c}$, $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$). Does the identity faithfulness constraint IDENT$_{\text{[nasal]}}$ satisfy the FIC$_{\text{comp}}$ (21)?

We can reason exactly as in the two preceding Sections 4.1 and 4.2. We assume that the antecedent of the FIC$_{\text{comp}}$ holds, namely that the candidate ($\mathbf{b}$, $\mathbf{c}$, $\rho_{\mathbf{b},\mathbf{c}}$) does not violate IDENT$_{\text{[nasal]}}$, as in (29a).

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(29) (a) $\text{ID}_{[\text{nas}]}(\mathbf{b},\mathbf{c})=0$ (b) $\text{ID}_{[\text{nas}]}(\mathbf{a},\mathbf{c})=1$ (c) $\text{ID}_{[\text{nas}]}(\mathbf{a},\mathbf{b})=1$

$$\begin{array}{lll}
 & \mathbf{a} = \text{ŋ a} & \mathbf{a} = \text{ŋ a} \\
\mathbf{b} = \text{g a} & \quad\;\; | \;\; | & \quad\;\; | \;\; | \\
\quad\;\; | \;\; | & \quad\;\; | \;\; | & \mathbf{b} = \text{g a} \\
\mathbf{c} = \text{g a} & \mathbf{c} = \text{g a} &
\end{array}$$

Moreover, we assume that the left-hand side of the inequality in the consequent of the FIC_{comp} is larger than zero, namely that the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ does violate $\text{IDENT}_{[\text{nasal}]}$, as in (29b). By definition of the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$, $\rho_{\mathbf{a},\mathbf{b}}$ must put in correspondence the underlying nasal /ŋ/ of **a** with the surface oral [g] of **b**, as represented in (29c). This says in turn that the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ that figures in the right-hand side of the FIC_{comp} inequality violates $\text{IDENT}_{[\text{nasal}]}$ as well, so that the inequality holds.

Also for $\text{IDENT}_{[\text{nasal}]}$, as for DEP, the FIC_{comp} requires no underlying segment to be broken by the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$, as shown by the counterexample (30), analogous to (27).

(30) (a) $\text{ID}_{[\text{nas}]}(\mathbf{b},\mathbf{c})=0$ (b) $\text{ID}_{[\text{nas}]}(\mathbf{a},\mathbf{c})=2$ (c) $\text{ID}_{[\text{nas}]}(\mathbf{a},\mathbf{b})=1$

$$\begin{array}{lll}
 & \mathbf{a} = \;\;\text{ŋ a} & \mathbf{a} = \text{ŋ a} \\
\mathbf{b} = \;\;\text{g a} & \quad\;\; / | \;\; | & \quad\;\; | \;\; | \\
\quad\;\; / | \;\; | & \quad\;\; / | \;\; | & \mathbf{b} = \text{g a} \\
\mathbf{c} = \text{g g a} & \mathbf{c} = \text{g g a} &
\end{array}$$

The antecedent of the FIC_{comp} holds: gemination preserves nasality in the candidate in (30a) (which could correspond, for instance, to the Japanese loan [fu.róg.gu] of English *frog*; Kubozono, Ito & Mester 2008). However, the inequality in the consequent of the FIC_{comp} fails: the composition candidate (30b) violates $\text{IDENT}_{[\text{nasal}]}$ twice because of the gemination while the candidate (30c) violates it only once, so that the left-hand side of the inequality exceeds the right-hand side.

These considerations extend from $\text{IDENT}_{[\text{nasal}]}$ to the identity faithfulness constraint IDENT_{φ} corresponding to any feature φ, independently of whether it is binary or multi-valued, as long as it is total. The case of partial features is indeed more delicate. Assume that the identity faithfulness constraint IDENT_{φ} corresponding to a partial feature φ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each pair $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}$ of corresponding segments such that the feature φ is defined for both segments and assigns them a different value. Thus, IDENT_{φ} is not violated when the feature φ is undefined for at least one of the two segments.[10] To illustrate, suppose that the feature [strident] is only defined

[10] Another option is to let IDENT_{φ} assign one violation also when the feature φ is undefined for one and only one of the two segments in the corresponding pair (a, b). This definition of IDENT_{φ} effectively treats φ as a total but multi-valued feature. Under this approach, feature partiality raises no additional complications for the FIC_{comp}.

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for coronals (Hayes 2009). The corresponding constraint $\text{IDENT}_{[\text{strident}]}$ does not satisfy the FIC_{comp} (21), as shown by the counterexample (31).

(31) (a) $\text{ID}_{[\text{str}]}(\mathbf{b}, \mathbf{c})=0$

b = f a
c = s a

(b) $\text{ID}_{[\text{str}]}(\mathbf{a}, \mathbf{c})=1$

a = θ a
c = s a

(c) $\text{ID}_{[\text{str}]}(\mathbf{a}, \mathbf{b})=0$

a = θ a
b = f a

The antecedent of the FIC_{comp} holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ in (31a) does not violate $\text{IDENT}_{[\text{strident}]}$, because the underlying /f/ of **b** is not coronal and is thus undefined for stridency. The right-hand side of the FIC_{comp} inequality is equal to 0: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (31c) does not violate $\text{IDENT}_{[\text{strident}]}$, because again [f] is undefined for stridency. The FIC_{comp} inequality fails because its left-hand side is equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ in (31b) does violate $\text{IDENT}_{[\text{strident}]}$, because of the two corresponding coronal segments /θ/ and [s].

These considerations lead to Lemma 4, which will be refined in Section 5 and proved in Appendix A.3.

Lemma 4 (provisional). *The identity faithfulness constraint* IDENT_{φ} *relative to a phonological feature* φ *satisfies the* FIC_{comp} *(21) provided that no correspondence relation in the candidate set breaks any underlying segment and furthermore the feature* φ *is total relative to the candidate set.* □

5. ESTABLISHING THE FIC_{COMP}: RESTRICTED CONSTRAINTS

The phonological literature has made use of *restricted* variants of MAX, DEP, and IDENT which are only offended when the deleted, epenthetic, or mismatching segments belong to a *privileged* segment set. Privilege can be determined by segmental quality or position in the string. This section investigates how these restricted constraints fare with respect to the FIC_{comp}.

5.1 MAX_R

A *restriction* R pairs a string **a** with a subset $R(\mathbf{a})$ of its segments. A segment of the string **a** *satisfies* the restriction provided that it belongs to $R(\mathbf{a})$. The faithfulness constraint MAX_R assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the underlying string **a** that satisfies the restriction R and is deleted. Deletion of underlying segments that do not satisfy the restriction is not penalized. To illustrate, consider the restriction R which pairs a string **a** with the set $R(\mathbf{a})$ of its consonants. The corresponding constraint MAX_R is the constraint MAX-C which militates against consonant deletion, but is not offended by vowel deletion (it thus assigns only one violation to the candidate (23), while unrestricted MAX assigns two violations).

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While we have seen that unrestricted MAX satisfies the FIC_{comp}, its restricted counterpart MAX_R can fail at the FIC_{comp}, as shown by the counterexample (32) for $\text{MAX}_R = \text{MAX-C}$.

(32) (a) $\text{MAX}_R(\mathbf{b}, \mathbf{c})=0$ (b) $\text{MAX}_R(\mathbf{a}, \mathbf{c})=1$ (c) $\text{MAX}_R(\mathbf{a}, \mathbf{b})=0$

(a) **b** = ee, **c** = e

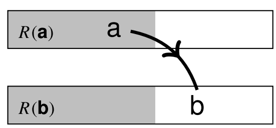
(b) **a** = es, **c** = e

(c) **a** = es, **b** = ee

The antecedent of the FIC_{comp} holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ in (32a) does not violate MAX-C, because the deleted segment is a vowel. The right-hand side of the FIC_{comp} inequality is equal to zero: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (32c) does not violate MAX-C, because it involves no deletion. The FIC_{comp} inequality thus fails, because its left-hand side is instead equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ in (32b) does violate MAX-C, because it deletes a consonant.

In order for MAX_R to fail at the FIC_{comp} in (32), it is crucial that $\rho_{\mathbf{a},\mathbf{b}}$ establishes a correspondence between the consonant /s/ and the vowel [e], namely between a segment that satisfies the restriction R and a segment that does not satisfy it. Given a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ is said to *exit from* the restriction R if it puts some underlying segment a that satisfies the restriction R in correspondence with some surface segment b that does not satisfy it, as in (33). The top and bottom rectangles represent the sets of segments of **a** and **b**, with the subsets selected by the restriction R highlighted in gray.

(33) $(\mathrm{a}, \mathrm{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathrm{a} \in R(\mathbf{a}), \quad \mathrm{b} \notin R(\mathbf{b})$



The following lemma ensures that MAX_R satisfies the FIC_{comp} provided that no correspondence relation in the candidate set exits from the restriction R. To illustrate, the lemma guarantees that MAX-C satisfies the FIC_{comp} provided that no underlying consonant is in correspondence with a surface vowel. The lemma will be further extended in Section 5.3.

Lemma 2 (provisional). *Assume that the candidate set contains no candidate $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ whose correspondence relation exits from the restriction R, so that condition (33) is impossible relative to the candidate set. The faithfulness constraint* MAX_R *then satisfies the FIC_{comp} (21).* □

A restriction is *trivial* provided that it pairs every string with the totality of its segments. The case of unrestricted MAX discussed in Section 4.1 follows as a special case of MAX_R with a trivial restriction R: no correspondence relation can exit from R in this case and (33) is thus contradictory.

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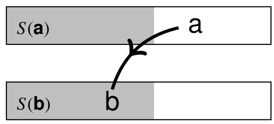
5.2 DEP^S

The reasoning in Section 5.1 extends straightforwardly from MAX to DEP. Given a restriction S, the corresponding faithfulness constraint DEP^S assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the surface string $\mathbf{b}$ that satisfies the restriction S and is epenthetic.[11] To illustrate, consider the restriction S which pairs a string with the set of its vowels. The corresponding constraint DEP^S is the constraint DEP-V which militates against vowel epenthesis, but is not offended by consonant epenthesis. Given a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ is said to *enter into* S provided that it puts some underlying segment a that does not satisfy the restriction S in correspondence with some surface segment b that does satisfy it, as in (34). Condition (34) is analogous to (33), only with the roles of underlying and surface segments switched.

(34) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{a} \notin S(\mathbf{a}), \quad \mathsf{b} \in S(\mathbf{b})$



In the case of MAX$_R$, the no-exiting assumption that (33) is impossible suffices to establish the FIC$_{\text{comp}}$. In the case of DEP^S, the no-entering assumption that (34) is impossible needs to be coupled with the no-breaking condition, as expected based on the discussion in Section 4.2. The following lemma guarantees that it suffices to require the no-breaking condition among the segments that satisfy the restriction S, intuitively because DEP^S only cares about those segments. To illustrate, the lemma guarantees that DEP-V satisfies the FIC$_{\text{comp}}$ provided that no surface vowel is in correspondence with an underlying consonant and furthermore no underlying vowel is diphthongized.[12] The lemma will be further extended in Section 5.4.

Lemma 3 (provisional). *Assume that no underlying segment that satisfies the restriction S is broken into two surface segments that both satisfy the restriction S, in the sense that the candidate set contains no candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ with two different corresponding pairs $(\mathsf{a}, \mathsf{b}_1), (\mathsf{a}, \mathsf{b}_2) \in \rho_{\mathbf{a},\mathbf{b}}$ for the same underlying segment a such that $\mathsf{a} \in S(\mathbf{a})$ and $\mathsf{b}_1, \mathsf{b}_2 \in S(\mathbf{b})$. Assume furthermore that the candidate set contains no candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ whose correspondence relation enters into the restriction S, so that condition (34) is impossible relative to the candidate set. The faithfulness constraint* DEP^S *then satisfies the FIC$_{comp}$ (21).* □

[11] Throughout this section, a restriction on the underlying segments is denoted by R and appears as a subscript on the constraint name (as in MAX$_R$) while a restriction on the surface segments is denoted by S and appears as a superscript (as in DEP^S).

[12] Assuming a breaking analysis of vowel diphthongization.

5.3 $\textsc{Max}_R^S$

The *doubly restricted* constraint $\textsc{Max}_R^S$ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the underlying string $\mathbf{a}$ that satisfies the restriction R (namely, it belongs to $R(\mathbf{a})$) and has no correspondent segment in the surface string $\mathbf{b}$ that satisfies the restriction S (namely, it belongs to $S(\mathbf{b})$), although it might have surface correspondents that do not satisfy the restriction S. To illustrate, consider the restriction R which pairs a string with the set of its consonants and the restriction S which pairs a string with the set of the segments in its initial syllable. $\textsc{Max}_R^S$ is Beckman's (1999) constraint $\textsc{Max}$-C-σ_1, which mandates that every consonant has a correspondent in the initial syllable. The following lemma extends the analysis of the singly restricted $\textsc{Max}_R$ to the doubly restricted $\textsc{Max}_R^S$. This lemma concludes the analysis of segmental $\textsc{Max}$ constraints. The proof is a straightforward verification, as shown in Appendix A.1.

Lemma 2. *Assume that the candidate set contains no candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ that satisfies condition (35), which is therefore impossible relative to the candidate set.*

(35) $(\mathrm{a}, \mathrm{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathrm{a} \in R(\mathbf{a}), \quad \mathrm{b} \notin R(\mathbf{b}), \quad \mathrm{b} \in S(\mathbf{b})$

The faithfulness constraint $\textsc{Max}_R^S$ then satisfies the FIC_{comp} (21). □

Condition (35) for $\textsc{Max}_R^S$ and condition (33) considered in Section 5.1 for $\textsc{Max}_R$ differ only in that the former has the additional fourth clause $\mathrm{b} \in S(\mathbf{b})$. Because of this additional clause, the assumption that (35) is impossible required for $\textsc{Max}_R^S$ to satisfy the $\mathrm{FIC_{comp}}$ is weaker than the assumption that (33) is impossible required for $\textsc{Max}_R$. To illustrate, the lemma says that the doubly restricted $\textsc{Max}$-C-σ_1 satisfies the $\mathrm{FIC_{comp}}$ provided that no consonant is in correspondence with the vowel of the initial syllable, while the singly restricted $\textsc{Max}$-C was shown in Section 5.1 to require the stronger assumption that no consonant is in correspondence with any vowel.

The case of $\textsc{Max}_R$ follows as a special case of $\textsc{Max}_R^S$ with a trivial restriction S: the additional clause $\mathrm{b} \in S(\mathbf{b})$ is trivially satisfied in this case and the two conditions (33) and (35) thus coincide. Furthermore, the constraint $\textsc{Max}^S$ always satisfies the $\mathrm{FIC_{comp}}$ because it coincides with $\textsc{Max}_R^S$ with a trivial restriction R, whereby the clause $\mathrm{b} \notin R(\mathbf{b})$ is impossible. As yet another interesting special case, suppose that the two restrictions R and S coincide. Condition (35) is then contradictory, because it cannot be the case that $\mathrm{b} \notin R(\mathbf{b})$ and $\mathrm{b} \in S(\mathbf{b})$. The constraint $\textsc{Max}_R^S$ thus satisfies the $\mathrm{FIC_{comp}}$ without additional assumptions when $S = R$. This observation will be used in Section 6.1 to establish the $\mathrm{FIC_{comp}}$ for featural $\textsc{Max}$ constraints.

5.4 $\textsc{Dep}_R^S$

The reasoning in Section 5.3 extends straightforwardly from $\textsc{Max}_R^S$ to $\textsc{Dep}_R^S$. The doubly restricted constraint $\textsc{Dep}_R^S$ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the surface string $\mathbf{b}$ that satisfies the restriction S (namely, it belongs to $S(\mathbf{b})$) and has no correspondent segment in the underlying string $\mathbf{a}$ that satisfies the restriction R (namely, it belongs to $R(\mathbf{a})$), although it might have underlying correspondents that do not satisfy the restriction R. The following lemma extends the analysis of the singly restricted DEP^S to the doubly restricted DEP^S_R and thus concludes the analysis of segmental DEP constraints. The assumption that (36) is impossible is weaker than the assumption that (34) is impossible, because of the additional clause $\mathsf{a} \in R(\mathbf{a})$. The proof of the lemma is a straightforward verification, as shown in Appendix A.2.

Lemma 3. *Assume that no underlying segment that satisfies the restriction S is broken into two surface segments that both satisfy the restriction S, in the sense that the candidate set contains no candidate* $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ *with two different corresponding pairs* $(a, b_1), (a, b_2) \in \rho_{\boldsymbol{a},\boldsymbol{b}}$ *for the same underlying segment* a *such that* $a \in S(\boldsymbol{a})$ *and* $b_1, b_2 \in S(\boldsymbol{b})$. *Assume furthermore that the candidate set contains no candidate* $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ *that satisfies condition (36), which is therefore impossible relative to the candidate set.*

(36) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{b} \in S(\mathbf{b}), \quad \mathsf{a} \notin S(\mathbf{a}), \quad \mathsf{a} \in R(\mathbf{a})$

The faithfulness constraint DEP^S_R *then satisfies the* FIC_{comp} *(21).* □

5.5 $\text{IDENT}_{\varphi,R}$

The faithfulness constraint $\text{IDENT}_{\varphi,R}$ corresponding to a total feature φ and a restriction R assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each corresponding pair $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}$ of segments that differ in the value of the feature φ such that the underlying segment a satisfies the restriction R (namely, it belongs to $R(\mathbf{a})$). To illustrate, consider the restriction R which pairs a string with the set of its nasal segments. The corresponding constraint $\text{IDENT}_{[\text{nasal}],R}$ is the constraint $\text{IDENTI} \rightarrow \text{O}[+\text{nasal}]$ of Pater (1999), which penalizes de-nasalization (i.e., an underlying nasal segment with an oral surface correspondent), but not nasalization. Lemma 4/A guarantees that $\text{IDENT}_{\varphi,R}$ satisfies the FIC_{comp} provided that the candidate set makes (37) impossible (and furthermore satisfies the usual no-breaking assumption).

Lemma 4/A. *Assume that no correspondence relation in the candidate set breaks any underlying segment. Consider a feature* φ *that is total relative to the candidate set. Assume furthermore that the candidate set contains no candidate* $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ *that satisfies condition (37), which is therefore impossible relative to the candidate set.*

(37) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{a} \in R(\mathbf{a}), \quad \mathsf{b} \notin R(\mathbf{b}), \quad \varphi(\mathsf{a}) = \varphi(\mathsf{b})$

The faithfulness constraint $\text{IDENT}_{\varphi,R}$ *then satisfies the* FIC_{comp} *(21).* □

Condition (37) coincides with condition (33) used above in the analysis of MAX_R, apart from the additional clause $\varphi(\mathsf{a}) = \varphi(\mathsf{b})$ that the two segments a and b are assigned the same value by the feature φ. The assumption that (37) is impossible thus means that the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ cannot exit from R without changing the value of the feature φ. The assumption that (37) is impossible is thus weaker than the assumption that (33) is impossible, which was needed above for MAX_R. To illustrate, consider again the feature $\varphi =$ [nasal] and the restriction R which pairs a string with the set of its nasals. Condition (37) is contradictory in this case, because the three clauses $\mathsf{a} \in R(\mathbf{a})$, $\mathsf{b} \notin R(\mathbf{b})$, and $\varphi(\mathsf{a}) = \varphi(\mathsf{b})$ cannot hold simultaneously. Pater's constraint IDENTI$\rightarrow$O[+nasal] $= \text{IDENT}_{[\text{nasal}],R}$ thus satisfies the FIC_{comp} (provided there is no breaking).

5.6 $\text{IDENT}_{\varphi}^{S}$

The faithfulness constraint $\text{IDENT}_{\varphi}^{S}$ is defined analogously, the only difference being that the restriction is applied to surface rather than underlying segments. To illustrate, consider the restriction S which pairs a string with the set of segments that belong to its initial syllable. The corresponding constraint $\text{IDENT}_{[\text{high}]}^{S}$ is the constraint $\text{IDENT}_{[\text{high}]}^{\sigma_1}$ of Beckman (1997, 1999), which is violated by a surface vowel in the initial syllable in correspondence with an underlying vowel that differs with respect to the feature [high]. As another example, consider the restriction S which pairs a string with the set of its nasal segments. The corresponding constraint $\text{IDENT}_{[\text{nasal}]}^{S}$ is the constraint IDENTO $\rightarrow$ I[+nasal] of Pater (1999), which penalizes nasalization (i.e., an underlying oral segment with a nasal surface correspondent), but not de-nasalization. Lemma 4/B guarantees that $\text{IDENT}_{\varphi}^{S}$ satisfies the FIC_{comp} provided that condition (38) is impossible. This assumption means that the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ cannot exit from R without changing the value of the feature φ. The only difference between Lemmas 4/A and 4/B is that the no-breaking assumption in the latter lemma is restricted to the segments that satisfy the restriction (as underlined). The proof of both lemmas is a straightforward verification, as shown in Appendix A.3.

Lemma 4/B. *Assume that no underlying segment <u>that satisfies the restriction S</u> can be broken into two surface segments <u>that both satisfy the restriction S</u>, in the sense that the candidate set contains no candidate* $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ *with two different candidate pairs* $(a, b_1), (a, b_2) \in \rho_{\boldsymbol{a},\boldsymbol{b}}$ *for the same underlying segment* a *such that* $a \in S(\boldsymbol{a})$ *and* $b_1, b_2 \in S(\boldsymbol{b})$. *Consider a feature* φ *that is total relative to the candidate set. Assume furthermore that the candidate set contains no candidate* $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ *that satisfies condition (38), which is therefore impossible relative to the candidate set.*

(38) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{b} \in S(\mathbf{b}), \quad \mathsf{a} \notin S(\mathbf{a}), \quad \varphi(\mathsf{a}) = \varphi(\mathsf{b})$

The faithfulness constraint $\text{IDENT}_{\varphi}^{S}$ *then satisfies the* FIC_{comp} *(21).* □

5.7 $\text{IDENT}^S_{\varphi,R}$

For completeness, let us also consider the faithfulness constraint $\text{IDENT}^S_{\varphi,R}$ corresponding to a total feature φ and two restrictions R, S, which assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each pair $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}$ of an underlying segment a that satisfies the restriction R (namely, it belongs to $R(\mathbf{a})$) and a surface segment b that satisfies the restriction S (namely, it belongs to $S(\mathbf{b})$) such that $\varphi(\mathsf{a}) \neq \varphi(\mathsf{b})$. To illustrate, consider the constraint *REPLACE (coronal, labial) proposed in Boersma (1998): it is violated by an underlying coronal consonant with a labial surface correspondent. It can be reinterpreted as the constraint $\text{IDENT}^S_{[\text{place}],R}$ corresponding to a tri-valued feature [place] where the restrictions R and S pair a string with the set of its coronal segments and the set of its labial segments, respectively. As another example, consider the *MAP constraint in (39), proposed by White (2013) and Hayes & White (to appear) building on Zuraw (2007, 2013): it is violated by an underlying voiceless stop which is in correspondence with a surface voiced fricative. This constraint can be reinterpreted as the constraint $\text{IDENT}^S_{[\text{voice}],R}$ or $\text{IDENT}^S_{[\text{cont}],R}$ where the restrictions R and S pair a string with the set of its voiceless stops and the set of its voiced fricatives, respectively.[13]

(39) $\text{*MAP}\left(\begin{bmatrix}-\text{voice}\\-\text{cont}\end{bmatrix}, \begin{bmatrix}+\text{voice}\\+\text{cont}\end{bmatrix}\right)$

No simple conditions on the correspondence relations and the restrictions R, S seem to suffice to ensure that the doubly restricted constraint $\text{IDENT}^S_{\varphi,R}$ satisfies the FIC_{comp}. In particular, it does not suffice to simply assume that the two conditions (37) and (38) for IDENT^S_{φ} and $\text{IDENT}_{\varphi,R}$ are both impossible. Here is a counterexample. Consider a feature φ that is partial and binary. Consider the corresponding feature $\widehat{\varphi}$ that is total and ternary, in the sense that $\widehat{\varphi}$ coincides with φ for any segment that φ is defined for, while $\widehat{\varphi}$ assigns the dummy value '0' to the segments that φ is undefined for. Consider the identity faithfulness constraint IDENT_{φ} relative to the partial feature φ, which only penalizes an underlying and a corresponding surface segment when the feature is defined for both and assigns them a different value (IDENT_{φ} does not assign a violation when the feature is defined for exactly one of the two corresponding segments). This constraint IDENT_{φ} is identical to the doubly restricted identity faithfulness constraint $\text{IDENT}^S_{\widehat{\varphi},R}$ relative to the total three-valued feature $\widehat{\varphi}$ and the restrictions $R = S$ which pair a string with the set of its segments for which the feature φ is defined (namely the set of segments to which the corresponding total feature $\widehat{\varphi}$ assigns values $\pm$ and not the dummy value). Since the identity constraint IDENT_{φ} corresponding to the partial feature φ has been shown not to satisfy the FIC_{comp} in Section 4.3, the doubly restricted constraint $\text{IDENT}^S_{\widehat{\varphi},R}$ cannot satisfy

[13] Zuraw (2013) actually assumes that *MAP applies to corresponding output segments. Output–output correspondence falls outside the scope of this paper.

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the FIC_{comp} either. Yet, conditions (37) and (38) are both contradictory, because the restrictions R, S are defined in terms of the values of the feature $\widehat{\varphi}$.

6. Establishing the FIC_{COMP}: Other constraints

This section completes the analysis of the FIC_{comp} within Correspondence Theory, by looking at a variety of other faithfulness constraints that naturally arise within that framework. For simplicity, only the unrestricted versions of these constraints are considered.

6.1 $\text{MAX}_{[+\varphi]}$, $\text{DEP}_{[+\varphi]}$

Let '+' be a designated value of a feature φ, either partial or total, either binary or multi-valued. The faithfulness constraint $\text{MAX}_{[+\varphi]}$ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the underlying string **a** that has the designated value + for the feature φ but has no correspondent in the surface string **b** that shares the value + for the feature φ (Casali 1997, 1998; Walker 1999; Lombardi 2001).[14] To illustrate, $\text{MAX}_{[+\text{voice}]}$ assigns two violations to the candidate (40), because both /b/ and /d/ lose their voicing (through devoicing and deletion, respectively).

(40)
$$\begin{array}{l} \mathbf{a} = \text{b a d} \\ \quad\quad\;\; | \;\; | \\ \mathbf{b} = \text{p a} \end{array}$$

The featural constraint $\text{MAX}_{[+\varphi]}$ coincides with the doubly restricted segmental constraint MAX_R^S when the two restrictions R and S both pair up a string with the set of its segments that have the value + for the feature φ. Since, in particular, $R = S$, condition (35) of Lemma 2 is impossible, because its last two clauses $\mathsf{b} \notin R(\mathbf{b})$ and $\mathsf{b} \in S(\mathbf{b})$ are contradictory. The following result thus follows as a special case of Lemma 2.

Lemma 5. *Let '+' be a designated value of a feature φ (either binary or multi-valued, either partial or total). The faithfulness constraint* $\text{MAX}_{[+\varphi]}$ *satisfies the* $\textit{FIC}_{\textit{comp}}$ *(21) under no additional assumptions.* □

Analogous considerations hold for the constraint $\text{DEP}_{[+\varphi]}$, which assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each segment of the surface string **b** that has the designated value + for the feature φ but has no correspondent in the underlying string **a** that shares the value +.

[14] This paper assumes correspondence relations to be defined among segments. Correspondence relations among feature values are then defined *indirectly*: two feature values are in correspondence provided that their segmental carriers are in correspondence. The investigation of idempotency in an auto-segmental framework where correspondence relations are defined *directly* among feature values is left for future research.

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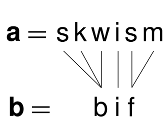
Lemma 6. *Let '+' be a designated value of a feature φ (either binary or multi-valued, either partial or total). Assume that no underlying segment with value + is broken into two surface segments that both have value +. The faithfulness constraint* $\textsc{Dep}_{[+\varphi]}$ *then satisfies the FIC$_{comp}$ (21).* □

The featural constraints $\textsc{Max}_{[+\varphi]}$/$\textsc{Dep}_{[+\varphi]}$ differ subtly from the restricted segmental constraints $\textsc{Max}_R$/$\textsc{Dep}^S$, where R and S both pair a string with the set of its segments that have the value '+' for the feature φ. In fact, $\textsc{Max}_R$/$\textsc{Dep}^S$ are violated by an underlying/surface segment that has the value '+' and is deleted/epenthesized, while $\textsc{Max}_{[+\varphi]}$/$\textsc{Dep}_{[+\varphi]}$ are violated by an underlying/surface segment that has the value '+' and is deleted/epenthesized *or* put in correspondence with segments with a different value for feature φ. This subtle difference is computationally substantial: $\textsc{Max}_{[+\varphi]}$/$\textsc{Dep}_{[+\varphi]}$ satisfy the FIC$_{\text{comp}}$ under no additional assumptions, but $\textsc{Max}_R$/$\textsc{Dep}^S$ require the additional no-entering and no-exiting assumptions that (33) and (34) are impossible, as seen in Sections 5.1–5.2. Formally, this difference is due to the fact that $\textsc{Max}_{[+\varphi]}$/$\textsc{Dep}_{[+\varphi]}$ coincide not with $\textsc{Max}_R$/$\textsc{Dep}^S$ but with $\textsc{Max}_R^S$/$\textsc{Dep}_R^S$ with $S = R$.

6.2 Uniformity *and* Integrity

The faithfulness constraint Uniformity assigns to a candidate ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) one violation for each surface *coalescence*, namely for each segment of the surface string $\mathbf{b}$ that has two or more correspondents in the underlying string $\mathbf{a}$ according to $\rho_{\mathbf{a},\mathbf{b}}$ (McCarthy & Prince 1995). To illustrate, Uniformity assigns two violations to the candidate ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) in (41), because of its two surface coalescences [b] and [f]. The constraint thus defined is *coarse* (Wheeler 2005): it does not distinguish between a coalescence of just two segments (such as [f] below) and a coalescence of more than two segments (such as [b]). This distinction can be captured through the following alternative *gradient* definition: the faithfulness constraint Uniformity$^{\text{grad}}$ assigns k violations for each coalescence of $k \geqslant 2$ underlying segments.

(41)


While Dep penalizes surface segments that have *no* underlying correspondents, Uniformity penalizes surface segments that have *too many*. The analysis of Dep in Section 4.2 extends to Uniformity, yielding the following lemma 7, whose simple verification is omitted for brevity.

Lemma 7. *The faithfulness constraints* Uniformity *and* Uniformity^{grad} *satisfy the FIC$_{comp}$ (21) provided that no correspondence relation in the candidate set breaks any underlying segment.* □

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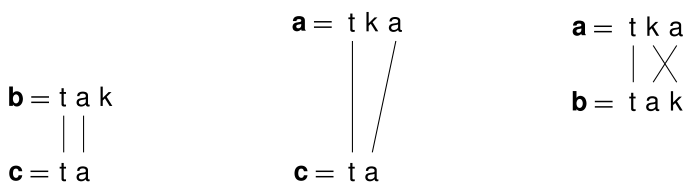
Analogous considerations hold for the faithfulness constraint INTEGRITY, which assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each *broken* underlying segment, namely for each segment of the underlying string $\mathbf{a}$ that has two or more correspondents in the surface string $\mathbf{b}$ according to $\rho_{\mathbf{a},\mathbf{b}}$ (see McCarthy & Prince 1995 and Staroverov 2014 for discussion). The corresponding gradient constraint INTEGRITY$^{\text{grad}}$ assigns instead k violations for each underlying segment that is broken into $k \geqslant 2$ surface segments. While MAX penalizes underlying segments that have *no* surface correspondents, INTEGRITY penalizes underlying segments that have *too many*. The analysis of MAX in Section 4.1 extends to INTEGRITY, yielding the following Lemma 8.

Lemma 8. *The faithfulness constraints* INTEGRITY *and* INTEGRITY^{grad} *satisfy the FIC$_{comp}$ (21) under no additional assumptions.* □

6.3 CONTIGUITY

The faithfulness constraint I-CONTIGUITY assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each *skipped* underlying segment, namely for each segment of the underlying string $\mathbf{a}$ that has no correspondents in the surface string $\mathbf{b}$ according to $\rho_{\mathbf{a},\mathbf{b}}$ and furthermore is flanked both on the left and on the right by underlying segments that instead do admit surface correspondents.[15] The faithfulness constraint I-CONTIGUITY fails at the FIC$_{\text{comp}}$, as shown by the counterexample (42). The antecedent of the FIC$_{\text{comp}}$ holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ in (42a) does not violate I-CONTIGUITY because it has no skipped segments (the deleted coda /k/ does not count as skipped because it is string-final). The right-hand side of the FIC$_{\text{comp}}$ inequality is small, namely equal to zero: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (42c) (modeled on metathesis in Rotuman; Carpenter 2002) does not violate I-CONTIGUITY, because it has no skipped segments (because no underlying segment is deleted). The FIC$_{\text{comp}}$ inequality fails because its left-hand side is large, namely equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ in (42b) violates I-CONTIGUITY because /k/ is skipped.

(42) (a) I-CON$(\mathbf{b}, \mathbf{c})=0$ (b) I-CON$(\mathbf{a}, \mathbf{c})=1$ (c) I-CON$(\mathbf{a}, \mathbf{b})=0$



[15] McCarthy & Prince (1995) consider a slightly different definition, whereby I-CONTIGUITY assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ a number of violations that is equal to 1 (equal to 0) if the candidate has at least one (does not have any) skipped segments. The choice between the two alternative definitions of I-CONTIGUITY is irrelevant to the point made in this subsection that it does not satisfy the FIC$_{\text{comp}}$, since the candidates in the counterexample (42) have no more than one skipped segment, so that the two definitions collapse.

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As McCarthy & Prince (1995) note, I-CONTIGUITY can in most applications be re-defined as penalizing the deletion of *internal* segments, namely underlying segments that are flanked on both sides by other underlying segments, no matter whether these flankers have correspondents. The counterexample (42) shows that the constraint thus re-defined fails at the $\mathrm{FIC}_{\mathrm{comp}}$ as well. This is not surprising, because the constraint thus redefined coincides with MAX_R, where the restriction R pairs a string with the set of its internal segments. As seen in Section 5.1, MAX_R fails at the $\mathrm{FIC}_{\mathrm{comp}}$ when the correspondence relations can exit from the restriction R. That is precisely the case in the counterexample (42), as the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$ establishes a correspondence between underlying /k/ (which satisfies the restriction R because it is internal to $\mathbf{a}$) and surface [k] (which does not satisfy the restriction R because it is not internal to $\mathbf{b}$). Analogous considerations hold for O-CONTIGUITY.

6.4 ADJACENCY

Carpenter (2002) argues that the faithfulness constraint I-CONTIGUITY should be replaced with O-ADJACENCY. The latter assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for any two pairs $(\mathsf{a}_1, \mathsf{b}_1)$, $(\mathsf{a}_2, \mathsf{b}_2)$ of underlying segments $\mathsf{a}_1, \mathsf{a}_2$ and surface segments $\mathsf{b}_1, \mathsf{b}_2$ that are in correspondence according to $\rho_{\mathbf{a},\mathbf{b}}$ despite the fact that $\mathsf{b}_1, \mathsf{b}_2$ are adjacent in the surface string $\mathbf{b}$ while $\mathsf{a}_1, \mathsf{a}_2$ are not adjacent in the underlying string $\mathbf{a}$. The faithfulness constraint I-ADJACENCY is defined analogously, by looking at adjacency relative to the underlying string. To appreciate the difference between I-CONTIGUITY and O-ADJACENCY, consider again the counterexample (42) used to show that I-CONTIGUITY fails at the $\mathrm{FIC}_{\mathrm{comp}}$. This counterexample raises no problems for O-ADJACENCY. The crucial difference is that O-ADJACENCY assigns one violation to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (42c), because of the two pairs of corresponding segments (/t/, [t]) and (/a/, [a]). Indeed, despite the fact that O-ADJACENCY and I-CONTIGUITY are shown by Carpenter to do much of the same work, they differ with respect to the $\mathrm{FIC}_{\mathrm{comp}}$: I-CONTIGUITY fails at the $\mathrm{FIC}_{\mathrm{comp}}$, as seen in the preceding subsection; O-ADJACENCY instead satisfies the $\mathrm{FIC}_{\mathrm{comp}}$, as stated by the following lemma, whose simple verification is omitted for brevity.

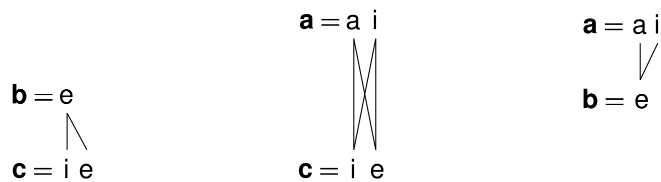
Lemma 9. *The faithfulness constraints* O-ADJACENCY *and* I-ADJACENCY *satisfy the FIC_{comp} (21) provided that no correspondence relation in the candidate set breaks any underlying segment.* □

6.5 LINEARITY

The faithfulness constraint LINEARITY penalizes metathesis. McCarthy (2008: 198) defines this constraint as follows: $\mathrm{LINEARITY}_{\mathrm{McCarthy}}$ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each pair of underlying segments a_1 and a_2 that admit two swapped surface correspondents, namely there exist two surface segments b_1 and b_2 such that a_1 corresponds through $\rho_{\mathbf{a},\mathbf{b}}$ to b_1, a_2 corresponds to

b_2, and yet a_1 precedes a_2 while b_1 follows b_2. Heinz (2005) offers the following alternative definition: LINEARITY$_{\text{Heinz}}$ assigns to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ one violation for each pair of underlying segments a_1 and a_2 that admit no non-swapped surface correspondents, namely there exist no two surface segments b_1 and b_2 such that a_1 corresponds through $\rho_{\mathbf{a},\mathbf{b}}$ to b_1, a_2 corresponds to b_2, and both a_1 precedes a_2 and b_1 precedes b_2.[16,17] The faithfulness constraint LINEARITY$_{\text{McCarthy}}$ fails at the FIC$_{\text{comp}}$ when the candidate set allows *both* coalescence and breaking, as shown by the counterexample (43). The antecedent of the FIC$_{\text{comp}}$ holds: the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ in (43a) does not violate LINEARITY$_{\text{McCarthy}}$ because it has a unique underlying segment. The right-hand side of the FIC$_{\text{comp}}$ inequality is small, namely equal to zero: the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ in (43c) does not violate LINEARITY$_{\text{McCarthy}}$, because it has a unique surface segment. The FIC$_{\text{comp}}$ inequality fails because its left-hand side is large, namely equal to 1: the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ in (43b) violates LINEARITY$_{\text{McCarthy}}$ because the two underlying segments /a/ and /i/ are in correspondence with the two surface segments [e] and [i] which have the opposite linear order.[18]

(43) (a) LIN$_{\text{MC}}(\mathbf{b}, \mathbf{c})=0$ (b) LIN$_{\text{MC}}(\mathbf{a}, \mathbf{c})=1$ (c) LIN$_{\text{MC}}(\mathbf{a}, \mathbf{b})=0$



This counterexample does not threaten LINEARITY$_{\text{Heinz}}$. Although the composition candidate (43b) violates LINEARITY$_{\text{McCarthy}}$, because the two underlying segments admit swapped surface segments, it does not violate LINEARITY$_{\text{Heinz}}$, because the two underlying segments also admit non-swapped correspondents. These considerations lead to the following lemma, whose proof is a simple verification which is omitted for brevity.

---

[16] Heinz suggests a further departure from McCarthy's formulation, namely the replacement of precedence with *immediate* precedence. The difference has implications for the comparison between short and long distance metathesis (Hume 1998: Section 4; Heinz 2005). Yet, the difference between precedence and immediate precedence has no implications for establishing the FIC$_{\text{comp}}$, and I thus ignore it here.

[17] McCarthy's formulation of LINEARITY counts over pairs of underlying segments a_1, a_2 that admit swapped surface correspondents. I am thus comparing it here with what Heinz calls MAXCONTIGUITY. Heinz also considers a constraint DEPCONTIGUITY, which is defined analogously by counting over pairs of surface segments with no non-swapped underlying correspondents.

[18] LINEARITY$_{\text{McCarthy}}$ counts over underlying segments, just like MAX. Based on the discussion in Section 4, one might thus have expected LINEARITY$_{\text{McCarthy}}$ to satisfy the FIC$_{\text{comp}}$ without requiring additional assumptions on the correspondence relations, just as MAX. The difference lies in the fact that MAX counts over *single* segments while LINEARITY$_{\text{McCarthy}}$ counts over *pairs* of segments.

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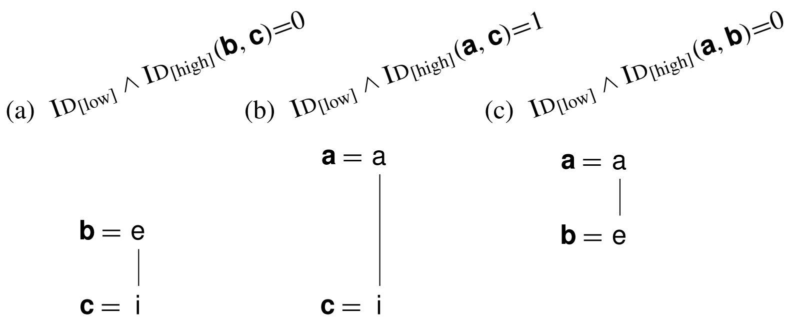
Lemma 10. *The faithfulness constraint* LINEARITY$_{Heinz}$ *satisfies the FIC$_{comp}$ (21) under no additional assumptions on the correspondence relations. The faithfulness constraint* LINEARITY$_{McCarthy}$ *satisfies the FIC$_{comp}$ (21) provided that no correspondence relation in the candidate set breaks any underlying segment into multiple surface segments or else no correspondence relation coalesces multiple underlying segments into a single surface segment.* □

McCarthy & Prince's (1995) CONTIGUITY and Carpenter's (2002) ADJACENCY are closely related constraints meant to serve the same purpose. The same holds for McCarthy's (2008) and Heinz's (2005) slightly different implementations of LINEARITY constraints. The discussion in the last two subsections has shown that these small differences in the definition of the constraints can have substantial formal consequences for idempotency.

6.6 *Constraint conjunction and disjunction*

The OT literature has made use of constraints defined as boolean combinations of other constraints (Crowhurst & Hewitt 1997, Wolf 2007). Two boolean operations that have figured prominently are constraint *conjunction* (Smolensky 1995, Moreton & Smolensky 2002) and *disjunction* (Downing 1998, 2000). Constraint conjunction fails at the FIC$_{\text{comp}}$, as shown by the counterexample in (44). The conjoined constraint IDENT$_{[\text{low}]}$ $\wedge$ IDENT$_{[\text{high}]}$ assigns one violation for each pair of corresponding segments that differ for *both* features [low] and [high]. The antecedent of the FIC$_{\text{comp}}$ holds: the candidate ($\mathbf{b}$, $\mathbf{c}$, $\rho_{\mathbf{b},\mathbf{c}}$) in (44a) does not violate the conjoined constraint, because /e/ and [i] only differ for the feature [high]. The right-hand side of the FIC$_{\text{comp}}$ inequality is small, namely equal to zero: the candidate ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$) in (44c) does not violate the conjoined constraint, because /a/ and [e] only differ for the feature [low]. The FIC$_{\text{comp}}$ inequality fails because its left-hand side is large, namely equal to 1: the composition candidate ($\mathbf{a}$, $\mathbf{c}$, $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$) in (44b) violates the conjoined constraint, because /a/ and [i] differ for both features [low] and [high].

(44)



The case of constraint disjunction is different. For concreteness, consider the disjunction IDENT$_\varphi$ $\vee$ IDENT$_\psi$ of two identity faithfulness constraints IDENT$_\varphi$ and IDENT$_\psi$ corresponding to two (total) features φ and ψ. This constraint assigns

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one violation for each pair of corresponding segments that differ for either feature φ or ψ (possibly both). Lemma 11 ensures that it satisfies the FIC_{comp}. The proof is a straightforward verification, which is omitted for brevity. This conclusion easily extends to the disjunction of other (disjoinable) faithfulness constraints: conditions on the FIC_{comp}-compliance of the constraint disjunction follow by combining conditions on the FIC_{comp}-compliance of the faithfulness constraints being combined in the disjunction.[19]

Lemma 11. *Assume that the features φ and ψ are total and that correspondence relations are not allowed to break any underlying segment. The disjunctive faithfulness constraint* $\text{IDENT}_{\varphi} \vee \text{IDENT}_{\psi}$ *then satisfies the FIC_{comp} (21).* □

The difference between constraint conjunction and disjunction with respect to the FIC_{comp} can be appreciated as follows. Suppose that the antecedent of the FIC_{comp} holds for the disjunction $\text{IDENT}_{[\text{low}]} \vee \text{IDENT}_{[\text{high}]}$. This means that the candidate ($\mathbf{b}$, $\mathbf{c}$, $\rho_{\mathbf{b},\mathbf{c}}$) does not violate it. This entails in turn that the candidate violates neither $\text{IDENT}_{[\text{low}]}$ nor $\text{IDENT}_{[\text{high}]}$. The FIC_{comp} for the disjunction thus follows from the FIC_{comp} previously established for the individual disjuncts. The case of conjunction is different: even if the candidate ($\mathbf{b}$, $\mathbf{c}$, $\rho_{\mathbf{b},\mathbf{c}}$) does not violate the conjunction $\text{IDENT}_{[\text{low}]} \wedge \text{IDENT}_{[\text{high}]}$ as required by the antecedent of the FIC_{comp}, it could nonetheless violate one of the two conjuncts $\text{IDENT}_{[\text{low}]}$ or $\text{IDENT}_{[\text{high}]}$. The fact that the conjuncts satisfy the FIC_{comp} thus provides no guarantees that their conjunction satisfies it as well.

7. Implications of idempotency for phonology

A grammar is idempotent provided that it displays no chain shifts, as explained in Section 2.4. Chain shifts have been widely documented in adult phonology (Moreton & Smolensky 2002, Moreton 2004a, Łubowicz 2011), child phonology (Velten 1943; Smith 1973; Macken 1980; Dinnsen & Barlow 1998; Cho & Lee 2000, 2003; Dinnsen, O'Connor & Gierut 2001; Jesney 2007), second language acquisition (Lee 2000, Jesney 2007), and delayed phonological acquisition (Dinnsen & Barlow 1998, Dinnsen, Green, Gierut & Morrisette 2011). Various approaches to chain shifts within Optimality Theory can be sorted into two groups. Some approaches trade some of the assumptions of (classical) OT, reviewed in Section 3.2. Such approaches include sympathy theory (McCarthy 1999), output–output correspondence (Burzio 1998, Benua 2000), targeted constraints (Wilson 2001), turbidity (Goldrick 2001), anti-faithfulness constraints (Alderete 2001, 2008), comparative markedness (McCarthy 2003a), candidate

[19] The situation is rather different for the disjunction of a faithfulness and a markedness constraint. Such a disjunction can yield a constraint that is neither a faithfulness nor a markedness constraint (see Wolf 2007 for broader discussion), contrary to what is required by the *classical* implementation of OT defined in Section 3.2.

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chains (McCarthy 2007), stratal OT (Bermúdez-Otero 2007), and contrast preservation constraints (Łubowicz 2012). Other OT approaches to chain shifts are instead framed squarely within the classical architecture. This section provides an overview of the latter approaches from the formal perspective of the results established in Sections 3–6.

7.1 *Only a sufficient condition?*

Consider an arbitrary (reflexive and transitive) candidate set, an arbitrary constraint set, and an arbitrary constraint ranking. Section 3 has established that the FIC_{comp} is a *sufficient* condition for the idempotency of the corresponding OT grammar. This statement contains three universal quantifications: over candidate sets, over constraint sets, and over rankings. At this level of generality, the FIC_{comp} is not only a sufficient but also a *necessary* condition for idempotency, in the following sense. Consider a faithfulness constraint F that does not satisfy the FIC_{comp} (21). This means that there exist two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ such that F assigns no violations to the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$, so that the antecedent of the FIC_{comp} holds; yet, F assigns more violations to the candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ than to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, so that the consequent fails. Suppose that the constraint set also contains a markedness constraint M that assigns more violations to the surface form $\mathbf{b}$ than to the surface form $\mathbf{c}$.[20] The OT grammar corresponding to the ranking $F \gg M$ displays the chain shift $\mathbf{a} \rightarrow \mathbf{b} \rightarrow \mathbf{c}$ and thus fails at idempotency: the string $\mathbf{b}$ is phonotactically licit, because the underlying form $\mathbf{a}$ is mapped to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, as shown in the left-hand side tableau in (45); yet, the string $\mathbf{b}$ does not surface faithfully, because the underlying form $\mathbf{b}$ is not mapped to the identity candidate $(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$, as shown in the right-hand side tableau.

(45)

	$\mathbf{a}$	F	M
☞	$(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$	$* \cdots *$	$*$
	$(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$	$* \cdots * *!$	

	$\mathbf{b}$	F	M
	$(\mathbf{b}, \mathbf{b}, \mathbb{I}_{\mathbf{b},\mathbf{b}})$		$*$
☞	$(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$		

The rest of this section shows how various approaches to chain shifts in the classical OT literature fit within the schema (45), where F is one of the faithfulness constraints shown in Sections 4–6 to fail the FIC_{comp}.

7.2 *Chain shifts through constraint conjunction*

As noted in Section 6.6, constraint conjunction yields faithfulness constraints that fail the FIC_{comp}. The use of constraint conjunction to model chain shifts within

[20] Furthermore, assume that there exists a markedness constraint (either M or a different markedness constraint ranked above M) that assigns more violations to the surface form $\mathbf{a}$ than to the surface form $\mathbf{b}$. The latter markedness constraint is responsible for ruling out the candidate $(\mathbf{a}, \mathbf{a}, \mathbb{I}_{\mathbf{a},\mathbf{a}})$, which is therefore ignored in the rest of this section.

classical OT has been pioneered by Kirchner (1996) and systematized by Moreton & Smolensky (2002). To illustrate, consider the chain shift in the Lena dialect of Spanish in (46) (data from Hualde 1989 and Gnanadesikan 1997: Section 5.4.3): because of the high final vowel in the masculine form, the underlying low and mid vowels (as revealed by the feminine form) raise to mid and high vowels, respectively.

(46)	/a/→[e]	gáta 'cat-FEM'	gétu 'cat-MAS'
		sánta 'saint-FEM'	séntu 'saint-MAS'
	/e/→[i]	néna 'child-FEM'	nínu 'child-MAS'
		séks 'dry-FEM'	síku 'dry-MAS'

The markedness constraint RAISE favors higher vowels before a high vowel. The conjunction of the two faithfulness constraints $\text{IDENT}_{[\text{high}]}$ and $\text{IDENT}_{[\text{low}]}$ penalizes underlying low vowels mapped to surface high vowels. The analysis (47) is an instance of the scheme (45), with the conjoined constraint playing the role of the non-FIC_{comp}-complying faithfulness constraint.

(47)

/gátu/	$\text{ID}_{[\text{high}]} \wedge \text{ID}_{[\text{low}]}$	RAISE
☞ [gétu]		*
[gítu]	*!	

/nénu/	$\text{ID}_{[\text{high}]} \wedge \text{ID}_{[\text{low}]}$	RAISE
[nénu]		*!
☞ [nínu]		

Other approaches proposed in the literature are equivalent to the approach based on the conjunction of identity faithfulness constraints (see also the discussion of V-HEIGHTDISTANCE in Kirchner 1995). For instance, Gnanadesikan (1997: chapter 3) accounts for the chain shift p → b → m (in post-nasal position) through the faithfulness constraint IDENT-ADJ, which is violated by a voiceless obstruent and a corresponding sonorant because they are separated by a distance larger than 2 on the *inherent voicing scale*. This constraint is thus equivalent to the conjunction $\text{IDENT}_{[\text{voice}]} \wedge \text{IDENT}_{[\text{son}]}$. Analogously, Dinnsen & Barlow (1998) account for the chain shift s → θ → f through the faithfulness constraint DISTFAITH, which is violated when the underlying and surface forms differ by more than 1 on the scale f = 1, θ = 2, and s = 3 and is thus equivalent to the conjunction $\text{IDENT}_{[\text{coronal}]} \wedge \text{IDENT}_{[\text{strident}]}$.

Yet, the approach based on constraint conjunction is more general than the latter approaches based on 'scales', as the former but not the latter extends to chain shifts that involve deletion (Moreton & Smolensky 2002). To illustrate, consider the Sea Dayak chain shift in (48) (data from Kenstowicz & Kisseberth 1979).

(48)	/ŋga/ → [ŋa]:	/naŋga/	[nãŋaʔ]	'set up a ladder'
	/ŋa/ → [ŋã]:	/naŋa/	[nãŋãʔ]	'straighten'

The analysis based on constraint conjunction extends as in (47) (based on Łubowicz 2011), which is another instance of the scheme (45).

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(49)

/naŋga/	ID$_{[nas]}$ ∧ MAX	*NV
☞ [nãŋaʔ]		*
[nãŋãʔ]	*!	

/naŋa/	ID$_{[nas]}$ ∧ MAX	*NV
[nãŋaʔ]		*!
☞ [nãŋãʔ]		

7.3 *Chain shifts through constraint restrictions*

As noted in Section 5, constraint restriction yields faithfulness constraints that fail the FIC$_{comp}$ when the correspondence relations are allowed to 'cross' the restriction. This observation systematizes various approaches to chain shifts proposed in the classical OT literature. For instance, Orgun (1995) considers the chain shift in (50) from Bedouin Hijazi Arabic: /a/ is raised to [i] but /i/ is deleted (both processes are restricted to short vowels in non-final open syllables; the data come from McCarthy 1993).

(50)	/a/ → [i]:	/katab/	[kitab]	'he wrote'
		/rafaagah/	[rifaagah]	'companions'
	/i/ → Ø:	/ʕarif+at/	[ʕarfat]	'she knew'
		/kitil/	[ktil]	'he was killed'

Orgun's analysis is summarized in (51), plus a markedness constraint [*a] which is omitted here. It relies on his constraint CORRESPOND(/a/), which mandates that 'every input /a/ has an output correspondent'. This constraint can be re-interpreted as MAX$_R$, where the restriction R pairs a string with the set of its a's. As shown in Section 5.1, MAX$_R$ fails at the FIC$_{comp}$ when correspondence relations are allowed to exit from the restriction R, namely to put in correspondence an underlying segment that satisfies the restriction with a surface segment that does not, as described in (33). That is precisely the case in (51), as the underlying /a/ (which satisfies the restriction R) corresponds to the surface [i] (which does not satisfy the restriction). Orgun's analysis (51) is thus an instance of the scheme (45), with the restricted constraint MAX$_R$ playing the role of the non-FIC$_{comp}$-complying faithfulness constraint.

(51)

/a/	CORR(/a/)	*V
☞ [i]		*
Ø	*!	

/i/	CORR(/a/)	*V
[i]		*!
☞ Ø		

As another example, Jesney (2005, 2007) considers the classical child chain shift in (52): coronal stridents are realized as coronal stops across the board, but coronal stops are velarized when followed by a lateral (data from Amahl age 2;2-2;11, as described in Smith 1973).

(52)	/s, z, ʃ, ʒ, tʃ, dʒ/ → [t, d]:	[pʌdəl] 'puzzle'		[paːtliː]	'parsley'
		[pɛtəl] 'special'		[ændələ]	'Angela'
	/t, d, n/ → [k, g, ŋ]:	[pʌgəl] 'puddle'		[bʌklə]	'butler'
		[tæŋgəl] 'sandal'		[bɔkəl]	'bottle'

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Jesney's analysis is summarized in (53). It relies on her 'specific' faithfulness constraint IDENTCORONAL/[+strident], which mandates that input stridents preserve their coronality. This constraint can be re-interpreted as $\text{IDENT}_{\varphi,R}$, where φ is the feature [coronal] and the restriction R pairs a string with the set of its stridents. As shown in Section 5.5, $\text{IDENT}_{\varphi,R}$ fails at the FIC_{comp} when correspondence relations are allowed to exit from the restriction R without changing the value of the feature φ, namely to put in correspondence an underlying segment that satisfies the restriction with a surface segment that does not and yet has the same value for the feature φ, as described in (37). That is precisely the case in (53), as the underlying /s/ (which satisfies the restriction R) corresponds to the surface [t] (which does not satisfy the restriction) and yet they are both coronals. Jesney's analysis (53) is thus an instance of the scheme (45), with the restricted constraint $\text{IDENT}_{[\text{cor}],R}$ playing the role of the non-FIC_{comp}-complying faithfulness constraint.[21]

(53)

/s/	IDCOR/[+strid]	*TL
☞ [t]		*
[k]	*!	

/t/	IDCOR/[+strid]	*TL
[t]		*!
☞ [k]		

7.4 *Chain shifts through breaking*

Let me close this section by discussing a fictional example. Kubozono et al. (2008) report that English *frog* is imported as [fu.róg.gu] into Japanese: the velar stop geminates (despite being voiced) because of a requirement on the placement of stress, captured here through a place-holder constraint STRESS. Assume an analysis of consonant gemination in terms of breaking of a single underlying consonant into two surface copies, as indicated by the correspondence relations in (54). Section 4.3 has shown that plain identity faithfulness constraints fail at the FIC_{comp} when the correspondence relations are allowed to break underlying segments. This fact could be used to derive a fictional chain shift such as ŋ → g → gg through the analysis (54), which is an instance of the scheme (45) with the identity constraint playing the role of the non-FIC_{comp} faithfulness constraint.

(54)

/ŋ/	$\text{IDENT}_{[\text{nas}]}$	STRESS
☞ (ŋ, g)	*	*
(ŋ, gg)	**!	

/g/	$\text{IDENT}_{[\text{nas}]}$	STRESS
(g, g)		*!
☞ (g, gg)		

[21] Assume that only coronals can be [+strident], while all non-coronals are [-strident]. Jesney's constraint IDENTCORONAL/[+strident] is then provably equivalent to the conjoined constraint $\text{IDENT}_{[\text{strident}]} \wedge \text{IDENT}_{[\text{coronal}]}$.

This example is fictional because I have not been able to find a realistic case of chain shift that involves an underlying segment broken into two surface segments.

7.5 *Summary*

Section 3 has shown that chain shifts require the FIC_{comp} to fail. Based on Sections 4–6, there are three major ways for the FIC_{comp} to fail. One option is to use a faithfulness constraint that flouts the FIC_{comp}, such as a faithfulness constraint obtained through constraint conjunction. A second option is to use the restriction of a faithfulness constraint that would otherwise comply with the FIC_{comp}. The third option is to let the correspondence relations break underlying segments. The former two options have been exploited in the literature on chain shifts.

8. Conclusions and extensions

A grammar is idempotent provided that it faithfully maps to itself any phonotactically licit phonological form. Equivalently, a grammar fails at idempotency provided that it displays at least one chain shift. Within constraint-based phonology, the typology of grammars is defined through a constraint set and a candidate set. Formal grammatical conditions such as idempotency must therefore be derivable from assumptions on the constraint and the candidate sets. This paper has pursued this idea within the (classical) OT implementation of constraint-based phonology. Building on Tesar's (2013) theory of output-drivenness, the theory of idempotency has been developed in this paper through two steps. First, Lemma 1 has distilled the FIC_{comp} as a general condition on the faithfulness constraints which suffices to ensure idempotency. Second, Lemmas 2–11 have established the FIC_{comp} for a number of faithfulness constraints that naturally arise within McCarthy & Prince's (1995) Correspondence Theory, under various assumptions on the correspondence relations in the candidate set. The overall picture obtained by combining these lemmas is summarized in the following theorem, which is the main result of this paper.

Theorem 1. *Consider a candidate set that consists of triplets $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ of an underlying segment string $\boldsymbol{a}$ and a surface segment string $\boldsymbol{b}$ together with a correspondence relation $\rho_{\boldsymbol{a},\boldsymbol{b}}$ between the segments of $\boldsymbol{a}$ and those of $\boldsymbol{b}$. Assume that this candidate set satisfies the reflexivity axiom (6) and the transitivity axiom (20), repeated below in (55) and (56).*

(55) *If the candidate set contains a candidate $(\boldsymbol{a}, \boldsymbol{b}, \rho_{\boldsymbol{a},\boldsymbol{b}})$ with a surface form $\boldsymbol{b}$, it also contains the corresponding identity candidate $(\boldsymbol{b}, \boldsymbol{b}, \mathbb{I}_{\boldsymbol{b},\boldsymbol{b}})$, where $\mathbb{I}_{\boldsymbol{b},\boldsymbol{b}}$ is the identity correspondence relation among the segments of $\boldsymbol{b}$.*

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(56) *If the candidate set contains two candidates* ($\boldsymbol{a}$, $\boldsymbol{b}$, $\rho_{\boldsymbol{a},\boldsymbol{b}}$) *and* ($\boldsymbol{b}$, $\boldsymbol{c}$, $\rho_{\boldsymbol{b},\boldsymbol{c}}$) *that share* $\boldsymbol{b}$ *as the surface and underlying forms, it also contains the composition candidate* ($\boldsymbol{a}$, $\boldsymbol{c}$, $\rho_{\boldsymbol{a},\boldsymbol{b}}\rho_{\boldsymbol{b},\boldsymbol{c}}$) *whose correspondence relation* $\rho_{\boldsymbol{a},\boldsymbol{b}}\rho_{\boldsymbol{b},\boldsymbol{c}}$ *is the composition of* $\rho_{\boldsymbol{a},\boldsymbol{b}}$ *and* $\rho_{\boldsymbol{b},\boldsymbol{c}}$.

Assume that no correspondence relation breaks any underlying segment, namely puts it in correspondence with two or more surface segments. Assume furthermore that the constraint set only contains faithfulness constraints drawn from the following list:

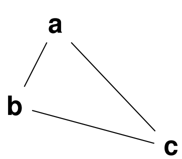
(57) (a) IDENT_{φ} *(when the feature* φ *is total, not necessarily binary);*
(b) *segmental* MAX *and* DEP*;*
(c) *featural* $\text{MAX}_{[\pm\varphi]}$ *and* $\text{DEP}_{[\pm\varphi]}$ *(for any feature* φ*);*
(d) UNIFORMITY*,* LINEARITY*, and* ADJACENCY.

The OT grammar corresponding to any ranking of this constraint set is idempotent. This conclusion extends to the case where the constraint set contains restricted variants of these constraints (such as MAX_R*,* DEP^S*,* $\text{IDENT}_{\varphi,R}$*, or* IDENT^S_{φ}*), as long as no correspondence relation crosses the restrictions, namely puts in correspondence a segment that satisfies the restrictions with a segment that does not satisfy them.* □

Theorem 1 provides the basis for further developments of the theory of idempotency in Magri (to appear, 2015), briefly sketched in the rest of this section.

8.1 *The FIC_{comp} and the metrical nature of the faithfulness constraints*

Theorem 1 places no restrictions on the markedness constraints and only looks at the faithfulness constraints. This result thus motivates a deeper look into the formal underpinning of the Correspondence Theory of faithfulness. Intuitively, faithfulness constraints measure the 'distance' between underlying and surface forms along various phonologically relevant dimensions. It thus makes sense to investigate whether faithfulness constraints satisfy formal properties of distances (or *metrics*). One such important property is the *triangle inequality* (58): it captures the intuition that the distance between any two points **a** and **c** is shorter than the distance between **a** and **b** plus the distance between **b** and **c**, for any choice of the intermediate point **b** (Rudin 1953).

(58)


$$\textit{distance}(\mathbf{a}, \mathbf{c}) \leqslant \textit{distance}(\mathbf{a}, \mathbf{b}) + \textit{distance}(\mathbf{b}, \mathbf{c})$$

A faithfulness constraint F is thus said to satisfy the *faithfulness triangle inequality* provided that condition (59) holds for any two candidates ($\mathbf{a}$, $\mathbf{b}$, $\rho_{\mathbf{a},\mathbf{b}}$)

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and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ that share the string $\mathbf{b}$ as the underlying and the surface form, respectively.

(59) $F\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leqslant F\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big) + F\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big)$

Magri (to appear) establishes an equivalence result between the triangle inequality (59) and the condition $\mathrm{FIC}_{\mathrm{comp}}$, which was established in this paper to be sufficient for idempotency. This equivalence thus provides an intuitive metric interpretation of the apparently very technical $\mathrm{FIC}_{\mathrm{comp}}$. This equivalence requires two assumptions: first, that the correspondence relations in the candidate set are one-to-one (no breaking nor coalescence); second, that the faithfulness constraints are *categorical*, in the sense of McCarthy (2003b). Building on Lemmas 2–11 established in this paper, this equivalence between the $\mathrm{FIC}_{\mathrm{comp}}$ and the triangle inequality yields a straightforward characterization of the faithfulness constraints that satisfy the triangle inequality and thus admit a metric interpretation.

8.2 *Idempotency in Harmonic Grammar*

This paper has focused on idempotency within the OT implementation of constraint-based phonology. Magri (to appear) extends the theory of idempotency to *Harmonic Grammar* (HG; Legendre et al. 1990a, b; Smolensky & Legendre 2006). Also in HG, idempotency is guaranteed by a condition on the faithfulness constraints, which can be referred to as the $\mathrm{FIC}^{\mathrm{HG}}_{\mathrm{comp}}$, to distinguish it from the OT condition obtained in this paper, which I will now refer to as the $\mathrm{FIC}^{\mathrm{OT}}_{\mathrm{comp}}$. In the general case, the $\mathrm{FIC}^{\mathrm{HG}}_{\mathrm{comp}}$ asymmetrically entails the $\mathrm{FIC}^{\mathrm{OT}}_{\mathrm{comp}}$. This makes sense: HG typologies properly contain OT typologies[22], so that a stronger condition is needed in order to discipline a larger typology of grammars to all comply with idempotency. The asymmetric relationship between the $\mathrm{FIC}^{\mathrm{HG}}_{\mathrm{comp}}$ and the $\mathrm{FIC}^{\mathrm{OT}}_{\mathrm{comp}}$ is revealed, for instance, by the following fact: while the no-breaking assumption on correspondence relations suffices to ensure that basic faithfulness constraints (such as DEP and IDENT) satisfy the $\mathrm{FIC}^{\mathrm{OT}}_{\mathrm{comp}}$, the $\mathrm{FIC}^{\mathrm{HG}}_{\mathrm{comp}}$ also requires no coalescence, effectively restricting all correspondence relations in the candidate set to be one-to-one. Yet, when correspondence relations are one-to-one, the $\mathrm{FIC}^{\mathrm{HG}}_{\mathrm{comp}}$ and the $\mathrm{FIC}^{\mathrm{OT}}_{\mathrm{comp}}$ can be shown to be equivalent for faithfulness constraints that are categorical in the sense of McCarthy (2003b), because both conditions are equivalent to the triangle inequality. McCarthy's categoricity conjecture thus entails that, although HG idempotency requires additional conditions on the correspondence relations relative to OT idempotency (the former requires neither breaking nor coalescence; the latter only requires no breaking), it effectively places no additional restrictions on the faithfulness constraints.

[22] For any set of constraints with bounded violations.

8.3 *Idempotency and output-drivenness*

Tesar (2013) investigates another structural condition on phonological grammars, which he calls *output-drivenness*. It formalizes the intuition that any discrepancy between an underlying and a surface (or *output*) form is *driven* exclusively by the goal of making the surface form fit the phonotactics. Tesar shows that output-drivenness holds provided that the faithfulness constraints satisfy two implications which generalize the FIC and are called the *faithfulness output-drivenness condition* (FODC). He then investigates which faithfulness constraints satisfy the FODC in the special case where all correspondence relations are one-to-one. The answer to this question is non-trivial. For instance, two pages of Tesar's book suffice to establish the FODC as a sufficient condition for OT output-drivenness, while the entire chapter 3 is devoted to verifying the FODC for just the three constraints MAX, DEP, and IDENT.

The results established in this paper on the FIC afford a substantial simplification of Tesar's theory. In fact, Magri (to appear) looks at the relationship between idempotency and output-drivenness and between the two corresponding sufficient conditions, the FIC and the FODC. In the general case, the FODC is stronger than the FIC, matching the fact that output-drivenness is a stronger condition than idempotency, as shown by *derived environment effects* or *saltations* (Łubowicz 2002, White 2013), which are idempotent but not output-driven. Yet, the FODC and the FIC are shown to be equivalent when the correspondence relations are all one-to-one (as assumed by Tesar) and the faithfulness constraints are all *categorical* (as conjectured in McCarthy 2003b), because both conditions are equivalent to the triangle inequality mentioned in Section 8.1. Crucially, all of the faithfulness constraints analyzed in this paper are categorical. The equivalence result between the FIC and the FODC thus allows the results obtained here concerning the faithfulness constraints that satisfy the FIC to be translated straightforwardly into results concerning the faithfulness constraints that satisfy the FODC. A measure of the improvement obtained is provided by the fact that a large array of faithfulness constraints (beyond the three faithfulness constraints considered by Tesar) are shown in a snap to satisfy the FODC.

8.4 *Benign chain shifts*

Magri (2015) explores the implications of the theory of idempotency developed in this paper for OT learnability. The literature on the early acquisition of phonotactics usually assumes that the child posits a fully faithful underlying form for each phonotactically licit training surface form (Gnanadesikan 2004, Hayes 2004, Prince & Tesar 2004). Is this assumption of faithful underlying forms computationally sound? Suppose that the target grammar is not idempotent because it displays the chain shift (60a). The form **b** is phonotactically licit (because it is the surface realization of **a**) and yet it is not faithfully mapped to itself. The assumption that every instance of **b** in the training set is the surface realization of a faithful underlying form could thus mislead the learner into positing a set of mappings that is not consistent with any grammar in the typology.

(60) (a)
$$\begin{tikzcd} \mathbf{a} \arrow[r] & \mathbf{b} \arrow[r] & \mathbf{c} \arrow[loop right] \end{tikzcd}$$

(b)
$$\begin{tikzcd} \mathbf{a} \arrow[r] & \mathbf{b} \arrow[loop right] & \mathbf{c} \arrow[loop right] \end{tikzcd}$$

Yet, the chain shift (60a) raises no issues for the learner's assumption of fully faithful underlying forms as long as the chain shift happens to be *benign*, in the sense that the typology entertained by the learner happens to contain the *companion* grammar (60b), which is idempotent (there is no chain shift) and yet makes the same phonotactic distinctions (**a** is illicit while **b** and **c** are licit for both grammars). Under which conditions are chain shifts benign?

The theory of idempotency developed in this paper and summarized above as Theorem 1 says that *basic* faithfulness constraints all satisfy the FIC_{comp}, thus explaining the well-known difficulty in modeling chain shifts within (classical) OT. In order to flout the FIC_{comp} and thus be able to derive chain shifts, we need to look at *derived* faithfulness constraints. As seen in Section 7, there are two main strategies to construct these derived constraints: through the restriction of some basic and thus FIC_{comp}-complying faithfulness constraint (such as MAX_R) or through the conjunction of two basic and thus individually FIC_{comp}-complying faithfulness constraints (such as the conjunction $\text{IDENT}_{[\text{high}]} \wedge \text{IDENT}_{[\text{low}]}$). Magri (2015) then formulates the conjecture that, whenever a chain shift (60a) is obtained through a ranking where one of these derived constraints occupies a prominent position, the ranking with the derived faithfulness constraint replaced by the corresponding basic one (say, MAX_R replaced by MAX, or $\text{IDENT}_{[\text{high}]} \wedge \text{IDENT}_{[\text{low}]}$ replaced by either $\text{IDENT}_{[\text{high}]}$ or $\text{IDENT}_{[\text{low}]}$) yields the idempotent and phonotactically equivalent companion grammar (60b). The exploration of this conjecture would provide a solid foundation for a variety of models of the acquisition of phonotactics that share the assumption of completely faithful underlying forms.

APPENDIX

Proofs

Throughout this appendix, I consider three strings **a**, **b**, and **c**, whose generic segments are denoted by a, b, and c. I use statements such as 'for every/some segment a' as a shorthand for 'for every/some segment a *of the string* **a**', thus leaving the domain of the quantifiers implicit.

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A.1 Proof of Lemma 2

A segment a violates the constraint $\textsc{Max}^S_R$ relative to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ provided that a belongs to $R(\mathbf{a})$ and it has no $\rho_{\mathbf{a},\mathbf{b}}$-correspondent in the surface string $\mathbf{b}$ that belongs to $S(\mathbf{b})$. $\textsc{Max}^S_R$ assigns one violation for each underlying segment that violates it. This appendix proves Lemma 2 repeated below, which establishes the FIC_{comp} for $\textsc{Max}^S_R$.

Lemma A.1. *The faithfulness constraint* $\textsc{Max}^S_R$ *satisfies the* FIC_{comp}

(61) **If:** $\textsc{Max}^S_R(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}) = 0$

Then: $\textsc{Max}^S_R(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}) \leqslant \textsc{Max}^S_R(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$

for any two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ *and* $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ *such that the former candidate* $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ *has no underlying segment* a *and no surface segment* b *such that*

(62) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{a} \in R(\mathbf{a}), \quad \mathsf{b} \notin R(\mathbf{b}), \quad \mathsf{b} \in S(\mathbf{b})$ □

Proof. Assume that the antecedent of the implication (61) holds, namely that the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate $\textsc{Max}^S_R$. The following chain of implications (63) then holds for any segment a of the string $\mathbf{a}$. In step (63a), I have used the definition of $\textsc{Max}^S_R$. In step (63b), I have used the definition of the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$. In step (63c), I have used the antecedent of the implication (61), which guarantees that (*) entails (**). In fact, suppose by contradiction that (*) is true but (**) is false. This means that $\mathsf{b} \in R(\mathbf{b})$, and furthermore b has no surface correspondent c that belongs to $S(\mathbf{c})$. In other words, the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ incurs at least one violation of $\textsc{Max}^S_R$, contradicting the antecedent of the implication (61). In step (63d), I have replaced (**) with (***) because of the assumption that (62) is impossible. In step (63e), I have used again the definition of $\textsc{Max}^S_R$.

(63)
$$
\begin{aligned}
& \mathsf{a} \text{ violates } \textsc{Max}^S_R \text{ relative to } (\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}) \\
\overset{(a)}{\Longleftrightarrow}\; & \mathsf{a} \in R(\mathbf{a}) \text{ and } \forall \mathsf{c}[(\mathsf{a}, \mathsf{c}) \in \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}} \rightarrow \mathsf{c} \notin S(\mathbf{c})] \\
\overset{(b)}{\Longleftrightarrow}\; & \mathsf{a} \in R(\mathbf{a}) \text{ and } \forall \mathsf{b}\Big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \underbrace{\forall \mathsf{c}\Big[(\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \rightarrow c \notin S(\mathbf{c})\Big]}_{(*)}\Big] \\
\overset{(c)}{\Longrightarrow}\; & \mathsf{a} \in R(\mathbf{a}) \text{ and } \forall \mathsf{b}\Big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \underbrace{\mathsf{b} \notin R(\mathbf{b})}_{(**)}\Big] \\
\overset{(d)}{\Longleftrightarrow}\; & \mathsf{a} \in R(\mathbf{a}) \text{ and } \forall \mathsf{b}\Big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \underbrace{\mathsf{b} \notin S(\mathbf{b})}_{(***)}\Big] \\
\overset{(e)}{\Longleftrightarrow}\; & \mathsf{a} \text{ violates } \textsc{Max}^S_R \text{ relative to } (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})
\end{aligned}
$$

The chain of implications (63) says that, if an underlying segment a violates $\textsc{Max}^S_R$ relative to the composition candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$, then a also violates $\textsc{Max}^S_R$ relative to the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. This conclusion establishes the inequality in the consequent of the FIC_{comp} (61). □

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By (63), if a violates $\textsc{Max}^S_R$ relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$, then a also violates $\textsc{Max}^S_R$ relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. Suppose that the restriction S is trivial, namely S pairs any string $\mathbf{a}$ with the complete set $S(\mathbf{a})$ of its segments. In this case, the reverse implication trivially holds as well: if a violates $\textsc{Max}_R$ relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, then a has no $\rho_{\mathbf{a},\mathbf{b}}$-correspondents in $\mathbf{b}$ and therefore a cannot have any $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$-correspondents in $\mathbf{c}$ either, thus violating $\textsc{Max}_R$ also relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$. In conclusion, the inequality in the consequent of the $\mathrm{FIC}_{\mathrm{comp}}$ (61) holds as an identity in the case where S is the trivial restriction.

A.2 **Proof of Lemma 3**

A segment b violates the constraint $\textsc{Dep}^S_R$ relative to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ provided that b belongs to $S(\mathbf{b})$ and has no $\rho_{\mathbf{a},\mathbf{b}}$-correspondent in the underlying string $\mathbf{a}$ that belongs to $R(\mathbf{a})$. $\textsc{Dep}^S_R$ assigns one violation for each surface segment that violates it. This appendix proves Lemma 3 repeated below, which establishes the $\mathrm{FIC}_{\mathrm{comp}}$ for $\textsc{Dep}^S_R$.

Lemma A.2. *The faithfulness constraint* $\textsc{Dep}^S_R$ *satisfies the* FIC_{comp}

(64) **If:** $\textsc{Dep}^S_R\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big) = 0$

Then: $\textsc{Dep}^S_R\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leqslant \textsc{Dep}^S_R\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big)$

for any two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ *and* $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ *such that the latter candidate* $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ *has no underlying segment* b *and no surface segment* c *such that*

(65) $(\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}}, \quad \mathsf{c} \in S(\mathbf{c}), \quad \mathsf{b} \notin S(\mathbf{b}), \quad \mathsf{b} \in R(\mathbf{b})$

and furthermore it enforces no breaking among the segments that satisfy the restriction S, namely there exist no underlying segment b *and no surface segments* $\mathsf{c}_1, \mathsf{c}_2$ *such that*

(66) $\mathsf{b} \in S(\mathbf{b}), \quad \mathsf{c}_1, \mathsf{c}_2 \in S(\mathbf{c}), \quad (\mathsf{b}, \mathsf{c}_1) \in \rho_{\mathbf{b},\mathbf{c}}, \quad (\mathsf{b}, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}, \quad \mathsf{c}_1 \neq \mathsf{c}_2.$ □

Proof. Assume that the antecedent of the implication (64) holds, namely that the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate $\textsc{Dep}^S_R$. The following chain of implications (67) then holds for any segment c of the string $\mathbf{c}$. In step (67a), I have used the definition of $\textsc{Dep}^S_R$. In step (67b), I have used the definition of the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$. In step (67c), I have used the antecedent of the implication (64), which guarantees that the surface segment $\mathsf{c} \in S(\mathbf{c})$ admits a correspondent underlying segment $\mathsf{b} \in R(\mathbf{b})$ according to $\rho_{\mathbf{b},\mathbf{c}}$. In step (67d), I have replaced (*) with (**) because of the assumption that (65) is impossible. In step (67e), I have used again the definition of $\textsc{Dep}^S_R$. I note that the chain of implications (67) makes no use of the no-breaking assumption that (66) is impossible.

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(67) $\quad \mathsf{c}$ violates DEP^S_R relative to $(\mathbf{a},\ \mathbf{c},\ \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$

$$\overset{(a)}{\Longleftrightarrow} \quad \mathsf{c} \in S(\mathbf{c}) \text{ and } \forall \mathsf{a}[(\mathsf{a},\mathsf{c}) \in \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}} \rightarrow \mathsf{a} \notin R(\mathbf{a})]$$

$$\overset{(b)}{\Longleftrightarrow} \quad \mathsf{c} \in S(\mathbf{c}) \text{ and } \forall \mathsf{b}[(\mathsf{b},\mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \rightarrow \forall \mathsf{a}[(\mathsf{a},\mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \mathsf{a} \notin R(\mathbf{a})]]$$

$$\overset{(c)}{\Longrightarrow} \quad \mathsf{c} \in S(\mathbf{c}) \text{ and } \exists \mathsf{b}\Big[(\mathsf{b},\mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \text{ and } \underbrace{\mathsf{b} \in R(\mathbf{b})}_{(*)} \text{ and } \forall \mathsf{a}\big[(\mathsf{a},\mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \mathsf{a} \notin R(\mathbf{a})\big]\Big]$$

$$\overset{(d)}{\Longrightarrow} \quad \mathsf{c} \in S(\mathbf{c}) \text{ and } \exists \mathsf{b}\Big[(\mathsf{b},\mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \text{ and } \underbrace{\mathsf{b} \in S(\mathbf{b})}_{(**)} \text{ and } \forall \mathsf{a}\big[(\mathsf{a},\mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}} \rightarrow \mathsf{a} \notin R(\mathbf{a})\big]\Big]$$

$$\overset{(e)}{\Longleftrightarrow} \quad \mathsf{c} \in S(\mathbf{c}) \text{ and } \exists \mathsf{b}\Big[(\mathsf{b},\mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \text{ and } \mathsf{b} \text{ violates } \text{DEP}^S_R \text{ relative to } (\mathbf{a},\ \mathbf{b},\ \rho_{\mathbf{a},\mathbf{b}})\Big]$$

By (67), for each segment c that violates DEP^S_R relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$, there exists a segment b that violates DEP^S_R relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ such that $(\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}}$. This is not sufficient to secure the inequality in the consequent of (64). In fact, it could still be the case that two *different* segments c_1 and c_2 that violate DEP^S_R relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ both correspond to the *same* segment b that violates DEP^S_R relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. In this case, $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ could have more epenthetic segments than $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, and the inequality in the consequent of (64) would fail.[23] In order to secure the inequality in the consequent of (64), I need to make sure that the mapping from the segments $\mathsf{c}_1, \mathsf{c}_2, \ldots$ that violate DEP^S_R relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ to the segments $\mathsf{b}_1, \mathsf{b}_2, \ldots$ that violate DEP^S_R relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ defined by (67) is *injective*: if two violating segments $\mathsf{c}_1, \mathsf{c}_2$ are different, the two corresponding violating segments $\mathsf{b}_1, \mathsf{b}_2$ are different as well.

The no-breaking assumption (66) serves precisely this purpose. Indeed, consider two different segments c_1 and c_2 that both violate the constraint DEP^S_R relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ and thus both belong to $S(\mathbf{c})$. By (67), there exist segments $\mathsf{b}_1, \mathsf{b}_2$ such that $(\mathsf{b}_1, \mathsf{c}_1), (\mathsf{b}_2, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}$ and furthermore $\mathsf{b}_1, \mathsf{b}_2$ violate DEP^S_R relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and thus belong to $S(\mathbf{b})$. If it were $\mathsf{b}_1 = \mathsf{b}_2 = \mathsf{b}$, it would be $(\mathsf{b}, \mathsf{c}_1) \in \rho_{\mathbf{b},\mathbf{c}}$ and $(\mathsf{b}, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}$, contradicting the no-breaking assumption (66). In conclusion, (67) establishes an *injective* mapping from the segments $\mathsf{c}_1, \mathsf{c}_2, \ldots$ that violate DEP^S_R relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ to the segments

[23] That is precisely what happens in the counterexample (27): the two segments $\mathsf{c}_1 = [\mathrm{e}]$ and $\mathsf{c}_2 =$ [i] are both epenthetic relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$; they both correspond to the same segment $\mathsf{b} =$ [ə] which is indeed epenthetic relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. The candidate $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ thus has more epenthetic segments than the candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$, and the inequality in the consequent of the FIC_{comp} thus fails.

$\mathsf{b}_1, \mathsf{b}_2, \ldots$ that violate DEP^S_R relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. This conclusion establishes the inequality in the consequent of the FIC_{comp} (64). □

The two chains of implications (63) and (67) in the proofs of the two Lemmas 2 and 3 for MAX^S_R and DEP^S_R are completely analogous. Yet, the no-crossing assumptions (62) and (65) target different correspondence relations: the former targets the correspondence relation $\rho_{\mathbf{a},\mathbf{b}}$; the latter targets the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$.

A.3 **Proof of Lemma 4**

A pair (a, b) of an underlying segment a and a surface segment b violates the faithfulness constraint $\text{IDENT}_{\varphi,R}$ relative to a candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ provided that the following three conditions hold. First, the two segments a and b are in correspondence: $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}$. Second, the two segments a and b differ with respect to feature φ: $\varphi(\mathsf{a}) \neq \varphi(\mathsf{b})$. Third, the underlying segment a satisfies the restriction R: $\mathsf{a} \in R(\mathbf{a})$. $\text{IDENT}_{\varphi,R}$ assigns one violation for each underlying/surface segment pair (a, b) that violates it. This appendix proves Lemma 4/A repeated below, which establishes the FIC_{comp} for $\text{IDENT}_{\varphi,R}$. The proof of Lemma 4/B for IDENT^S_φ is analogous.

Lemma A.3. *Consider a total feature φ. The identity faithfulness constraint $\text{IDENT}_{\varphi,R}$ satisfies the FIC_{comp}*

(68) **If:** $\text{IDENT}_{\varphi,R}\big(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}}\big) = 0$
Then: $\text{IDENT}_{\varphi,R}\big(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}\big) \leqslant \text{IDENT}_{\varphi,R}\big(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}}\big)$

for any two candidates $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ such that the former candidate $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ has no underlying segment a and no surface segment b such that

(69) $(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}}, \quad \mathsf{a} \in R(\mathbf{a}), \quad \mathsf{b} \notin R(\mathbf{b}), \quad \varphi(\mathsf{a}) = \varphi(\mathsf{b})$

and furthermore the correspondence relation $\rho_{\mathbf{b},\mathbf{c}}$ does not break any underlying segment into two or more surface segments, namely there exist no underlying segment b and no surface segments $\mathsf{c}_1, \mathsf{c}_2$ such that

(70) $(\mathsf{b}, \mathsf{c}_1) \in \rho_{\mathbf{b},\mathbf{c}}, \quad (\mathsf{b}, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}, \quad \mathsf{c}_1 \neq \mathsf{c}_2.$ □

Proof. Assume that the antecedent of the implication (68) holds, namely that the candidate $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$ does not violate $\text{IDENT}_{\varphi,R}$. The following chain of implications (71) then holds for any segment a of the string $\mathbf{a}$ and any segment c of the string $\mathbf{c}$. In step (71a), I have used the definition of the constraint $\text{IDENT}_{\varphi,R}$. In step (71b), I have used the definition of the composition correspondence relation $\rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}}$. In step (71c), I have added the conjunct (*). This is licit because the antecedent of (68) guarantees that the pair of segments (b, c) does not violate the constraint $\text{IDENT}_{\varphi,R}$ relative to $(\mathbf{b}, \mathbf{c}, \rho_{\mathbf{b},\mathbf{c}})$. Since they are in correspondence through $\rho_{\mathbf{b},\mathbf{c}}$, this means that either $\varphi(\mathsf{b}) \notin R(\mathbf{b})$ or else $\varphi(\mathsf{b}) = \varphi(\mathsf{c})$. In step

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(71d), I have replaced (*) with the equivalent (**). In fact, if it is the first condition $\varphi(\mathsf{b}) = \varphi(\mathsf{c})$ of the disjunction (*) that holds, then it must be $\varphi(\mathsf{a}) \neq \varphi(\mathsf{b})$, because $\varphi(\mathsf{a}) \neq \varphi(\mathsf{c})$. If it is instead the second condition $\mathsf{b} \notin R(\mathbf{b})$ of the disjunction (*) that holds, then it must be $\varphi(\mathsf{a}) \neq \varphi(\mathsf{b})$, because (69) is impossible. Finally, in step (71e), I have used again the definition of $\textsc{Ident}_{\varphi,R}$ relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. It should be noted that the chain of implications (71) does not make use of the no-breaking assumption (??).

(71) (a, c) violates $\textsc{Ident}_{\varphi,R}$ relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$

$$
\begin{aligned}
&\overset{(a)}{\Longleftrightarrow} \varphi(\mathsf{a}) \in R(\mathbf{a}),\ \varphi(\mathsf{a}) \neq \varphi(\mathsf{c}),\ (\mathsf{a}, \mathsf{c}) \in \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}} \\
&\overset{(b)}{\Longleftrightarrow} \varphi(\mathsf{a}) \in R(\mathbf{a}),\ \varphi(\mathsf{a}) \neq \varphi(\mathsf{c}),\ \exists \mathsf{b}\big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}},\ (\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}}\big] \\
&\overset{(c)}{\Longleftrightarrow} \varphi(\mathsf{a}) \in R(\mathbf{a}),\ \varphi(\mathsf{a}) \neq \varphi(\mathsf{c}),\ \exists \mathsf{b}\Big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}},\ (\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}}, \\
&\qquad\qquad \underbrace{\big[\varphi(\mathsf{b}) = \varphi(\mathsf{c}) \text{ or } \mathsf{b} \notin R(\mathbf{b})\big]}_{(*)}\Big] \\
&\overset{(d)}{\Longrightarrow} \varphi(\mathsf{a}) \in R(\mathbf{a}),\ \varphi(\mathsf{a}) \neq \varphi(\mathsf{c}),\ \exists \mathsf{b}\Big[(\mathsf{a}, \mathsf{b}) \in \rho_{\mathbf{a},\mathbf{b}},\ (\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}}, \\
&\qquad\qquad \underbrace{\varphi(\mathsf{a}) \neq \varphi(\mathsf{b})}_{(**)}\Big] \\
&\overset{(e)}{\Longleftrightarrow} \exists \mathsf{b}\Big[(\mathsf{b}, \mathsf{c}) \in \rho_{\mathbf{b},\mathbf{c}} \text{ and } (\mathsf{a}, \mathsf{b}) \text{ violates } \textsc{Ident}_{\varphi,R} \text{ relative to } (\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})\Big]
\end{aligned}
$$

Consider two different pairs $(\mathsf{a}_1, \mathsf{c}_1)$ and $(\mathsf{a}_2, \mathsf{c}_2)$ that both violate the faithfulness constraint $\textsc{Ident}_{\varphi,R}$ relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$. The chain of implications (71) guarantees that there exist $\mathsf{b}_1, \mathsf{b}_2$ such that the two pairs $(\mathsf{a}_1, \mathsf{b}_1)$ and $(\mathsf{a}_2, \mathsf{b}_2)$ violate the faithfulness constraint $\textsc{Ident}_{\varphi,R}$ relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$ and furthermore $(\mathsf{b}_1, \mathsf{c}_1) \in \rho_{\mathbf{b},\mathbf{c}}$ and $(\mathsf{b}_2, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}$. If $\mathsf{a}_1 \neq \mathsf{a}_2$, also the two pairs $(\mathsf{a}_1, \mathsf{b}_1)$ and $(\mathsf{a}_2, \mathsf{b}_2)$ are different. Thus, assume that $\mathsf{a}_1 = \mathsf{a}_2 = \mathsf{a}$, whereby $\mathsf{c}_1 \neq \mathsf{c}_2$. If it were $\mathsf{b}_1 = \mathsf{b}_2 = \mathsf{b}$, then the latter two conditions would say that $(\mathsf{b}, \mathsf{c}_1) \in \rho_{\mathbf{b},\mathbf{c}}$ and $(\mathsf{b}, \mathsf{c}_2) \in \rho_{\mathbf{b},\mathbf{c}}$, contradicting the no-breaking assumption that (70) is impossible. In conclusion, (71) defines an injective mapping from the pairs (a, c) that violate the constraint $\textsc{Ident}_{\varphi,R}$ relative to $(\mathbf{a}, \mathbf{c}, \rho_{\mathbf{a},\mathbf{b}}\rho_{\mathbf{b},\mathbf{c}})$ to the pairs (a, b) that violate it relative to $(\mathbf{a}, \mathbf{b}, \rho_{\mathbf{a},\mathbf{b}})$. This conclusion establishes the inequality in the consequent of the implication (68). □

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