UNTERBERGER, A. and UPMEIER, H. *Pseudodifferential analysis on symmetric cones* (CRC Press, Boca Raton–New York–London–Tokyo, 1996), 216pp., 0 8493 7873 7, \$59.95.

The theory of pseudodifferential operators (ΨDOs) is one of the most powerful technical tools in the study of partial differential equations and corresponding boundary value problems. In the framework of the theory of pseudodifferential operators one can effectively construct the resolvent of an elliptic differential operator or even its non-integer power. The ΨDOs are also used for many other purposes; for example, in microlocal analysis they often play the same role as the cut-off functions in classical analysis.

The classical theory of ΨDOs deals with operators acting in the Euclidean space $\mathbb{R}^n$ where a ΨDO A is defined by the formula

$$Au(x) = \int_{\mathbb{R}^{2n}} a\left(\frac{x+y}{2}, \eta\right) e^{2\pi i(x-y)\cdot\eta} u(y)\, dy\, d\eta = \int_{\mathbb{R}^{2n}} a(y, \eta)(\sigma_{y,\eta} u)(x)\, dy\, d\eta.$$

Here $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$, $\eta \in \mathbb{R}^n$, $(\sigma_{y,\eta}u)(x) := e^{2\pi i(x-S_y x)\cdot\eta} u(S_y x)$ and $S_y x = 2y - x$ is the middle point on the straight line joining x and y. The function a is called the (Weyl) symbol of the ΨDO A. If a is a polynomial in η with smooth coefficients then A is a differential operator. In the general case the symbol a may be quite an arbitrary function. Obviously the properties of a ΨDO are determined by that of its symbol. Roughly speaking, the classical theory of ΨDOs is a series of results which describe the properties of operators in terms of symbols. In particular, if a is smooth and bounded with all its derivatives, then A is a bounded operator in $L_2(\mathbb{R}^n)$ (Calderon–Vaillancourt theorem).

There are many versions of the theory of ΨDOs (for operators acting on smooth manifolds with or without boundary, in domains with conical singularities, and so on), most of which are designed for applications to particular classes of problems appearing in the theory of partial differential equations.

Yet another version of the theory of ΨDOs is developed in the book being reviewed. The authors consider operators acting in a cone $\Lambda \subset \mathbb{R}^n$ which is self-dual and homogeneous under its linear automorphism group $GL(\Lambda)$ (such cones are called symmetric). On a symmetric cone Λ there exists a unique $GL(\Lambda)$-invariant Riemannian metric $g(\Lambda)$ with respect to which Λ is a global symmetric space. Moreover, for any two points $x, y \in \Lambda$ one can define "the geodesic middle" $\operatorname{mid}(x, y)$ (an analogue of $S_y x$ in the Euclidean case) and the map

$$(x, y) \to (\operatorname{mid}(x, y), x - y)$$

is a global diffeomorphism from $\Lambda \times \Lambda$ onto $\Lambda \times \mathbb{R}^n$. This allows the authors to define ΨDOs on symmetric cones in the same manner as in $\mathbb{R}^n$ and to construct a similar theory. They show that the classical Euclidean results (including the Calderon–Vaillancourt theorem) can be generalized to the ΨDOs on symmetric cones.

It should be emphasized that the symmetric cone with the metric $g(\Lambda)$ is a smooth Riemannian manifold without boundary. Therefore the book has nothing to do with the boundary value problems in domains with conical singularities (as one might think from the title). Moreover, the authors deal with a very special "smooth case" where a smooth manifold (symmetric cone) has an additional global structure. This enables one to develop a global theory of ΨDOs on such a manifold.

The book leaves an impression that the authors do not care very much about applications, but are rather interested in formal generalization of the classical Euclidean results. Therefore the theory developed in the book looks like a thing-in-itself. On the other hand it involves a number of algebraic results and shows the connection between algebra and analysis on the symmetric cones. Those interested in the classical theory of ΨDOs and their applications may find the book not very useful, but it can be recommended to the mathematicians working in between algebra and analysis or willing to learn the theory of symmetric cones.

Y. SAFAROV