RESULTS ON THE LARGE SCALE DISTRIBUTION OF EXTRAGALACTIC OBJECTS OBTAINED BY THE METHOD OF STATISTICAL REDUCTION

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1. INTRODUCTION

The phenomenon of clustering of galaxies was known earlier than the nature of the so called extragalactic nebulae. It was discovered as early as the end of XVIII century by W. Herschel and confirmed by his followers. After the introduction of Schmidt cameras into astronomy, Zwicky suggested that all galaxies participate in clustering. After the discovery of extragalactic radio sources some astronomers have been trying, in different ways, to apply the concept of clustering to them also.

The concept of clustering was, until the last two decades, taken to mean that all the objects under consideration belong to some individual systems called clusters. Those individuals may or may not belong to individual "clusters" of the second order, and those to the clusters of third and higher orders. Kiang (1967) drew attention to the fact that the overall existence of individual clusters is not the only possible model of clustering. He proposed another picture called continuous clustering. This picture is based, in principle, on the concept of irregularities or non random fluctuations in the distribution of objects on all possible scales. Configurations of regions of higher or lower density (number of objects per unit volume) can in some places manifest themselves as well defined individual clusters, but these are rather exceptions. In general, the distribution of extragalactic objects in Kiang's picture can be described by the statistical parameters of the fluctuations, and not by the characteristic parameters of individual clusters, clusters of clusters, etc., as was attempted in the classical papers of Abell, Neyman, Zwicky and their collaborators.

It is no wonder that the idea of the overall existence of individual clusters was considered first historically. Early investigators were fascinated by such systems as the Coma, Hydra I or Perseus clusters. These were the first to be elaborated and described in detail. Hundreds of others, similar to them, were discovered in farther regions of space. Therefore, general catalogues of galaxies such as those of

M. S. Longair and J. Einasto (eds.), The Large Scale Structure of the Universe, 229-239.

https://doi.org/10.1017/S0074180900144614 Published online by Cambridge University Press

Abell (1958) and Zwicky (Zwicky et al. 1961-1968) were made assuming that clustering means the existence of individual clusters. Even in the last few years single, field galaxies were considered sometimes to be clusters with one member. This is rather a psychological problem and its practical consequences were discussed in depth by Rudnicki (1976). Only after first attempting to describe whole regions of the sky by listing individual clusters and possible superclusters and finding it impossible to see any notion of "a cluster as an individual" which would be adequate to the real distribution was our attention drawn (Flin et al. 1974, see also Rudnicki 1976) to Kiang's concept of continuous clustering which had been overlooked.

One cannot say that today the controversy between the concept of the overall existence of individual clusters (Zwicky-Neyman proposition) and of continuous clustering (Kiang-Flin proposition) is in any sense resolved. One of the strongest observational arguments in favour of the existence, in general, of individual clusters is the peculiarity of first rank galaxies in almost all the strong condensations of galaxies. This is explained theoretically by the concept of the multiplication of galaxies described by Arp and Ambartsumian. This postulates that all the galaxies are offsprings of protogalaxies, ejected from them directly or from their descendants. A region consisting of offsprings of a single protogalaxy is certainly an individual cluster, at least at the beginning but subsequently irregular ejections and individual motions may blur the picture. There is another philosophical argument, namely we are placed inside a well defined system, our Supergalaxy (the Virgo cluster, in Zwicky's terminology). One would rather dislike the idea that we inhabit a very particular part of the Universe. On the other hand, in addition to the difficulties already mentioned with the division of the space between individual cluster cells, the strongest argument in favour of continuous clustering is the spectrum of characteristic dimensions of irregularities in the distribution of extragalactic objects. In the course of more detailed investigations it gets more and more complicated. Also the controversy about the existence of higher order clusters, especially the fact that there is a smooth transition between what some investigators call a superclusters and a single regular cluster of the same size (Rudnicki 1967) can be easily resolved by the Kiang-Flin proposition.

There are two ways of seeking a solution to this fundamental question of which proposal fits the real Universe better. One is to search for a better notion and a better practical criterion for picking out individual clusters in the sky; the other - to develop more sensitive statistical methods. We shall limit ourselves here to the second approach only.

2. SHORTCOMINGS OF FORMER METHODS

Excluding some methods of only historical value and a few others of limited applicability, the most widespread methods today are the

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following four:

(i) Correlation method (called also autocorrelation method) is very simple and permits one to obtain the characteristic dimensions of clustering provided that they are very distinct.

(ii) Dividing the investigated field into cells and comparing the standard deviations of numbers of objects in these cells with a statistical distribution which is given a priori. By changing the sizes of cells, characteristic dimensions of the fluctuations can be obtained (the method of subsequent subdivisions, cf Zwicky 1957). This method has various names and various mathematical realisations and gives also one parameter of distribution (dimension). There arise difficulties of interpretation when comparing the results based on different numbers of objects or of cells.

(iii) Analysis of the distribution of distances between the nearest (or second, third, etc. nearest) objects (Pilkington and Scott 1965). This, when applied to the nearest neighbours only, gives rather primitive information. When extended to more distant neighbours it brings difficulties of interpretation of the results. Besides it is very sensitive to all possible selection effects which bring serious difficulties in comparing the results based on different sources of observational material.

(iv) Power spectrum (cf Webster 1976a,b) is certainly one of the best methods used today. Its shortcomings lie in the complicated mathematics which makes practical programming and preparation of observational material difficult. It needs much computer time. In addition, in this method some effects of the general gradient of the distribution over the surveyed field may be mixed with the effects of clustering.

3. METHOD OF STATISTICAL REDUCTION

The method of statistical reduction, also called the reductional method, was elaborated by Andrzej Zieba and his collaborators and has already been published (A. Zieba 1975, S. Zieba 1977, Garbaj 1976) and there is no need to present it here in detail. We want, however, to present its simplest and most frequently applied variant in extragalactic astronomy for those who have not had the opportunity to get acquainted with it earlier.

An area of celestial sphere, which may or may not be rectangular, is divided into quadratic cells called elementary domains. An area so divided is called a chart of population. We form pairs of these cells. Each pair, called a fundamental domain, consists of two numbered cells (elementary domains). Those, in every fundamental domain, lie at the same distance and the same direction from each other (Figure 1).

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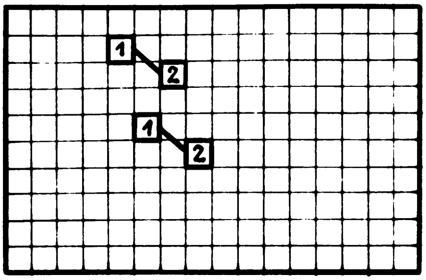

Figure 1. Scheme of elementary and fundamental domains.

We consider now all the possible pairs of objects distributed in the surveyed field and calculate the probabilities that in a fundamental area chosen at random the pair of objects drawn at random is distributed in every one out of six possible ways: 0,0; 0,1; 1,0; 1,1; 0,2; 2,0. The formulae, worked out by A. Zieba, for the probabilities of such configurations for a given chart and fundamental domains are unexpectedly simple. We call a set of such probabilities the disposition of the population reduced to two objects. Only four out of these six probabilities are independent. Then, we calculate the same probabilities for the so called randomized distribution which is the disposition of a chart where the numbers of objects in individual elementary domains are the same as in the real chart of the population, but the elementary domains are shuffled at random. In addition we calculate a standard disposition of objects distributed at random over the elementary domains. So we have three sets of parameters:

	distribution of two objects chosen at random in a fundamental domain chosen at random					
	0,0	0,1	1,0	1,1	0,2	2,0
	probabilities of the above distributions					
for the real chart (disposition of a chart)	P_{00}	P_{01}	P_{10}	P_{11}	P_{02}	P_{20}
for a shuffled chart (randomized disposition)	S_{00}	S_{01}	S_{10}	S_{11}	S_{02}	S_{20}
for a chart populated at random (standard disposition)	R_{00}	R_{01}	R_{10}	R_{11}	R_{02}	R_{20}

Four independent parameters can be formed out of P_{11} for the description of the real chart. The following parameters have simple physical interpretations.

(a) Concentration index c. It is defined as:

$$c = \frac{P_{02} + P_{20}}{R_{02} + R_{20}} = k^2 \frac{P_{02} + P_{20}}{2} ,$$

where k is the number of elementary domains. When there are more highly populated elementary domains on the real chart than the random distribution gives, c is larger than 1. On the other hand, c smaller than 1 indicates that there is a deficiency of highly populated cells in comparison with objects distributed at random.

(b) Index of grouping:

$$g = \frac{P_{11}}{S_{11}} = k^2 \frac{k-1}{k-c} \frac{P_{11}}{2} .$$

When highly populated areas avoid each other the value of g is less than 1. When they show a tendency to gather together it is larger than 1.

(c) Index of weak anisotropy:

$$a_1 = \frac{k^2}{2} \frac{P_{01} - P_{10}}{k - (1+c)} .$$

When scantily populated areas ("holes") in fundamental domains appear more frequently in the first elementary domains, a_1 has a negative value. In the opposite case – the value is positive. This index indicates the degree of overall anisotropy (i.e. gradients) in the general background of the surveyed objects.

(d) Index of strong anisotropy:

$$a_2 = \frac{P_{02} - P_{20}}{R_{02} + R_{20}} = k^2 \frac{P_{02} - P_{20}}{2} .$$

The physical sense of this index is similar to a_1 but shows the general anisotropy in the distribution of "condensations" and not of "holes". It shows the anisotropy (gradient) in the distribution of dense fluctuations. The last two indices are, of course, direction dependent.

In addition, the so-called structural index s = c/g was introduced by Stanislaw Zieba. When the sizes of elementary domains are of the same order as the sizes of the fluctuations, the concentration index tends toward its maximum, the grouping index toward its minimum. Thus the structural index is very sensitive to characteristic sizes of fluctuations and is a powerful tool for studying the distribution of these sizes.

4. ADVANTAGES OF THE METHOD OF STATISTICAL REDUCTION

The main advantages of the reductional method are as follows:

(i) Because every real population is reduced to two "mean" objects only, the results for populations of various numbers of objects can be compared immediately to each other without special discussion.

(ii) There exist easy formulae for calculating the standard deviations of each of the defined parameters. Therefore any discussion of confidence level is simple.

(iii) The effect of anisotropy of the background and anisotropy of clustering (the existence of which is still under discussion) is automatically separated from the effects of clustering as such.

(iv) Application of the concentration index gives results of the same kind as the method of subsequent subdivision.

(v) In application to the clustering problem, the grouping index calculated for different distances of elementary cells gives equivalent information (not identical in mathematical sense!) to the correlation method.

(vi) The structural index permits one to obtain the same results as in the power spectrum method but, as it appeared in practical application, is more sensitive, i.e. shows more individual maxima at a sufficiently high confidence level.

(vii) Since the observational material is reduced to two objects, any observational selection effect is meaningless for the results, provided this effect is independent of the coordinates.

(viii) The formulae of the method are rather simple, easy for numerical calculation, and do not require much computing time. If needed, calculation for division into cells of intermediate sizes, additional configurations of elementary domains within the fundamental domains, etc., may be performed at once.

(ix) When the four independent parameters are not sufficient for describing the real distribution, additional parameters may be easily formed by applying the reduction to a number of objects greater than two and/or by using fundamental domains containing a larger number of elementary domains (cf Zieba 1975 and Garbaj 1976).

The method described here has all the advantages of the other methods and in addition some features which are characteristic of it alone. We are sure it deserves more attention among astronomers working on the distribution of extragalactic objects, not to mention other applications.

5. RESULTS

In recent years, a group of about a dozen astronomers, working at the Jagiellonian University Observatory or associated scientifically with it, have been working on the theoretical development and practical application of the reductional method. Some of these results have been published already, some others are to be presented here separately as short contributions. We want to confine ourselves here to some other results obtained by Miss Jolanta Burczyk, Stanislaw Zieba, Marek Urbanik and Konrad Rudnicki.

(a) Subclustering in well defined individual clusters

The phenomenon of subclustering inside a large, even fairly regular cluster is well known. This problem was studied in Cracow by means of the concentration index. A field of 52 square degrees was covered by overlapping plates of two colours and different exposure times taken with the Palomar Schmidt Telescope. The field contains two large clusters: 1105.3 + 2835 and 1115.2 + 3013 (clusters Nos 5 and 14 in field 156 of Zwicky's Catalogue). It was divided into squares 75 x 75 $(\text{arcmin})^2$ which corresponds roughly to the size of the nuclear areas of these clusters. Smaller squares 15 x 15 $(\text{arcmin})^2$ were also studied inside every large square separately. By comparing plates with different exposures, galaxies in different magnitude ranges were studied separately (Burczyk and Rudnicki 1978). The results are as follows:

magnitude range	concentration index	
	15' x 15' (mean value)	75' x 75'
blue magnitudes		
$m < 15.8$	2.9	1.6
$15.8 < m < 18.0$	1.5	1.3
$18.0 < m < 19.4$	1.3	1.1
yellow magnitudes		
$m < 15.0$	1.5	1.5
$15.0 < m < 16.3$	1.7	1.6
$16.3 < m < 19.0$	1.6	1.3

The formalism of the reductional method allows one to compare immediately the results of large and small scale clustering. In terms of the hierarchical clustering model one can interpret these numbers to mean that subclustering inside clusters is stronger than the "main" clustering itself. In a continuous clustering model the same numbers say that the larger the structures are the less prominent they are. I have to confess, however, that we have found a region of the sky where concentration indices for small cells are smaller than for larger ones.

(b) Distribution of objects in large regions of the sky

A large set of statistical reductions was performed for a quadratic region of the sky centered on the north galactic pole which included the entire area down to b = +66 (S. Zieba 1977). Galaxies from the Uppsala Catalogue (divided into 7 magnitude classes and into 7 morphological types), clusters of galaxies from the Abell (divided into 3 distance classes) and Zwicky (divided into 4 classes) catalogues, as well as the radio sources from catalogue 4C (4 intensity classes) have been studied.

For individual galaxies, the spectrum of characteristic dimensions is continuous with some maxima and no distinct new results have been obtained except that, for distances larger than those of large individual clusters, spiral galaxies show a distribution tending to random, whereas elliptical, SO and irregular galaxies show complicated structures in all dimensions investigated. The fact that irregular and elliptical galaxies show a similar distribution on the celestial sphere is surprising and has certainly some cosmogonic implications.

The distribution of radio sources was found to be random. This confirms what was previously generally accepted. However, it should be pointed out that for smaller sizes the structural index is systematically less than unity, and for larger sizes it tends to one. This means that for small, and only for small distances, the distribution of radio sources tends to uniformity (Figure 2).

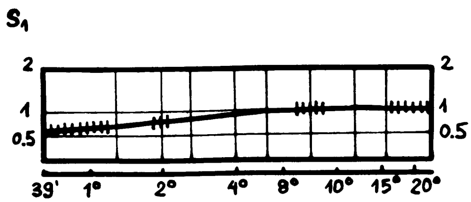

Figure 2. Structural index for radio sources with $S \geqslant 2.0 \times 10^{-26}$ Jy. Sections of the line which do not differ significantly (more than 3σ) from neighbouring sections are marked with vertical strokes.

An additional analysis has been made for the PKS and GB radio sources lying outside the north galactic cap (Urbanik 1977). It was shown that the trend toward uniformity in distribution cannot be explained by a confusion effect alone. On the other hand, individual classes of radio sources studied separately, exhibited a tendency toward clustering in small isolated regions. However this property is not common to any of these individual classes over the whole sky nor to all types of sources in any of the selected regions.

When selecting clusters of galaxies from existing catalogues, the assumption is implicitly made that the individual cluster model is accepted. The picture obtained here is rather complicated. A continuous distribution with many maxima is visible for every class of cluster. Figure 3 shows a review of the most distinct maxima. Besides the 6

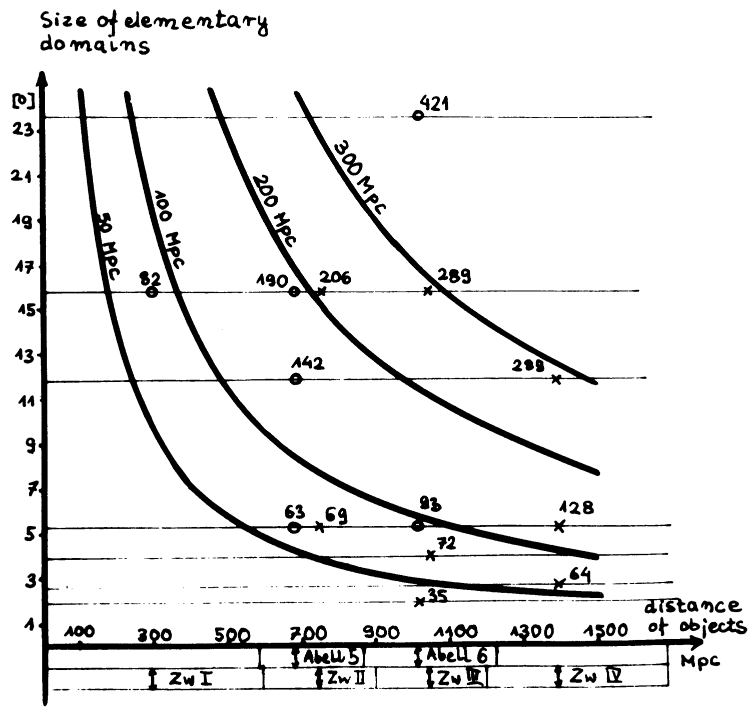

Figure 3. Significant (more than 3σ) maxima (characteristic sizes of clustering) in the distribution of clusters of galaxies. Circles - maxima known from other authors. Crosses - maxima newly obtained.

maxima already known earlier (Kiang 1967, Kiang and Saslaw 1969, Bogart and Wagoner 1973, Kalinkov 1974) marked by circles, 8 new maxima were found. The multiplicity of maxima seems to speak in favour of the continuous clustering picture. The anisotropy indices for all the investigated objects and regions show no significant deviations from zero.

Thus, using the statistical reduction formalism, all the main features known already in the distribution of objects are found in one set of calculations, and besides some new effects appear visible, which is consistent with the picture of continuous clustering rather than with the hierarchical clustering of individual systems.

The problem still remains open, but if we insist on remaining with the hierarchical picture, a very complicated model for structures of different order has to be created.

The author gratefully acknowledges the assistance of Prof. Andrzej Zieba, Mr Walter Murawski and Mr Marek Urbanik in preparing this review.

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DISCUSSION

Ostriker: It may be that the characteristics of the brightest cluster galaxies will not be helpful in distinguishing between different models of clustering. This is due to the fact that in a local extreme concentration of galaxies (whatever the origin of the maximum is), interactions between the galaxies can produce objects like cD systems. Tidal stripping, cannibalism and equipartition combine to produce a centrally located, low surface brightness supergiant system.

Rudnicki: I am very glad to hear it because I considered the first rank galaxies to be the strongest argument against my personal opinion on the clustering.

Holmberg: Your analysis does not take into account local variations in the galactic absorption, which may seriously affect the numbers of identified clusters. On account of the limited time, I refer to my paper of 1974, published in *Astronomy and Astrophysics*.

de Vaucouleurs: I should like to remind statisticians that the study of irregularities in the distribution of galaxies should take into account the patchiness of the obscuring interstellar medium in our Galaxy following the methods first developed by Ambartsumian some 40 years ago and first applied by Agekian and a few others in this country.

Fessenko: There are indications that the majority of the clusters of Zwicky and Abell are a result of the chance projection of independent galaxy systems with smaller dimensions and at different distances from us. Consequently, superclusters have been discovered among objects, which are not real clusters. Variability of the observational conditions can explain not only the dependence of the angular diameters of the apparent false clusters on distance, but also explains the analogous dependence for false superclusters.

Rudnicki: All the three last remarks are made on the same topic. We have investigated the observed distribution of objects. The final interpretation of the results has, of course, to take into account the galactic extinction also. I know Fessenko's paper and I am sure he is right to some extent, but I hope not completely. If he is, then everything we are doing is without any meaning.

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