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PROJECTIVE CHARACTERS OF DEGREE ONE AND THE INFLATION-RESTRICTION SEQUENCE

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Abstract

Let G be a finite group, α be a fixed cocycle of G and $\text{Proj}(G, \alpha)$ denote the set of irreducible projective characters of G lying over the cocycle α.

Suppose N is a normal subgroup of G. Then the author shows that there exists a G-invariant element of $\text{Proj}(N, \alpha_N)$ of degree 1 if and only if $[\alpha]$ is an element of the image of the inflation homomorphism from $M(G/N)$ into $M(G)$, where $M(G)$ denotes the Schur multiplier of G. However in many situations one can produce such G-invariant characters where it is not intrinsically obvious that the cocycle could be inflated. Because of this the author obtains a restatement of his original result using the Lyndon-Hochschild-Serre exact sequence of cohomology. This restatement not only resolves the apparent anomalies, but also yields as a corollary the well-known fact that the inflation-restriction sequence

$$\{1\} \to M(G/N) \to M(G) \to M(N)$$

is exact when N is perfect.

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All groups, G, considered in this paper are finite and all representations of G are defined over the complex numbers. The reader unfamiliar with projective representations is referred to [3] for basic definitions and elementary results.

The purpose of this paper is to investigate under which circumstances the following well-known corollary to Clifford's theorem can be generalized to projective characters.

Let $N \trianglelefteq G$, and $\chi \in \mathrm{Irr}(G)$ such that $[\chi_N, 1_N] \neq 0$, where 1_N denotes the trivial character of N. Then $N \leq \mathrm{Ker}(\chi)$.

Our generalization will take the form of answering the following question.

Let α be a cocycle of G and $N \trianglelefteq G$. Under which necessary and sufficient conditions does there exist a G-invariant projective character of N with degree 1 and cocycle α_N?

Our motivation for investigating this problem is provided by Haggarty and Humphreys [1] who said "Given a subgroup L of G, a cocycle α of G determines a cocycle α_L of L by restriction. However elements which are α_L-regular in L need not be α-regular in G. This fact complicates the theory of projective characters."

The implication here can be taken to be that given that every element of L is α-regular (see 1.1 for this definition), then the theory of projective characters is similar to ordinary character theory. Indeed as the theory applies to induction from L to G this is the case, however we shall show in Section 1 that this is certainly not the case when looking at the restriction from G to L, where $L \trianglelefteq G$.

1. Projective characters of degree one

To avoid repetition we fix the following notation for the rest of this paper. Let G be a group, α be a cocycle of G, and $\mathrm{Proj}(G, \alpha)$ denote the set of irreducible projective characters of G with cocycle α. We shall also use without further reference the fact due to Schur that $o([\alpha])$ in $M(G)$ divides $\xi(1)$ for all $\xi \in \mathrm{Proj}(G, \alpha)$, where $M(G)$ denotes the Schur multiplier of G. We thus have that there is an element of $\mathrm{Proj}(G, \alpha)$ of degree 1 if and only if $[\alpha] = [1]$. We now recall some more well-known facts about projective characters in a series of lemmas. We start however with a basic definition.

DEFINITION 1.1. An element, x, of G is said to be *α-regular* if $\alpha(x, g) = \alpha(g, x)$ for all $g \in C_G(x)$.

It is easy to check that if $[\alpha] = [\beta]$, then x is α-regular if and only if x is β-regular. Also any conjugate of an α-regular element is α-regular, so that we may speak of the α-regular conjugacy classes of G.

LEMMA 1.2. (i) *There exists $\beta \in [\alpha]$ such that*

$$\frac{\beta(g, x)\beta(gx, g^{-1})}{\beta(g, g^{-1})} = 1$$

for all β-regular $x \in G$, and all $g \in G$.

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(ii) *An element, x, of G is α-regular if and only if there exists $\xi \in \operatorname{Proj}(G,\alpha)$ such that $\xi(x) \neq 0$.*

PROOF. See (7.2.4) and (7.2.5) of [3].

We call a cocycle satisfying the condition of 1.2(i) a class-function cocycle, in the sense that by (ii) the elements of $\operatorname{Proj}(G,\beta)$ are then class functions.

Let $N \trianglelefteq G$, for $\varsigma \in \operatorname{Proj}(N,\alpha_N)$ we define the g-conjugate, ς^g, of ς by

$$\varsigma^g(x) = f_\alpha(g,x)\varsigma(gxg^{-1})$$

for $g \in G$, and all $x \in N$; where $f_\alpha(g,x) = \alpha(g,x)\alpha(gx,g^{-1})/\alpha(g,g^{-1})$. This defines an action of G on $\operatorname{Proj}(N,\alpha_N)$ for which Clifford's theorem holds. Having defined our action we can now being to look at the relationship between α_N-regular elements of N and $\operatorname{Proj}(N,\alpha_N)$.

LEMMA 1.3. *Let $N \trianglelefteq G$ and x be an α_N-regular element of N. Then for all $g \in G, x^g$ is α_N-regular.*

PROOF. Let $\varsigma \in \operatorname{Proj}(N,\alpha_N)$ such that $\varsigma(x) \neq 0$, and $g \in G$. Then $\varsigma^g \in \operatorname{Proj}(N,\alpha_N)$ and $\varsigma^g(x^g) = c\varsigma(x)$ for some $c \neq 0$. Thus x^g is α_N-regular.

Our next result shows that G-invariance, not surprisingly, does not depend upon the choice of cocycle from $[\alpha]$.

LEMMA 1.4. *Let $N \trianglelefteq G$, $\mu\colon G \to \mathbb{C}^*$ be a mapping with $\mu(1) = 1$, and*

$$\beta(g,h) = \frac{\mu(g)\mu(h)}{\mu(gh)}\alpha(g,h) \quad \textit{for all } g,h \in G.$$

Suppose $\operatorname{Proj}(N,\alpha_N) = \{\varsigma_1,\dots,\varsigma_t\}$. *Then*

(i) $\operatorname{Proj}(N,\beta_N) = \{\mu_N\varsigma_1,\dots,\mu_N\varsigma_t\}$;

(ii) *for $g \in G$, $\varsigma_i^g = \varsigma_j$ if and only if $(\mu_N\varsigma_i)^g = \mu_N\varsigma_j$.*

PROOF. (i) See pages 72–73 of [3].

(ii) Let $g \in G$. Then for all $x \in N$,

$$\begin{aligned}(\mu\varsigma_i)^g(x) &= f_\beta(g,x)(\mu\varsigma_i)(gxg^{-1})\\ &= \frac{\beta(g,x)\beta(gx,g^{-1})}{\beta(g,g^{-1})}\mu(gxg^{-1})\varsigma_i(gxg^{-1})\\ &= f_\alpha(g,x)\mu(x)\varsigma_i(gxg^{-1}) = \mu(x)\varsigma_i^g(x).\end{aligned}$$

PROPOSITION 1.5. *Let $N \trianglelefteq G$, and let* inf *denote the inflation homomorphism from $M(G/N)$ into $M(G)$. Then there exists a G-invariant $\delta \in \operatorname{Proj}(N,\alpha_N)$ with $\delta(1) = 1$ if and only if $[\alpha] \in \operatorname{Im}(\inf)$.*

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PROOF. Suppose there exists a G-invariant $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$. Let P be an irreducible projective representation of G with cocycle α which has δ as a constitutent of P_N. Then, if P has degree n, we have that P induces a homomorphism $\overline{P}\colon G/N \to PGL(n, \mathbb{C})$ defined by $\overline{P}(gN) = \Pi(P(g))$ for all $g \in G$, where Π denotes the natural homomorphism from $GL(n, \mathbb{C})$ onto $PGL(n, \mathbb{C})$. Now any section of $\overline{P}$ will be a projective representation of G/N with some cocycle β of G/N, moreover $[\beta]$ does not depend on the choice of section, and it is clear that $\inf([\beta]) = [\alpha]$.

Conversely, suppose $[\alpha] = \inf([\beta])$ for some $[\beta] \in M(G/N)$. Then regarding β as a cocycle of G we have that the trivial character, 1_N, of N is an element of $\text{Proj}(N, \beta_N)$. As such 1_N is G-invariant since β is constant on the cosets of N in G, and hence there exists a G-invariant $\delta \in \text{Proj}(N, \alpha_N)$ with $\delta(1) = 1$ by 1.4.

This result would appear to have easily answered our original question. However we shall now demonstrate that in certain circumstances it is possible to produce G-invariant projective characters of degree 1 in situations where it is not intrinsically obvious that the cocycle could be inflated.

LEMMA 1.6. *Let $N \trianglelefteq G$ such that $[\alpha_N] = [1]$. Then there exists $\delta \in \text{Proj}(N', \alpha_{N'})$ with $\delta(1) = 1$ such that δ is G-invariant.*

PROOF. Since $[\alpha_N] = [1]$, $\mathscr{A} = \{\delta \in \text{Proj}(N, \alpha_N) \colon \delta(1) = 1\}$ is non-empty and G acts upon it. Now for $\delta \in \mathscr{A}$ we have that $\delta_{N'}$ is irreducible, and so by Clifford's theorem there exists a bijection from $\text{Irr}(N/N')$ onto $\mathscr{A}$ defined by $\lambda \mapsto \lambda\delta$. Now if $\xi \in \text{Proj}(G, \alpha)$ such that δ is a constitutent of ξ_N, we have that $\xi_N = e(\delta_1 + \cdots + \delta_t)$, where $\delta = \delta_1, \ldots, \delta_t$ are the distinct G-conjugates of δ. Thus $\xi_{N'} = et\delta_{N'}$, and so $\delta_{N'}$ is G-invariant.

It is obvious that if there is a G-invariant element of $\text{Proj}(N, \alpha_N)$ of degree 1, then necessarily every element of N is α-regular. One could conjecture, falsely as it happens, that this was also a sufficient condition, but our next major result shows that to some extent this conjecture would be justified.

LEMMA 1.7. *Let $N \trianglelefteq G$ and x be an α_N-regular element of N. Suppose that α_N is class-function cocycle of N. Then for each $g \in G$ and all $y \in N$,*

$$f_\alpha(g, x) = f_\alpha(g, x^y).$$

PROOF. By 1.2 and 1.3 we may let $\varsigma \in \text{Proj}(N, \alpha_N)$ such that $\varsigma(gxg^{-1}) \neq 0$. Now for $g \in G$ and all $y \in N$ we have that

$$\varsigma^g(x) = f_\alpha(g, x)\varsigma(gxg^{-1})$$

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and

$$\begin{aligned}(\varsigma^g)^{y^{-1}}(x) &= f_\alpha(y^{-1},x)\varsigma^g(x^y)\\ &= \varsigma^g(x^y), \qquad \text{since } \alpha_N \text{ is a class-function cocycle;}\\ &= f_\alpha(g,x^y)\varsigma(gx^yg^{-1}).\end{aligned}$$

But gxg^{-1} and gx^yg^{-1} are conjugate in N, since G permutes the classes of N. Thus since α_N is a class-function cocycle we have that both $\varsigma(gx^yg^{-1}) = \varsigma(gxg^{-1}) \neq 0$, and $\varsigma^g(x) = \varsigma^g(x^y)$, hence $f_\alpha(g,x) = f_\alpha(g,x^y)$.

THEOREM 1.8. *Let $N \trianglelefteq G; \mathscr{C}_1,\dots,\mathscr{C}_r$ be the α_N-regular conjugacy classes of N fixed by $g \in G$, and $x_i \in \mathscr{C}_i$. Suppose that α_N is a class-function cocycle of N. Then $\sum_{i=1}^r f_\alpha(g,x_i)$ is the number of g-invariant elements of* $\mathrm{Proj}(N,\alpha_N)$.

PROOF (BRAUER, ISAACS). Let $\varsigma_i, \mathscr{C}_i$ for $1 \le i \le t$ denote respectively the elements of $\mathrm{Proj}(N,\alpha_N)$ and the α_N-regular classes of N. Let $x_i \in \mathscr{C}_i$, if $\mathscr{C}_i^g = \mathscr{C}_j$ we shall write $x_i^g = x_j$. For $g \in G$, we define $A(g) = (a_{ij})$, where $a_{ij} = 1$, if $\varsigma_i^g = \varsigma_j$ and is zero otherwise. We also define $B(g) = (b_{ij})$, where $b_{ij} = f_\alpha(g,x_j)$ if $\mathscr{C}_i^g = \mathscr{C}_j$ and is zero otherwise. We note from 1.7 that $f_\alpha(g,x_j)$ is independent of the choice of $x_j \in \mathscr{C}_j$. Finally let $P = (p_{ij})$, where $p_{ij} = \varsigma_i(x_j)$. Then we have that the (l,m)th entry of $A(g)P$ is $\sum_{j=1}^t a_{lj}\varsigma_j(x_m) = \varsigma_l^g(x_m)$; whereas the (l,m)th entry of $PB(g)$ is

$$\sum_{j=1}^t \varsigma_l(x_j)b_{jm} = f_\alpha(g,x_m)\varsigma_l(gx_mg^{-1}) = \varsigma_l^g(x_m).$$

Thus $P^{-1}A(g)P = B(g)$ and so $\mathrm{trace}(A(g)) = \mathrm{trace}(B(g))$. But $\mathrm{trace}(A(g))$ is the number of g-invariant elements of $\mathrm{Proj}(N,\alpha_N)$, whereas $\mathrm{trace}(B(g)) = \sum_{i\in I} f_\alpha(g,x_i)$ where $I = \{i : \mathscr{C}_i^g = \mathscr{C}_i\}$.

As applications of the above theorem we have the following results.

COROLLARY 1.9. *Let $N \trianglelefteq G$, and suppose that every element of N is α-regular. Then each $g \in G$ fixes at least one element of* $\mathrm{Proj}(N,\alpha_N)$.

PROOF. By 1.4(ii) we may assume that α is a class-function cocycle of G. Let $g \in G$, then by 1.8 the number of g-invariant elements of $\mathrm{Proj}(N,\alpha_N)$ equals the number of classes of N fixed by g.

COROLLARY 1.10. *Let N be a normal abelian subgroup of G such that $G/C_G(N)$ is cyclic. Suppose that every element of N is α-regular. Then there exists $\delta \in \mathrm{Proj}(N,\alpha_N)$ with $\delta(1) = 1$ which is G-invariant.*

PROOF. Let $C = C_G(N)$ and $\delta \in \mathrm{Proj}(N,\alpha_N)$. Then $\delta(1) = 1$ and C is a subgroup of the inertia subgroup, $I_G(\delta)$, of δ in G, since N is abelian and every element of N is α-regular. Let $g \in G$ such that $\langle gC \rangle = G/C$, then by 1.9 g fixes some $\delta' \in \mathrm{Proj}(N, \alpha_N)$. Thus $G = \langle g, C \rangle \leq I_G(\delta')$.

It is interesting to note that Mangold in (5.1) of [7] claimed that every element of G is α-regular if and only if $[\alpha] = [1]$. The first of the following examples demonstrates that this is in fact false in general for a non-abelian group, and hence also shows that the condition of every element of G being α-regular is not even sufficient generally, to guarantee the existence of an element of $\mathrm{Proj}(G, \alpha)$ of degree 1.

EXAMPLES. Let p be a prime number, and H be the "einfachste" representation group for $(C_p)^4$ as in (3.5.4) of [3], so that $|H| = p^{10}$ and $H = \langle x_1, x_2, x_3, x_4 \colon x_i^p = [x_i, x_j, x_k] = 1, \text{ for } 1 \leq i, j, k \leq 4 \rangle$.

Let $s = [x_1, x_2][x_3, x_4]$ and $A = \langle s \rangle$, so that $A \leq Z(H) \cap H'$ and $|A| = p$. It is easy to show that no non-trivial element of A is a commutator, see [5] for a generalization of this result. Now let $\lambda \in \mathrm{Irr}(A)$ be defined by $\lambda(s^j) = \omega^j$ for $\omega = e^{2\pi i/p}$, and let α be the cocycle of $G_1 = H/A$ constructed in the normal way from λ, see pages 180–182 of [2] for example. Then by construction $o([\alpha]) = p$. Now with the definition and results of pages 195–197 of [2], we have that every element of G_1 is 'λ-special' trivially, and hence every element of G_1 is α-regular.

For a different type of example let $B = \langle s, t \rangle$ where $t = [x_1, x_3]$, $M = \langle x_1, x_3, A \rangle$, and define $\mu \in \mathrm{Irr}(B)$ by $\mu(s^j t^k) = \lambda(s^j)$. One can then check that every element of $N = M/B$ is μ-special, but that not every element of $MZ(H)/Z(H)$ is ν-special for any extension, ν, of μ to $Z(H)$. So if β is the cocycle of $G_2 = H/B$ constructed from μ, we have shown that every element of the abelian group N is β-regular, but that no element of $\mathrm{Proj}(N, \beta)$ can be G_2-invariant.

2. The inflation-restriction sequence

Let $N \trianglelefteq G$. Then we have the Lyndon-Hochschild-Serre exact sequence of cohomology:

$$\{1\} \to H^1(G/N, \mathbb{C}^*) \xrightarrow{\mathrm{inf}_1} H^1(G, \mathbb{C}^*) \xrightarrow{\mathrm{res}_1} H^1(N, \mathbb{C}^*)^G \xrightarrow{\mathrm{tra}} M(G/N) \xrightarrow{\mathrm{inf}_2} M(G)$$

where the action of all groups on $\mathbb{C}^*$ is trivial, see [6, page 354].

It is clear that we may replace $M(G)$ in this exact sequence by $M(G)^\# = \{[\alpha] \in M(G) \colon [\alpha_N] = [1]\}$. In this section we shall extend this new sequence one term to the right, and in doing so we shall give a practical test to see whether an element of $M(G)^\#$ is in the image of inf_2. Thus it is 1.5 which connects the results of Section 1 to those of this section.

LEMMA 2.1. *Let $N \trianglelefteq G$, α be a cocycle of G such that $[\alpha] \in M(G)^{\#}$, and $\delta \in \mathrm{Proj}(N, \alpha_N)$ with $\delta(1) = 1$. Then*

(i) *the mapping $\alpha' \colon G/N \to H^1(N, \mathbb{C}^*)$ defined by $\alpha'(gN) = \delta/\delta^g$ is a crossed homomorphism;*

(ii) *the mapping $\tau \colon M(G)^{\#} \to H^1(G/N, H^1(N, \mathbb{C}^*))$ defined by $\tau([\alpha]) = [\alpha']$ is a homomorphism.*

PROOF. (i) Let $g \in G$. Then $\delta^g, \delta \in \mathrm{Proj}(N, \alpha_N)$, and so since $\delta(1) = 1$ we have that $\delta/\delta^g \in H^1(N, \mathbb{C}^*)$. Now let $g_1, g_2 \in G$, and suppose that $g_1 x = g_2$ for $x \in N$. Then

$$\alpha'(g_2 N) = \frac{\delta}{\delta^{g_1 x}} = \frac{\delta}{(\delta^{g_1})^x} = \frac{\delta}{\delta^{g_1}} = \alpha'(g_1 N),$$

since $N \le I_G(\delta^{g_1})$. Thus α' is well defined. Finally let $g_1, g_2 \in G$. Then

$$\alpha'(g_1 g_2 N) = \frac{\delta}{\delta^{g_1 g_2}} = \left(\frac{\delta}{\delta^{g_1}}\right)^{g_2} \frac{\delta}{\delta^{g_2}} = (\alpha'(g_1 N))^{g_2 N} \alpha'(g_2 N).$$

(ii) Suppose $\beta \in [\alpha]$, and let $\mu \colon G \to \mathbb{C}^*$ be a mapping with $\mu(1) = 1$ such that

$$\beta(g, h) = \frac{\mu(g)\mu(h)}{\mu(gh)} \alpha(g, h) \quad \text{for all } g, h \in G.$$

Let $\nu \in \mathrm{Proj}(N, \beta_N)$ with $\nu(1) = 1$. Then by 1.4 we have that $\nu = \mu_N \delta_1$ for some $\delta_1 \in \mathrm{Proj}(N, \alpha_N)$. But $\delta_1 = \lambda\delta$ for $\lambda \in H^1(N, \mathbb{C}^*)$ as in the proof of 1.6. Thus

$$\frac{\nu}{\nu^g} = \frac{\lambda}{\lambda^g} \frac{\mu_N \delta}{(\mu_N \delta)^g} = \frac{\lambda}{\lambda^g} \frac{\delta}{\delta^g}$$

as in the proof of 1.4, and so τ is well defined. Clearly τ is a homomorphism.

THEOREM 2.2. *Let $N \trianglelefteq G$. Then the sequence*

$$M(G/N) \xrightarrow{\text{inf}} M(G)^{\#} \xrightarrow{\tau} H^1(G/N, H^1(N, \mathbb{C}^*))$$

is exact.

PROOF. By 1.5 we have that $\mathrm{Im}(\mathrm{inf}) \le \mathrm{Ker}(\tau)$. Let $[\alpha] \in \mathrm{Ker}(\tau)$. Then for $\delta \in \mathrm{Proj}(N, \alpha_N)$ with $\delta(1) = 1$, we have that $\delta/\delta^g = \lambda/\lambda^g$ for some $\lambda \in H^1(N, \mathbb{C}^*)$. But then $\delta\lambda^{-1}$ is G-invariant, and so by 1.5 we obtain that $[\alpha] \in \mathrm{Im}(\mathrm{inf})$.

The above theorem can be regarded as a generalization of a result of Read, see (4.4.5) of [3], which deals with the special case when N is a central subgroup of G. We now mention some applications of 2.2, the first being well known.

COROLLLARY 2.3. *Let N be a perfect normal subgroup of G. Then the sequence*

$$\{1\} \to M(G/N) \xrightarrow{\text{inf}} M(G) \xrightarrow{\text{res}} M(N)$$

is exact.

PROOF. We start by noting that $H^1(N, \mathbb{C}^*) = \{1\}$, since $N' = N$. Thus by 2.2 we have that $\text{Ker}(\text{res}) = M(G)^{\#} = \text{Im}(\text{inf})$.

Our next result was used by Liebler and Yellen in (2.4) of [4] to help prove that groups of central type are solvable.

COROLLARY 2.4. *Let $N \trianglelefteq G$, and suppose that $(|G/N|, |N/N'|) = 1$. Then $M(G)^{\#} = \text{Im}(\text{inf})$.*

PROOF. By the Schur-Zassenhaus theorem we have that $H^1(G/N, H^1(N, \mathbb{C}^*))$ is trivial, and so the desired result is immediate from 2.2.

COROLLARY 2.5. *Suppose G is metacyclic, and let $N \trianglelefteq G$ such that both N and G/N are cyclic. Then $M(G)$ is isomorphic to a subgroup of $H^1(G/N, N)$.*

PROOF. From 2.2 we have that the sequence

$$\{1\} \to M(G) \xrightarrow{\tau} H^1(G/N, \text{Irr}(N))$$

is exact, since $M(G)^{\#} = M(G)$. Thus τ is a monomorphism.

For our last application we can now explain the result of 1.6.

COROLLARY 2.6. *Let $N_1, N_2 \trianglelefteq G$ with $N_2 \leq N_1, T$ denote the image of* res: $H^1(N_1, \mathbb{C}^*) \to H^1(N_2, \mathbb{C}^*)$, *and $M(G)^{\#_i} = \{[\alpha] \in M(G)\colon [\alpha_{N_i}] = [1]\}$. Then the homomorphism $\tau\colon M(G)^{\#_2} \to H^1(G/N_2, H^1(N_2, \mathbb{C}^*))$ defined in 2.1, induces by restriction to $M(G)^{\#_1}$ a homomorphism from $M(G)^{\#_1}$ into $H^1(G/N_2, T)$.*

PROOF. Let $[\alpha] \in M(G)^{\#_1}$, and $\delta \in \text{Proj}(N_1, \alpha_{N_1})$ with $\delta(1) = 1$. Then by 2.1 we have that $\tau([\alpha]) = [\alpha']$, where $\alpha'(gN_2) = (\frac{\delta}{\delta^g})_{N_2} \in T$.

In the situation of 1.6 we have that $N_2 = N_1'$ and so $T = \{1\}$, with the above notation. Thus by 2.2 we obtain that $M(G)^{\#_1}$ is a subgroup of inf: $M(G/N_1') \to M(G)$.

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