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DEEP HABITS AND THE MACROECONOMIC EFFECTS OF GOVERNMENT DEBT

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In this paper, we study the effects of government debt on macroeconomic aggregates in a non-Ricardian framework. We develop a microfounded framework that combines time-varying markups, endogenous labor supply, and overlapping generations based on infinitely lived families. The main contribution of this paper is to provide a new transmission mechanism for public debt through the countercyclical markup movements induced by external deep habits. We analyze the effects of a positive shock to public debt. We show that the interest rate rises, entailing higher markups and a fall in employment and consumption. Interestingly, even without capital, a crowding-out effect of government debt is obtained in the long run. In addition, we show that when prices are sticky, public debt has a short-run expansionary effect, which is strengthened by the presence of deep habits.

Keywords: Wealth Effects, Public Debt Shock, Deep Habits, Overlapping Generations

1. INTRODUCTION

The latest global economic crisis has pushed many governments to intervene to fight the recession. The active use of fiscal policy has raised concerns about debt and revived the old debate about the impact of government debt on economic activity. Indeed, public debt in advanced economies has reached levels not seen before in peacetime. According to the IMF, the ratio of public debt to GDP in the G20 countries surged from 78% in 2007 to 97% in 2009, and is projected to rise to 115% in 2011.[1] The purpose of this paper is to contribute to the literature aiming at evaluating the short-run and long-run effects of public debt on macroeconomic aggregates.

The question of whether public debt is detrimental to a country's growth is an old one, and many economists have attempted to address it. As summarized by Bernheim (1989) and Elmendorf and Mankiw (1999), the conventional economic effects of government debt seem to be expansionary in the short run (the traditional Keynesian view) and contractionary in the long run (the neoclassical view). For

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instance, a raise in public debt to finance tax cuts stimulates aggregate demand, causing output to increase when prices (and/or wages) are sticky. This is the short-run effect. However, the real interest rate must rise to bring the securities market into balance. Consequently, investment is crowded out and capital and output eventually decrease. This is the long-run effect. These two views, however, abstract from an important dimension along which the economy adjusts to shocks, namely, endogenous variations in the markup.

For instance, Fitoussi and LeCacheux (1988) and Fitoussi and Phelps (1988) show that episodes of high real interest rates are often associated with high markups. One explanation is that a higher real interest rate means that future profits are more heavily discounted, thereby increasing firms' markups. On the other hand, Bils (1989) and Rotemberg and Woodford (1991), among others, show that markups are countercyclical. During recession, imperfectly competitive firms charge high markups because they compete less aggressively.

Together, these findings suggest an additional/alternative transmission mechanism through which public debt affects the macroeconomy, one that operates through countercyclical markup variations. The aim of this paper is to develop a microfounded model able to take into account this new transmission mechanism. In other words, this paper offers a microfounded general equilibrium non-Ricardian model with time-varying markups. Our model builds on the deep-habit model of Ravn et al. (2006). These authors show that the assumption of external deep habits profoundly alters the supply side of the economy. Under external deep habits, households do not simply form habits from a benchmark consumption level, but rather feel the need to keep up with the Joneses on a good-by-good basis.[2] Households that consume a large amount of a particular good today are more likely to buy this kind of good in the future by force of habit. Such behavior influences firms' pricing strategy. Indeed, under deep habits, the demand for goods faced by firms becomes dynamic, implying time-varying markups. In particular, a higher real interest rate implies higher markups because firms discount future profits more heavily. As a consequence, labor demand, output, and consumption decrease. In addition, the decline in aggregate demand entails lower elasticity of demand, inducing higher markups. This is a price–elasticity of demand effect that strengthens the decline in output.

However, Ravn et al.'s model cannot be used to analyze the effects of public debt because it adopts the assumption of infinitely lived representative households, implying the neutrality of government debt.

A departure from Ricardian equivalence is necessary to study the effects of public debt. In our model we break Ricardian equivalence by assuming an overlapping-generations structure. The model also assumes monopolistic competition and external deep habit formation. Importantly, it abstracts from capital accumulation. We show that an increase in government debt to finance government transfers has a long-run contractionary effect despite the lack of private capital. On the other hand, the short-run effect of debt-financed government transfers is contractionary in a flexible-price framework, whereas it is expansionary in a sticky-price framework.

At this stage, it is of interest to note that, in the non-Ricardian framework, the short-run and the long-run effects on output depend on the assumptions made about price adjustment, labor supply, and capital.

First, if labor is supplied inelastically, there is no short-run effect on output even when prices are sticky. In the long run, output decreases because of capital adjustment. Indeed, Annicchiarico (2007) shows that the increase in aggregate demand caused by the rise in government debt entails higher consumption and higher real interest rates in the short run. The real interest rate rise crowds out investment and output falls in the long run.

Second, if labor supply is endogenous and physical capital is absent, when prices are flexible, increased government debt will have no short-run or long-run effects on output. Labor supply and thus output are determined by an intratemporal first-order condition. Thus, government debt is neutral despite the non-Ricardian framework. However, if prices are sticky, a short-run expansionary effect on output is evident, but there is no long-run effect. This result is found in Devereux (2010). He analyzes the effect of government debt increase in a non-Ricardian framework without capital and with sticky prices and shows that higher government debt leads to a consumption and output rise in the short run.

In this paper, even in the flexible-price model, the government-debt neutrality expected to occur does not hold. Instead, we find that government debt increase implies a decline in output because of the countercyclical markup movements induced by the assumption of external deep habits. Hence, this paper offers a new transmission mechanism for government debt through countercyclical markup movements. The transmission mechanism can be summarized as follows. Debt-financed government transfers raise the interest rate, entailing higher markups, which in turn induce a fall in employment and consumption.

In the sticky-price model, an increase in government debt induces an increase in consumption and aggregate demand. Because prices cannot fully adjust to balance the goods market, output increases. Thus a short-run expansionary effect is obtained. In addition, we show that deep habits specification strengthens the short-run expansionary effect of government debt because increased aggregate demand entails higher elasticity of demand and thus markups decrease, implying high output.

The remainder of the paper is organized as follows. The next section develops the model. Sections 3 and 4 study the symmetric equilibrium, the steady state equilibrium, and the implications of an increase in government debt in the long run. Section 5 investigates the impact of temporary and public debt shocks. Section 6 concludes.

2. THE MODEL

The economy consists of three types of agents: infinitely lived dynasties (or families), monopolistically competitive firms, and the fiscal authority. Each period, new and identical infinitely lived families (a component of a generation) appear in the economy without financial wealth and owing a monopolistically competitive firm that produces a specific good using labor. It is assumed that the firm's ownership is not transferable. Therefore, the profit of the family firms is transferred in full to the owner-manager (the infinitely lived family). On the other hand, labor moves freely in this economy.

Moreover, there is uncertainty in the economy, caused by fiscal shocks. However, we assume that agents have access to complete markets. In addition, as in most of the recent New Keynesian literature, we assume a cashless economy à la Woodford (2003). Here, money is only a unit of account.

2.1. Consumers

A generation j consists of many identical infinitely lived families (or agents) of type j, where j belongs to the interval $[1, N_t]$. Accordingly, we can consider a representative-agent framework in a generation. In this economy, agents care about their own consumption of a specific good compared to the benchmark level of the consumption of that specific good. We start by giving the aggregation rule that will be used to aggregate individual variables,

$$z_t = \sum_{j \leq t-1} \frac{(N_j - N_{j-1})}{N_{t-1}} z_{j,t-1}, \tag{1}$$

where z is a generic variable. Notice that $N_j - N_{j-1}$ is the number of agents of the representative generation $j \leq t$, where N_j is the number of agents born in period $j \leq t$.

We adopt an extended version of the CES habit-adjusted consumption index, $x_{j,t}$, used by Ravn et al. (2006),

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left\{ \sum_{m=1}^{M_t} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where

$$\tilde{c}_{t-1}(m) = \begin{cases} c_{t-1}(m) & \forall m \leq M_{t-1} \\ c_{t-1} & \forall m \in]M_{t-1}, M_t]. \end{cases}$$

Here $x_{j,t}$ denotes the CES habit-adjusted consumption index with elasticity of substitution, $\varepsilon > 1$. The parameter ϕ measures the degree of external habit formation in the consumption of each variety. When $\phi = 0$, consumption externalities disappear. $c_{j,t}(m)$ is the consumption of good $m \in [1, M_t]$ by agent j, born in period $j \leq t$, and $c_{t-1}(m)$ denotes the per capita aggregate consumption of good m in period $t-1$. $c_{j,t-1}$ and c_{t-1} denote individual consumption of a basket of goods in period $t-1$ and per capita aggregate consumption of the basket of goods in period $t-1$, respectively.[3]

Notice that the consumption reference used in (2) differs from the one used in Ravn et al. (2006). The reason is the following. Remember that each agent is the owner of a monopolistically competitive firm, so the number of specific goods grows at the same rate as the population. The appearance of new specific goods in each period raises a new difficulty in developing a deep-habits non-Ricardian model. Indeed, new goods appearing in period t were not consumed in period $t-1$. Consequently, the benchmark level cannot be the average level of past consumption of those goods. Therefore, we assume that agents observe per capita aggregate consumption of the basket of goods in period $t-1$, which will be considered as the benchmark level of the consumption of goods appearing between periods $t-1$ and t. This assumption is important in developing a deep-habits non-Ricardian model precluding a life cycle of goods and eliminating any discontinuity between the first period and the next periods. This is also helpful in restore symmetry in the firm's decisions.

Letting β denote the constant subjective discount factor and E_t the mathematical expectations operator conditional on information available in period t, the lifetime utility of a representative agent j is

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \ln[x_{j,s} - d(l_{j,s})], \tag{3}$$

where $d(l_{j,t})$ is an increasing and convex function measuring the disutility of the labor supply of agent j, $l_{j,t}$; more specifically

$$d(l_{j,t}) \equiv \alpha \frac{l_{j,t}^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}, \tag{4}$$

with $\sigma > 0$ representing the Frisch elasticity of labor supply.

The specification given by (3) features preferences à la Greenwood et al. (1988) (henceforth "GHH"). The reason is twofold. First, it helps to make aggregation feasible. We will show later that the GHH specification makes labor age-independent, which is necessary to aggregate individual human wealth. Second, in our overlapping-generations structure, the labor supply is endogenous, which raises a potential problem of negative labor supply. Actually, if leisure is a normal good, wealthier agents will supply less labor. Indeed, if labor is not constrained by a lower positive bound, then labor supply may be negative. As shown by Ascari and Rankin (2007), the GHH specification makes labor supply independent of wealth.[4]

In each period, agents supply labor, $l_{j,t}$, in a competitive market and receive nominal wages, $P_t w_t$, which are independent of the agents' age.

Agent j maximizes its expected utility subject to its intertemporal budget constraint,

$$\sum_{m=1}^{M_t} P_t(m) c_{j,t}(m) + E_t Q_{t,t+1} V_{j,t+1} \leq V_{j,t} + P_t w_t l_{j,t} + \Psi_{j,t} + T_{j,t}. \tag{5}$$

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where $P_t(m)$ is the nominal price of the differentiated good m. Agent j receives an average nominal profit $\Psi_{j,t}$ from the family's ownership of a monopolistic firm and receives lump-sum government transfers $T_{j,t}$. $V_{j,t}$ represents agent j's financial asset holding. $Q_{t,t+1}$ is the stochastic discount factor, and more generally,

$$Q_{t,T} = Q_{t,t+1} \times Q_{t+1,t+2} \times \ldots \times Q_{T-1,T} \quad \text{and } Q_{t,t+1} = 1. \tag{6}$$

In addition, because markets are complete, there is a risk-free one-period interest rate defined by

$$1 + i_t = [E_t Q_{t,t+1}]^{-1}. \tag{7}$$

First, we start by solving the dual problem. For any given level of $x_{j,t}$, the agent j demand for individual goods varieties must solve the cost-minimization problem

$$\min \sum_{m=1}^{M_t} P_t(m) c_{j,t}(m)$$

subject to the aggregate constraint (2). Solving this problem yields the demand functions

$$c_{j,t}(m) = \frac{1}{M_t}\left[\frac{P_t(m)}{P_t}\right]^{-\varepsilon} x_{j,t} + \phi \tilde{c}_{t-1}(m), \quad \text{for all } m \in [1, M_t]. \tag{8}$$

The price index is defined by

$$P_t \equiv \left[\frac{1}{M_t}\sum_{m=1}^{M_t} P_t(m)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{9}$$

where $P_t(m)$ denotes the nominal price of good m.

Using equation (8) and the definition of the price index (9), we define the total expenditure on habit-adjusted consumption as[5]

$$P_t x_{j,t} = \sum_{m=1}^{M_t} P_t(m) c_{j,t}(m) - \phi \sum_{m=1}^{M_t} P_t(m) \tilde{c}_{t-1}(m). \tag{10}$$

Notice that demand for good m by agent j, equation (8), features a dynamic component, as it depends not only on current-period habit-adjusted consumption, $x_{j,t}$, but also on the lagged value of consumption of good m. This, in turn, makes the pricing decision of firm $m \in [1, M_t]$ intertemporal. Indeed, as pointed out by Ravn et al. (2006), the deep-habits assumption makes the price elasticity of demand procyclical. From equation (8), we can easily see that an increase in the level of $x_{j,t}$ raises the relative importance of the price-elastic term, $1/M_t(P_t(m)/P_t)^{-\varepsilon} x_{j,t}$, and reduces the relative importance of the price-inelastic demand component, $\phi \tilde{c}_{t-1}(m)$. As a result, the price elasticity of demand for good m increases with aggregate demand.

The second stage of household j's problem consists of choosing its demand for $x_{j,t}$ and its financial asset holdings $V_{j,t+1}$ resulting from the maximization of lifetime utility (3) subject to the dynamic budget identity (5). The first-order conditions for this maximizing problem yield the following optimality conditions:

$$x_{j,t} - d(l_{j,t}) = \beta^{-1} Q_{t,t+1} \frac{P_{t+1}}{P_t} [x_{j,t+1} - d(l_{j,t+1})], \quad \forall j \text{ and } \forall s^t, \tag{11}$$

$$d_l(l_{j,t}) = w_t, \tag{12}$$

$$P_t x_{j,t} + \phi \sum_{m=1}^{M_t} P_t(m) \tilde{c}_{t-1}(m) + E_t Q_{t,t+1} V_{j,t+1} = V_{j,t} + P_t w_t l_{j,t} + \Psi_{j,t} + T_{j,t}, \tag{13}$$

$$\lim_{T \to +\infty} E_t Q_{t,T} V_{j,T} = 0. \tag{14}$$

We note from equation (12) that labor is independent of the agent's age and also independent of the agent's consumption. This is a consequence of the GHH preferences, which feature no wealth effect on hours. Equation (13) is the intertemporal budget constraint of agent j, which is obtained by introducing (10) into (5). Equation (14) represents the transversality condition.

Moreover, we notice, from (11), that the standard Euler equation is modified in two ways. First, it is expressed in terms of individual habit-adjusted consumption $x_{j,t}$ rather than individual consumption $c_{j,t}$. Second, the term $d(l_{j,t})$ is subtracted from the individual habit-adjusted consumption $x_{j,t}$. As we have already mentioned, the term $d(l_{j,t})$ is independent of agents' age; i.e., it is identical for all agents. Consequently, we can drop the subscript j. In addition, Ascari and Rankin (2007) assume that $d(l_t)$ acts as a subsistence level of consumption. For this reason, they define adjusted consumption as individual habit-adjusted consumption minus its subsistence level $[d(l_t)]$. We follow Ascari and Rankin (2007) and define adjusted consumption as[6]

$$a_{j,t} \equiv x_{j,t} - d\,(l_t). \tag{15}$$

Moreover, let us define the stochastic gross interest rate by

$$R_{t,t+1} = \left(Q_{t,t+1} \frac{P_{t+1}}{P_t} \right)^{-1}. \tag{16}$$

Accordingly, using (15) and (16), equation (11) and (13) can be rewritten in real terms as

$$a_{j,t} = \beta^{-1} R_{t,t+1}^{-1} a_{j,t+1} \tag{17}$$

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and

$$x_{j,t}+\phi\sum_{m=1}^{M_t}\frac{P_t(m)}{P_t}\tilde{c}_{t-1}(m)+E_t\frac{v_{j,t+1}}{R_{t,t+1}}=v_{j,t}+w_tl_{j,t}+\psi_{j,t}+\tau_{j,t}, \quad \textbf{(18)}$$

where $v_{j,t}=V_{j,t}/P_t$, $\psi_{j,t}=\Psi_{j,t}/P_t$, and $\tau_{j,t}=T_{j,t}/P_t$.

In addition, we define human wealth (discounted present value of labor income and profits plus government transfers) as

$$h_{j,t}=E_t\sum_{s=t}^{\infty}R_{t,s}^{-1}[w_sl_{j,s}+\psi_{j,s}+\tau_{j,s}]. \quad \textbf{(19)}$$

Iterating the budget constraint (18) forward (from t to infinity), taking into account the no–Ponzi game restriction, and using (17) iterated forward (from t to infinity) and the definition of human wealth (19) yields[7]

$$a_{j,t}=(1-\beta)(v_{j,t}+h_{j,t}-\chi_t), \quad \textbf{(20)}$$

where

$$\chi_t=\phi E_t\sum_{s=t}^{\infty}R_{t,s}^{-1}\sum_{m=1}^{M_t}\frac{P_s(m)}{P_s}\tilde{c}_{s-1}(m)$$

denotes the future time path of reference consumption.

We note from (20) that in the absence of a consumption externality ($\phi=0$ and thus $\chi_t=0$), individuals condition their consumption solely on their consolidated wealth $(v_{j,t}+h_{j,t})$, with a ratio of $(1-\beta)$ of total wealth. With a nonzero consumption externality, however, individual adjusted consumption is also directly affected by the future time path of economywide per capita consumption of good m.

So far, we have focused on individual variables. Now we consider aggregate variables. Variables without the subscript j represent a per capita aggregate value. We apply the aggregation rule used in (1) to $x_{j,t}$ and $v_{j,t}$. Agents are assumed to receive the same amount of government transfers independent of their age, so $\tau_{j,t}=\tau_t$. Moreover, we will show later that, as firms display the same behavior, the average profits received from firms are independent of the agents' age; i.e., $\psi_{j,t}=\psi_t$. Accordingly, because $l_{j,t}$ is the same for all age cohorts, human wealth is also the same for all, namely $h_{j,t}=h_t$.

Finally, notice that applying the aggregation rule used in (1) in period t to the variable $v_{j,t+1}$ yields

$$\sum_{j\le t}\frac{(N_j-N_{j-1})}{N_t}v_{j,t+1}=(1+n)v_t,$$

as generation $j=t+1$ has no financial wealth in period $t+1$; i.e., $v_{t,t}=0$. Here, n denotes the population growth rate, i.e., $N_t=(1+n)\,N_{t-1}$.

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Using this result and aggregating equation (11), where we replace $a_{j,t+1}$ with its expression given by equation (20) expressed in $t+1$, one obtains[8]

$$a_t = \beta^{-1} R_{t,t+1}^{-1} a_{t+1} + n(\beta^{-1} - 1) R_{t,t+1}^{-1} v_{t+1}. \tag{21}$$

This equation is the aggregate Euler equation, which differs from the individual Euler condition (11) as long as the population growth rate is different from zero. The last term on the right-hand side reflects a real wealth effect, which is characteristic of a non-Ricardian economy. Indeed, the growth rate of aggregate adjusted consumption is negatively correlated with the aggregate financial wealth. An increase in beginning-of-period financial wealth in period $t+1$ cannot be proportionally allocated to present and future aggregate adjusted consumption, because only those consumers alive during this period benefit.

2.2. Firms

This section focuses on the supply side. Here we describe the problem of a firm m that appeared before $t-1$. Later on, we will show that new firms behave in the same way as old firms.

The differentiated good $m \in [1, M_{t-1}]$ is produced by a monopolist, m, who uses labor input $l_t(m)$ and specific human capital—normalized to one—to produce a quantity $y_t(m)$ using linear production technology:

$$y_t(m) = l_t(m). \tag{22}$$

Firms are assumed to be price setters. We assume that monopolistic firms are subject to Rotemberg's (1982) convex adjustment costs associated with changing nominal prices,

$$\frac{\kappa}{2}\left[\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right]^2, \tag{23}$$

where $\bar{\Pi}$ denotes the steady state inflation rate and $\kappa \geq 0$ measures the degree of nominal rigidities. When $\kappa = 0$ prices are flexible, whereas positive values of κ imply that firms find it costless to adjust their prices in line with the central bank inflation target.

Letting

$$\psi_t(m) = \frac{P_t(m)}{P_t} y_t(m) - w_t y_t(m) - \frac{\kappa}{2}\left[\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right]^2 \tag{24}$$

define firm m's real profits in period t, using (6), the owner-manager m's problem is to maximize the discounted value of the sum of its present and future cash flows,

$$E_t \sum_{s=t}^{T} R_{t,s}^{-1} \psi_s(m),$$

subject to (22), and

$$y_t(m) = \left[\frac{P_t(m)}{P_t}\right]^{-\varepsilon} \frac{N_t x_t}{M_t} + \phi(1+n)\tilde{y}_{t-1}(m), \tag{25}$$

where equation (25) is given by the aggregation of (8) expressed in level terms. x_t is per capita habit-adjusted consumption.

Note that the marginal costs of firm m are equal to real wages, w_t. The first-order conditions corresponding to firm m's optimization problem give the following equilibrium equations: (25),

$$\lambda_t(m) = \frac{P_t(m)}{P_t} - w_t + \phi(1+n)E_t\frac{\lambda_{t+1}(m)}{R_{t,t+1}}, \tag{26}$$

and

$$y_t(m) - \kappa\frac{P_t}{P_{t-1}(m)}\left[\frac{P_t(m)}{P_{t-1}(m)} - \bar{\Pi}\right] + \kappa E_t\frac{P_{t+1}(m)}{R_{t,t+1}P_t(m)}\left[\frac{P_{t+1}(m)}{P_t(m)} - \bar{\Pi}\right]$$
$$= \varepsilon\lambda_t(m)\frac{N_t}{M_t}x_t\left[\frac{P_t(m)}{P_t}\right]^{-\varepsilon-1}. \tag{27}$$

$\lambda_t(m)$ is the Lagrangian multiplier associated with (25) and represents the shadow value of selling an extra unit of good m in period t. Equation (26) states that the value of selling an extra unit of good m in period t, $\lambda_t(m)$, has two components. The first term on the right-hand side represents the short-run profit margin of firm m in period t. The second term on the right-hand side corresponds to the future expected profits associated with selling an extra unit of good m in period t.

Furthermore, remember that demand for new goods—appearing in t, i.e. $m \in]M_{t-1}, M_t]$— features a dynamic component, the same as the old goods, which is the average of goods supplied in period $t-1$. Consequently, firms appearing in period t are also subject to (25) and have the same optimality conditions as firms appearing in periods before $t-1$.

Let $\eta_t(m)$ denote the relative markup, i.e., the ratio between profit margin (prices minus marginal cost) and prices charged by firm m,

$$\eta_t(m) \equiv \frac{P_t(m) - P_t w_t}{P_t(m)}, \tag{28}$$

and define the absolute value of price elasticity of demand as

$$\epsilon_t(m) \equiv \varepsilon\left[1 - \phi(1+n)\frac{y_{t-1}(m)}{y_t(m)}\right]. \tag{29}$$

To easily understand the effects of time-varying markup induced by the deep-habits assumption, let us consider the case $\kappa = 0$. Rearranging equation (27) using

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(25) and the definition (29) yields

$$\lambda_t(m) = \frac{P_t(m)}{P_t}\epsilon_t^{-1}(m). \tag{30}$$

Equation (30) states that the value of selling an extra unit of good m in period t equals the inverse of the price elasticity of demand. Now, combining (26) and (30) leads to

$$\eta_t(m) = \frac{P_t(m)}{\epsilon_t(m)P_t} - \phi(1+n)E_t R_{t,t+1}^{-1}\lambda_{t+1}(m). \tag{31}$$

Notice that in the absence of deep habits, i.e., $\phi = 0$, the price elasticity of demand and the relative markup lose their dynamic component and equal ε^{-1}.

Equation (31) shows that the short-run relative markup of the firm m in period t is inversely related to the price elasticity of demand for good m, $\epsilon_t(m)$, and it is negatively related to the future expected profits associated with selling an extra unit of good m in period t, $\lambda_{t+1}(m)$. Also, it is positively related to the discount factor $R_{t,t+1}$. Moreover, the deep-habit assumption gives rise to two effects, a price elasticity effect and an intertemporal effect. Ravn et al. (2006) explain these effects clearly.

First, when aggregate demand for good m, $y_t(m)$, increases, the price elasticity of demand, $\epsilon_t(m)$, decreases, inducing a decline in the short-run profit margin of firm m in period t, and thus a decline in markups: this is what Ravn et al. (2006) call the price-elasticity effect of deep habits on markup. Second, today's price decisions will affect future demand, and so when the present value of future per-unit profit is expected to be high, firms have an incentive to invest in the customer base today. Therefore, they induce higher current sales by lowering the current markups. Ravn et al. (2006) call this effect the intertemporal effect of deep habits on markup. The intertemporal effect is also driven by the change in the real interest rate. Indeed, if the real interest rate goes up, then the firm discounts future profits more, and thus has less incentive to invest in market share today.

2.3. Government

In period t, the government gives lump-sum transfers to households and issues one-period risk-free government bonds. Government expenditures are assumed to be zero. Therefore, government revenues are obtained from debt issue. The flow budget constraint of the government reads as

$$\frac{B_{t+1}}{(1+i_t)} = B_t + T_t, \tag{32}$$

where B_t and T_t are nominal governmentbonds issued at the start of period $t-1$ and total lump-sum government transfers, respectively. i_t is the one-period

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risk-free nominal interest rate. For the government to remain solvent, the no-Ponzi condition must be satisfied.[9]

Letting $b_t = \frac{B_t}{N_t P_{t-1}}$, $\tau_t = \frac{T_t}{N_t P_t}$, the government budget constraint is rewritten in real terms as

$$b_{t+1} = \frac{1+i_t}{(1+n)}\left(\frac{b_t}{\Pi_t} + \tau_t\right), \tag{33}$$

where b_t, R_t, and τ_t are the number of per capita government bonds issued at the start of period $t-1$, the risk-free return, and the per capita lump-sum transfers, respectively.[10]

In this paper, we focus on the effects of a change in public debt. Actually, the fiscal shock used in the analysis is a public debt shock. For this reason, we specify a fiscal rule such that the law of motion of public debt follows a first-order autoregressive process,

$$b_{t+1} = \rho_b b_t + (1-\rho_b)\,\bar{b} + \xi_t, \tag{34}$$

where ξ_t reflects a public debt shock, $\bar{b}$ is the target level of long-run debt, and $0 < \rho_b < 1$ denotes the speed of debt adjustment.

Specifically, using (33) and (34), our debt-stabilizing fiscal rule is as follows:

$$\tau_t = \left(\rho_b \frac{1+n}{1+i_t} - \Pi_t^{-1}\right) b_t + \frac{1+n}{1+i_t}[(1-\rho_b)\bar{b} + \xi_t]. \tag{35}$$

2.4. Monetary Authority

The monetary authority controls the nominal interest rate. Specifically, monetary policy is assumed to be described by a simple Taylor rule, given by

$$1 + i_t = \rho_i\,(1 + i_{t-1}) + (1-\rho_i)\left[\bar{R}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\varphi}\right]. \tag{36}$$

$\bar{\Pi}$ represents the long-run target level for the inflation rate. $\bar{R}$ is the steady state equilibrium gross real interest rate. Note that the Taylor formulation (36) is modified to allow interest rate smoothing, as proposed by Clarida et al. (1998). Indeed, Woodford (1999) argues that central bank behavior has an inertial character that shows up in estimated central bank reaction functions.

In particular, the parameter $\rho_i \in [0, 1]$ captures the degree of interest rate smoothing. $\varphi > 1$ is the Taylor rule coefficient, describing the degree of responsiveness of interest rates to inflation.

It is noteworthy to point out that, as was shown by Aloui and Guillard (2009), taking into account the zero lower bound (ZLB, henceforth) on nominal interest rates in a non-Ricardian model leads to multiple steady-state solutions. Precisely, the authors find four steady state equilibria. However, this is not the issue in this paper. We want to focus on the transmission mechanism of government debt through the time-varying markups. Therefore, the ZLB is not incorporated into the specification of (36).

3. SYMMETRIC EQUILIBRIUM

Firms are different in their dates of appearance. Recall that, in our model, even firms appearing in t face dynamic (backward) demand for goods. Thus, assuming that all firms make the same decisions, in $t-1$, implies that firms display the same behavior and make the same decisions also in period t. As we have already mentioned, agents are owner-managers of monopolistically competitive firms; i.e., $M_t = N_t$. Accordingly, $P_t(m) = P_t$, $c_t(m) = c_t$, $y_t(m) = y_t$, $l_t(m) = l_t$, $\eta_t(m) = \eta_t$, and $\epsilon_t(m) = \epsilon_t$. In addition, the equilibria in the financial market, in the goods market, and the labor market are given by

$$v_t = \frac{b_t}{\Pi_t},$$

$$y_t = c_t + \frac{\kappa}{2}(\Pi_t - \bar{\Pi})^2,$$

$$l_t = y_t.$$

It follows that we can describe the symmetric equilibrium using the following set of equations:

$$a_t = \beta^{-1}\frac{a_{t+1}}{R_{t,t+1}} + \zeta\frac{b_{t+1}}{\Pi_{t+1}R_{t,t+1}}, \tag{37}$$

$$a_t = y_t - \tilde{\phi}y_{t-1} - d\left(y_t\right), \tag{38}$$

$$\eta\left(y_t\right) = \lambda_t - \tilde{\phi}E_t\frac{\lambda_{t+1}}{R_{t,t+1}}, \tag{39}$$

$$\lambda_t \times \epsilon_t(y_t, y_{t-1}) = \kappa\frac{\Pi_t}{y_t}(\Pi_t - \bar{\Pi}) - \kappa E_t\frac{\Pi_{t+1}}{y_t R_{t,t+1}}(\Pi_{t+1} - \bar{\Pi}) - 1, \tag{40}$$

$$b_{t+1} = \rho_b b_t + (1-\rho_b)\,\bar{b} + \xi_t, \tag{41}$$

$$1 + i_t = \rho_i\left(1 + i_{t-1}\right) + \left(1-\rho_i\right)\bar{R}\bar{\Pi}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\varphi}, \tag{42}$$

$$1 + i_t = \left[E_t\frac{1}{R_{t,t+1}\Pi_{t+1}}\right]^{-1}, \tag{43}$$

where

$$\epsilon_t\left(y_t, y_{t-1}\right) \equiv -\varepsilon\left(\frac{y_t - \tilde{\phi}y_{t-1}}{y_t}\right), \tag{44}$$

$$\eta\left(y_t\right) \equiv 1 - d_l\left(y_t\right), \tag{45}$$

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$\tilde{\phi} = \phi(1+n)$, $\zeta = n(\beta^{-1} - 1)$, and

$$d\,(y_t) = \alpha \frac{y_t^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}}.$$

Equation (43) is the Fisher equation. (38) gives the definition of adjusted consumption. (38) is obtained by replacing per capita habit-adjusted consumption with its expression given by (25) in the definition of aggregate adjusted consumption given by the aggregation of (15). (37) is the modified aggregate Euler equation. (41) states that government debt is stabilized, in each period, with an adjustment speed ρ_b. (44) is obtained from (29), and states that the elasticity of demand is negatively related to aggregate demand, y_t. In the symmetric equilibrium (31) becomes (39), which states that the relative markup is dynamic.

In the absence of deep habits and price rigidities, i.e., $\phi = 0$ and $\kappa = 0$, the relative markup is invariant and equals ε^{-1}. We note that using the definition (45) with equation (39) gives the equilibrium level of labor. As a consequence, the level of output is determined, as is consumption. In this case, fiscal policy is neutral despite the non-Ricardian structure. Accordingly, wealth effects are nonsignificant. In fact, a change in government debt affects only the real interest rate.

In the deep habit case, i.e., $\phi \neq 0$, equation (39) does not solely determine the equilibrium level of employment. We notice, from equation (39), that the markup depends on the present value of future marginal profits induced by a unit increase in current sales, $\tilde{\phi} E_t \frac{\lambda_{t+1}}{R_{t,t+1}}$, and the short-run price elasticity of demand. In this case, wealth effects matter. For instance, an increase in debt to finance government transfers in period t implies a rise in the real interest rate, which has an impact on the markup. The description of this new mechanism is illustrated in Section 5, which gives the response of the economy to public debt shock.

4. STEADY STATE EQUILIBRIUM

In this section we analyze the long-run effects of fiscal policy on the steady state levels of consumption, output, and real interest rates. If we drop the time subscript and use (7), the system of equations (37)–(43) becomes

$$R = \beta^{-1} + \zeta \frac{\bar{b}}{a\bar{\Pi}}, \tag{46}$$

$$d_l(y) = 1 - \left(\frac{1 - \frac{\tilde{\phi}}{R}}{1 - \tilde{\phi}} \right) \varepsilon^{-1}, \tag{47}$$

$$a = (1 - \tilde{\phi}) y - d(y), \tag{48}$$

$$b = \bar{b}, \tag{49}$$

with

$$d(y) = \alpha \frac{y^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \tag{50}$$

and

$$d_l(y) = \alpha y^{\frac{1}{\sigma}}.$$

First of all, we notice that $\eta \geq \varepsilon^{-1}$. The steady state markup in the presence of deep habits, i.e., when $\phi \neq 0$, is greater than the steady state markup in the absence of deep habits, i.e., when $\phi = 0$. Firms have more market power in the presence of deep habits. Indeed, charging a low markup in the short run implies high market power in the long run because of the habit effect.

The preceding steady state system, (46) and (47), can be rewritten as

$$R \equiv \Re(y) = \beta^{-1} + \zeta \frac{\bar{b}}{a\bar{\Pi}}, \tag{51}$$

$$y \equiv \Upsilon(R) = \left\{ \left[1 - \frac{1 - \frac{\tilde{\phi}}{R}}{\varepsilon\left(1-\tilde{\phi}\right)} \right] \alpha^{-1} \right\}^{\sigma}, \tag{52}$$

with

$$a = (1-\tilde{\phi})y - \frac{y^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}},$$

where (52) is obtained by substituting the derivative of $d\,(l)$ in (50) into (47).

We show in Appendix D that the necessary and sufficient condition for the existence and the uniqueness of the steady state equilibrium is

$$0 < \underline{y} < \overline{y}, \tag{53}$$

where

$$\underline{y} \equiv \left[\frac{1 - \frac{1}{\varepsilon(1-\tilde{\phi})}}{\alpha} \right]^{\sigma}$$

and

$$\overline{y} \equiv \left[\frac{(1-\tilde{\phi})\left(1+\frac{1}{\sigma}\right)}{\alpha} \right]^{\sigma}.$$

Equivalently to (53), we have

$$\begin{cases} \tilde{\phi} < \tilde{\phi}_{\max} \equiv (1-\varepsilon^{-1}), & \text{for } \sigma < 4(\varepsilon-4)^{-1}, \\ \tilde{\phi} \in [0, \tilde{\phi}_1) \cup (\tilde{\phi}_2, \tilde{\phi}_{\max}), & \text{for } \sigma < 4\,(\varepsilon-4)^{-1}, \end{cases}$$

where

$$\tilde{\phi}_1, \tilde{\phi}_2 = \frac{2d-1}{2d} \mp \sqrt{\frac{\varepsilon-4d}{4\varepsilon d^2}}, \quad \text{with } d = 1 + \frac{1}{\sigma}.$$

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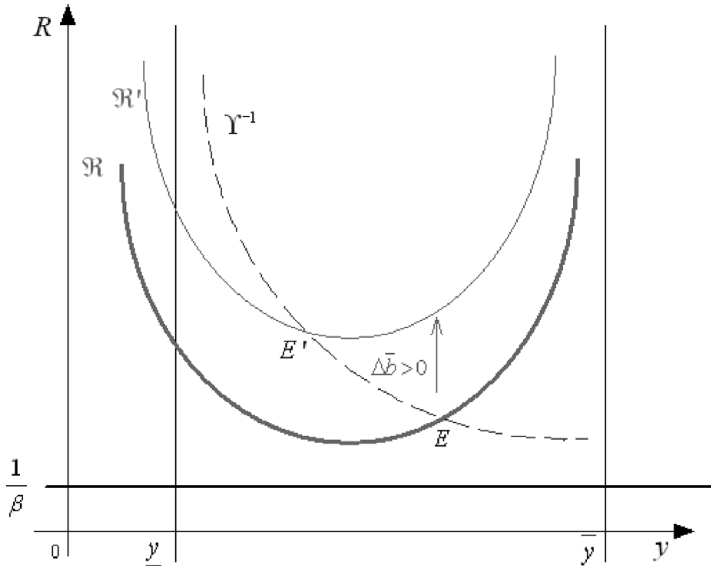

FIGURE 1. Steady state equilibrium.

Equations (51) and (52) are graphed in Figure 1. The functions Υ and $\Re$ are represented by the dashed-line curve and the solid-line curve in the yR plane, respectively. We easily see that in the interval $(\underline{y}, \overline{y})$, the two curves intersect once. The steady state equilibrium is given by E.

Figure 1 also displays the qualitative effects of a change in the long-run level of public debt, $\bar{b}$. If $\bar{b}$ increases ($\Delta\bar{b} > 0$), the $\Re$ curve moves upward, entailing an increase in long-run gross interest rates, R, and a decrease in long-run output, y. The new steady state equilibrium is given by E'.

We emphasize that in the long run, the crowding-out effect of public debt is obtained even without capital. Our paper proposes a new transmission mechanism of fiscal policy that is based on the absence of Ricardian equivalence and the countercyclical movement of the markup. In the next section, we analyze the effects of temporary debt-financed public transfers.

We also give a numerical illustration of the long-run effects of deep habits. We therefore give values to the parameters. We adopt the calibration used in Ravn et al. (2006).[11] Notice that accordingly $\sigma < 4(\varepsilon - 4)^{-1}$, so the necessary and sufficient condition for steady state equilibrium is $\tilde{\phi} < \tilde{\phi}_{\max}$. Figure 2 displays the effects of the variation of ϕ from 0 to

$$\phi_{\max} = \frac{(1 - \varepsilon^{-1})}{(1 + n)}.$$

Figure 2 shows that a greater degree of habit formation implies lower long-run levels of consumption and output, higher long-run levels of markup and real

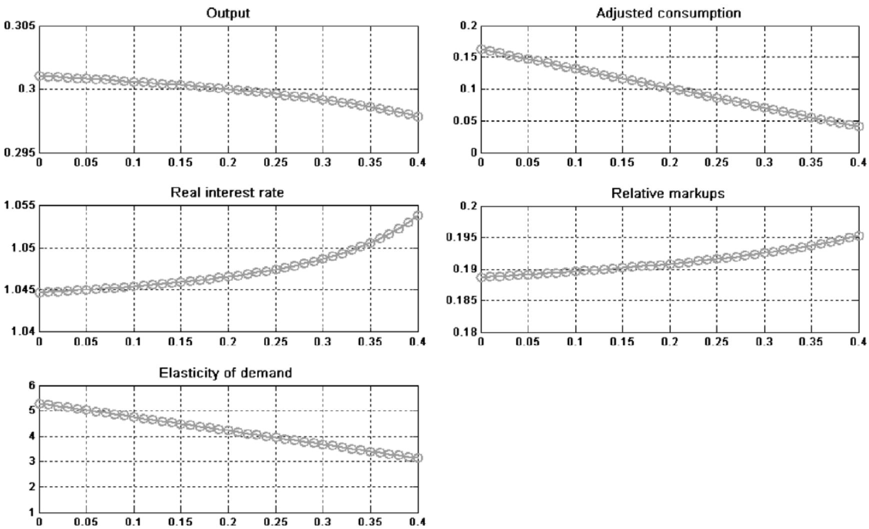

FIGURE 2. Increase in ϕ from 0 to 0.4.

interest rates, and lower elasticity of demand. We observe that the variation is nonlinear. In fact, the variation is sharp for values of ϕ between approximately 0.2 and 0.4.

The intuition behind the effects of the change in the degree of deep habits is the following. The higher the degree of habit formation, the more agents care about the difference between their consumption of a specific brand and the average consumption of that brand in the last period. This is a catching–up with the Joneses mechanism on a specific-brand basis. Agents who have low consumption (the young) are willing to sacrifice future consumption to increase their consumption today. They do so by lowering their saving today in order to catch up with the benchmark level of consumption. The decrease in saving entails higher real interest rates, implying a higher markup. As a result employment decreases, entailing lower consumption and output. This result is in line with Fisher and Heijdra (2009), who show that in a Blanchard–Yaari framework with exogenous labor supply, consumption externalities cause the long-run level of consumption and capital to drop. In our framework this result is preserved even without capital because of the effects of the time-varying markup.

5. PUBLIC DEBT SHOCKS

In this section, we calibrate our model and investigate the implications of temporary debt-financed government transfers. This exercise aims to shed light on the response of the economy to public debt shock in a deep-habits non-Ricardian model.

In Table 1 we summarize the information on our calibrated parameters. We assume that each period corresponds to a year. We set the discount factor β to

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TABLE 1. Parameter values

Definition	Parameter	Value
Discount factor	β	0.96
Elasticity of substitution across varieties	ε	5.3
Population growth rate	n	0.02
Frisch elasticity of labor supply	σ	1.3
Degree of habit formation	ϕ	0.2
Public debt adjustment speed	ρ_b	0.9
Degree of interest rate inertia	ρ_i	0 or 0.9
Degree of price stickiness	κ	0 or 14.5/4
Taylor rule coefficient	φ	1.5
Labor long-run level	$\bar{l}$	0.3
Inflation long-run target level	$\bar{\Pi}$	1.02
Public debt long-run target level	$\bar{b}$	0.6

0.96, implying an annual discount rate of approximately 4%. We follow Ravn et al. (2006) and set the elasticity of substitution, ε, equal to 5.3 and the Frisch labor supply elasticity, σ, equal to 1.3. In addition, the parameter α is calibrated so that the long-run level of labor equals 0.3. The population growth rate, n, is set equal to 0.02, which high than the values observed in the data. The reason is that this value is supposed to take into account all the wealth effects that would affect the real economy. The degree of habit formation ϕ is set to 0.2. This value is definitely much lower than the value estimated by Ravn et al. (2006), which is 0.86. The reason is that $\phi = 0.86$ induces a value of the gross real interest, R, of 3. This value is unrealistic. Moreover, the eigenvalues depend on the parameter ϕ. As a consequence, the determinacy of the equilibrium depends on ϕ. As shown in Blanchard and Kahn (1980), a necessary condition for the uniqueness of a stable solution in the neighborhood of the steady state is that there are as many eigenvalues larger than one in modulus as there are nonpredetermined variables in the model. Therefore, we choose the value of 0.2, which makes it possible to verify Blanchard and Kahn's conditions. In addition, $\phi = 0.2$ gives a plausible value for R, namely 1.046. We assume that the monetary authority reacts to the fluctuations in inflation. Thus we set the Taylor rule coefficient at 1.5. We follow Clarida et al. (1998) and set the degree of interest rate inertia to 0.9. The degree of price stickiness is set equal to the value estimated by Ravn et al. (2010), i.e., 14.5/4.

We solve the model and simulate the model using DYNARE[12]

5.1. Temporary Public Debt Increase

Here we simulate a temporary increase in government transfers financed by an increase in public debt. We assume that the public debt rises from 60% to 90%. In other words, ξ is set equal to 0.3. Notice that all variables are expressed as deviation (percentage) from the steady state. Figures 3a, 3b, and 3c represent the

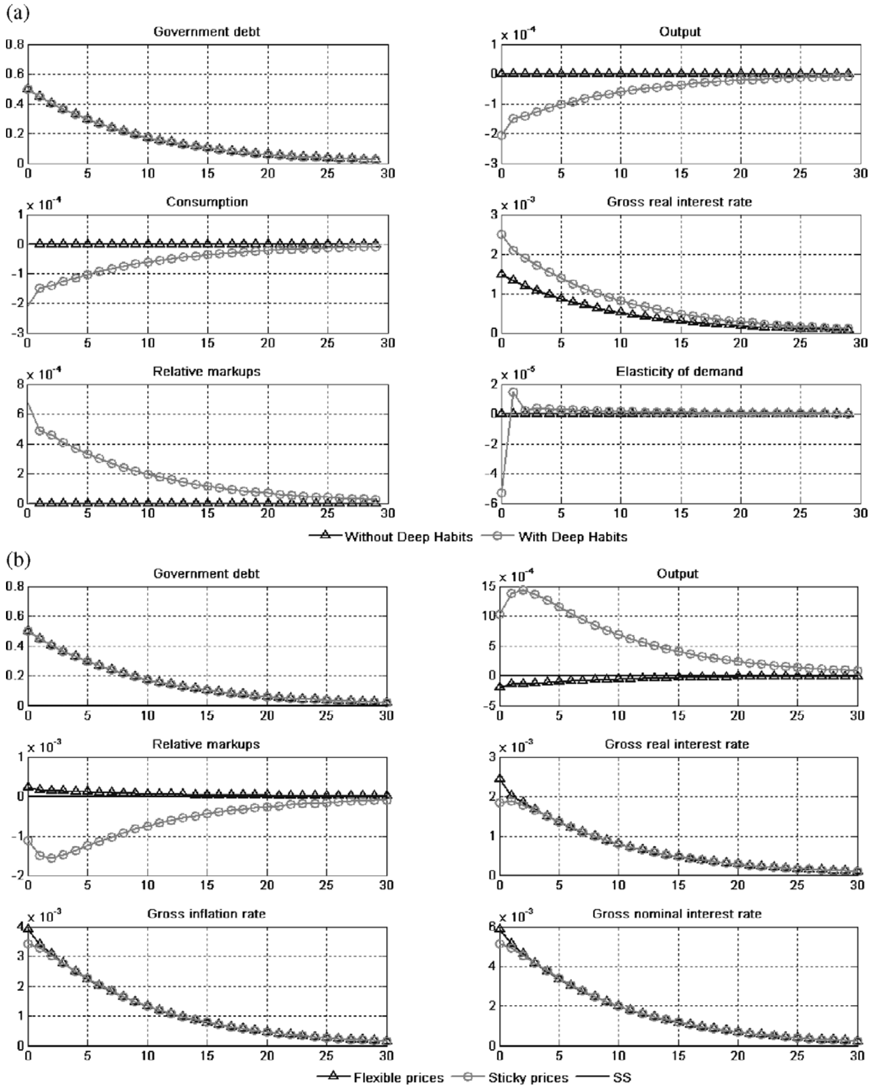

FIGURE 3. Temporary public debt shock.

time paths in response to a one-period public debt shock. Figure 3a contrasts the effects of public debt shock with and without deep habits. Figure 3b contrasts the effect of public debt shock when prices are fully flexible with its effect when prices are sticky.[13] Figure 3c compares the effect of public debt shock with and without–nominal interest rate smoothing.

Figure 3a shows that, in the absence of deep habits, the public debt increase only affects the real interest rate, which rises. In this case, fiscal policy is neutral despite

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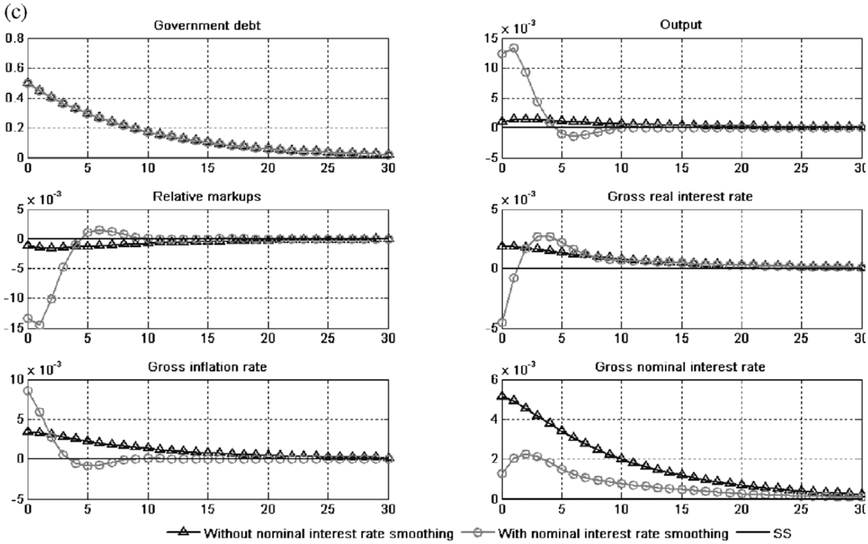

FIGURE 3. Continued.

the non-Ricardian framework. This is a consequence of using GHH preferences. In fact, the usual wealth effect on labor supply has been eliminated. Thus an increase in government debt does not affect labor supply or output. Labor supply is determined by intratemporal first-order condition.

Figures 3a and 3b show that, in the presence of deep habits, when prices are fully flexible, higher public debt entails lower consumption and consequently output. Consumption, employment, and output fall on impact. Relative markups and the real interest rate jump on impact. Inflation increases in line with the nominal interest rate. In addition, the elasticity of demand decreases, then increases, and then falls to reach its steady state value.

These results can be explained as follows. First, the increasing government debt makes current agents feel wealthier and want to consume more today, all other things being equal. Second, in the securities market, the supply of public bonds outstrips the demand for government bonds. As the economy is non-Ricardian, agents do not fear a future decrease in transfers (or increase in taxes). Consequently, they do not raise the demand for government bonds by the same amount as that by which the government bond supply rises. Thus, an interest rate increase is necessary to balance the securities market. Third, a higher real interest rate reduces the present value of future per-unit profits. As a result, firms have less incentive to invest in the customer base today and hence they are willing to increase markups today. In addition, higher markups entail lower employment and consequently lower consumption. Besides, lower consumption today implies lower price elasticity of demand and thus higher markups today, all other things being equal. At the same time, lower consumption today implies higher elasticity of demand at $t + 1$. As a result, firms have less incentive to invest in the customer base and will increase their markups today. As we can easily notice, there is no ambiguity: an increase in government debt, $\xi_t > 0$, implies an increase in markup, a decrease in employment, and a drop in consumption. In the next period, firms facing lower demand for their products will set a lower markup in order to increase the demand for goods. Consequently, employment increases and consumption goes up. Finally, the economy converges toward the steady state equilibrium. However, the convergence takes time because of the persistence of the government debt process (34). Furthermore, higher inflation is explained by the fact that during the adjustment the real interest rate targeted by the monetary authority is below the natural real interest rate, implying an inflationary bias.

Second, when prices are sticky, Figure 3b shows that public debt increase leads to an increase in output on impact, whereas relative markups decline. The real interest rate rises gradually and then starts to adjust to its steady state value from above. Inflation increases on impact. Nominal interest rates go up on impact. These results can be explained as follows. As the economy is non-Ricardian, public debt increase stimulates aggregate demand. For this reason, total consumption jumps upward on impact and then starts to adjust to its steady state value from above. As prices are sticky, output also jumps on impact. At the same time, the real interest rate increases gradually in order to balance the securities market. After the shock, the nominal interest rate increases and then adjusts gradually toward its steady state value. This is consistent with the behavior of the inflation rate.

Consider now the effects on markups. Here the elasticity effect dominates the intertemporal effect. Indeed, markups decrease on impact, despite the increase in the real interest rate. In fact, higher aggregate demand entails higher elasticity of demand, implying lower markups. But, as long as real interest rates rise and output decreases, the intertemporal effect on markups starts to dominate the elasticity effect, implying an increase in markup below its steady state level. It is clear that the introduction of the sticky-prices assumption restores the short-run expansionary effect of fiscal policy.

Moreover, we notice from Figure 3c that nominal–interest rate smoothing strengthens the short-run expansionary effect. In fact, the real interest rate declines, strengthening the intertemporal effect on the markup, and so the output increases more. The reason is as follows. The increase in the nominal interest rate in response to the first-period increase in inflation is smoothed over time. As the first-period increase in the nominal interest rate is not sufficient to balance the Fisher equation, the real interest rate decreases. Consequently, markup declines more, entailing higher employment, output, and consumption.

6. CONCLUSION

The goal of this paper is to contribute to the macroeconomic debate on the effects of growing and high public indebtedness. We develop a micro-founded general-equilibrium, non-Ricardian model with time-varying markups. Our principal motivation for adopting the OLG approach is to break down Ricardian equivalence in order to study the impact of government debt on macroeconomic aggregates. Specifically, we develop an extended stochastic version of overlapping generations based on Weil (1987) with a monopolistically competitive structure, an endogenous labor supply, and agents' preferences that feature external habit formation.

The main contribution of this paper is to provide a new transmission mechanism for public debt through the endogenous variation in markups induced by external deep habits. We show that positive shock to public debt raises the interest rate, entailing higher markups, which implies a decline in employment and consumption when prices are flexible. The crowding-out effect of government debt on output is preserved even without capital in the economy. Furthermore, when prices are sticky, deep-habits specification strengthens the short-run expansionary effect of government debt.

Rather than reiterating the rest of our findings, let us briefly indicate some possible extensions of this model. Given the recent economic crisis, such a model may be a useful tool to explore the role of government debt and deficits in an economy constrained by the zero lower bound on nominal interest rates, and to investigate the efficiency of fiscal stimulus.

NOTES

1. IMF Survey online, "Moving Public Debt Onto a Sustainable Path," September 1, 2010.
2. In this paper, deep habits refer to external deep habits. They are the catching–up with the Joneses described by Abel (1990), but on a good-by-good basis. A recent work using external habit formation in an overlapping-generations model is Wendner (2010).
3. We might use a Dixit–Stiglitz aggregator to obtain the consumption basket of goods, that is (2) when ϕ equals zero.
4. This issue is discussed in more detail in Ascari and Rankin (2007).
5. See Appendix A for further details.
6. From (3), we note that individuals' preferences are undefined for habit-adjusted consumption values below the subsistence level. $a_{j,t}$ needs to be positive for all j, t.
7. See Appendix A for further details.
8. See Appendix B for further details.
9. In this paper we abstract from government spending. However, it may be interesting to analyze the effects of government spending shock in this framework when monetary policy is passive. Indeed, Kim (2003) analyzes the effects of fiscal shocks in a passive monetary–active fiscal regime. He shows that an increase in government spending leads to a consumption rise in the model that predicts a consumption fall based on conventional theory.
10. The real public debt is a predetermined value.
11. We assume that $\bar{b} = 0.6$, $\varepsilon = 5.3$, $\sigma = 1.3$, $n = 0.02$, and $\beta = 0.96$. We will give more details about the calibration exercise in the next section.
12. See Juillard (2004).
13. Notice that here we set $\rho_i = 0$.

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APPENDIX A: OPTIMALITY CONDITIONS FOR THE CONSUMER

Here we present the optimality conditions for the agents j.

A.1. THE DEMAND FUNCTION OF GOOD *m*

Household j minimizes total expenditure $\sum_{m=1}^{M_t} P_t(m)c_{j,t}(m)$ subject to the aggregate constraint

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left\{ \sum_{m=1}^{M} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.1}$$

where $P_t(m)$ denotes the nominal price of good m at time t.

The Lagrangian for this problem is

$$\min \sum_{m=1}^{M_t} P_t(m)c_{j,t}(m) + \zeta_t \left(x_{j,t} - M_t^{\frac{1}{1-\varepsilon}} \left\{ \sum_{m=1}^{M_t} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}} \right),$$

where ζ_t is the Lagrange multiplier.

The first-order conditions of this problem for $c_{j,t}(m)$ and ζ_t are

$$\frac{P_t(m)}{\zeta_t} = M_t^{\frac{1}{1-\varepsilon}} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{-\frac{1}{\varepsilon}} \left\{ \sum_{m=1}^{M_t} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{1}{\varepsilon-1}}, \tag{A.2}$$

$$x_{j,t} = M_t^{\frac{1}{1-\varepsilon}} \left\{ \sum_{m=1}^{M_t} [c_{j,t}(m) - \phi \tilde{c}_{t-1}(m)]^{\frac{\varepsilon-1}{\varepsilon}} \right\}^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.3}$$

Rearranging (A.2) using (A.3) yields

$$c_{j,t}(m) = \frac{1}{M_t} \left[\frac{P_t(m)}{\zeta_t} \right]^{-\varepsilon} x_{j,t} + \phi \tilde{c}_{t-1}(m). \tag{A.4}$$

From the definition of the composite level of consumption (A.1), this implies

$$\zeta_t = \left\{ \frac{1}{M_t} \sum_{m=1}^{M} [P_t(m)]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}.$$

We define P_t as a price index that verifies

$$P_t c_t = \sum_{m=1}^{M} P_t(m) \sum_{j \leq t} (N_j - N_{j-1}) c_{j,t}(m).$$

The accounting definition of c_t is given by

$$c_t = M_t^{\frac{1}{1-\varepsilon}} \left[\sum_{m=1}^{M} c_t(m)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

which, combined with (A.2) and (A.4), allows us to write

$$P_t = \left\{ \frac{1}{M_t} \sum_{m=1}^{M} [P_t(m)]^{1-\varepsilon} \right\}^{\frac{1}{1-\varepsilon}}.$$

Moreover, we multiply (8) by $p_t(m)$,

$$P_t(m)c_{j,t}(m) = \frac{1}{M_t} P_t(m)^{1-\varepsilon} x_{j,t} + \phi P_t(m)\tilde{c}_{t-1}(m),$$

and then we sum the resulting equation over the variety of goods m, which yields

$$\sum_{m=1}^{M} P_t(m)c_{j,t}(m) = \frac{1}{M_t} \sum_{m=1}^{M} P_t(m)^{1-\varepsilon} x_{j,t} + \phi \sum_{m=1}^{M} P_t(m)\tilde{c}_{t-1}(m).$$

Finally, using the definition of the price index, we obtain

$$P_t x_{j,t} = \sum_{m=1}^{M} P_t(m)c_{j,t}(m) - \phi \sum_{m=1}^{M} P_t(m)\tilde{c}_{t-1}(m). \quad \textbf{(A.5)}$$

A.2. THE INDIVIDUAL EULER EQUATION

We build the following Lagrangian function corresponding to the consumer's program:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \ln[x_{j,s} - d(l_{j,s})]$$

$$- \rho_s \left[P_s x_{j,s} + \phi \sum_{m=1}^{M_t} P_s(m)\tilde{c}_{s-1}(m) - V_{j,s} - P_s w_s l_{j,s} - T_{j,s} - \Psi_{j,s} + Q_{s,s+1} V_{j,s+1} \right],$$

where λ_t is a Lagrange multiplier.

The first-order conditions of this problem for $x_{j,t}$, $l_{j,t}$, $V_{j,t+1}$ and ρ_t are

$$\frac{1}{x_{j,t} - d(l_{j,t})} = P_t \rho_t, \quad \textbf{(A.6)}$$

$$-\frac{d_l(l_{j,t})}{x_{j,t} - d(l_{j,t})} = -\rho_t P_t w_t, \quad \textbf{(A.7)}$$

$$E_t Q_{t,t+1} \rho_t = \beta \rho_{t+1}, \quad \textbf{(A.8)}$$

$$P_t x_{j,t} + \phi \sum_{m=1}^{M_t} P_t(m)\tilde{c}_{t-1}(m) + E_t Q_{t,t+1} V_{j,t+1} = V_{j,t} + P_t w_t l_{j,t} + T_{j,t} + \Psi_{j,t}. \quad \textbf{(A.9)}$$

Eliminating ρ_t by combining (A.6) and (A.8), we obtain the individual Euler equation

$$\beta \frac{[x_{j,t} - d(l_{j,t})]}{x_{j,t+1} - d(l_{j,t+1})} = \frac{P_{t+1}}{P_t} Q_{t,t+1}. \quad \textbf{(A.10)}$$

Then we combine (A.6) and (A.7) to get the labor supply function

$$d_l(l_{j,t}) = w_t. \quad \textbf{(A.11)}$$

Let us call $a_{j,t} (\equiv x_{j,t} - d(l_{j,t}))$ the "adjusted consumption" of agent j, (A.10) is rewritten

$$a_{j,t} = \beta^{-1} \frac{P_{t+1}}{P_t} Q_{t,t+1} a_{j,t+1}. \quad \textbf{(A.12)}$$

APPENDIX B: AGGREGATION

We remember that individual "adjusted" consumption is defined by

$$a_{j,t} = (1-\beta)(v_{j,t} + h_t - \chi_t). \tag{B.1}$$

Iterating the equation (B.1) once,

$$a_{j,t+1} = (1-\beta)(v_{j,t+1} + h_{j,t+1} - \chi_{t+1}), \tag{B.2}$$

and then introducing

$$a_{j,t} = \beta^{-1} R_{t,t+1}^{-1} a_{j,t+1}$$

into (B.2) leads to

$$a_{j,t} = (1-\beta)\beta^{-1} R_{t,t+1}^{-1}(v_{j,t+1} + h_{j,t+1} - \chi_{t+1}).$$

Now, aggregating this last equation, and using the fact that $h_{j,t+1}$ is age-independent, yields

$$a_t = (1-\beta)\beta^{-1} R_{t,t+1}^{-1}\left[(1+n)\, v_{t+1} + h_{t+1} - \chi_{t+1}\right]. \tag{B.3}$$

In addition, aggregating (B.2) yields

$$a_{t+1} = (1-\beta)\,(v_{t+1} + h_{t+1} - \chi_{t+1}). \tag{B.4}$$

Finally, we obtain the aggregate Euler equation

$$a_t = \beta^{-1} R_{t,t+1}^{-1} a_{t+1} + n(\beta^{-1} - 1) R_{t,t+1}^{-1} v_{t+1}$$

by combining (B.3) and (B.4).

APPENDIX C: OPTIMALITY CONDITIONS FOR THE FIRM

The Lagrangian function corresponding to the firm's problem is

$$E_t \sum_{s=t}^{T} R_{t,s}^{-1}\left[\frac{P_s(m)}{P_s} y_s(m) - w_s y_s(m)\right]$$
$$+ \lambda_s(m)\left\{\left[\frac{P_s(m)}{P_s}\right]^{-\varepsilon} \frac{N_s}{M_s} x_s + \phi\,(1+n)\, y_{s-1}(m) - y_s(m)\right\}.$$

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The first-order conditions of this problem for $y_t(m)$, $P_t(m)$, and λ_t are

$$\frac{P_t(m)}{P_t} - w_t - \lambda_t(m) + \phi(1+n)E_t\frac{\lambda_{t+1}(m)}{R_{t,t+1}} = 0, \quad \textbf{(C.1)}$$

$$y_t(m) = \varepsilon\lambda_t(m)\frac{N_t}{M_t}x_t\left[\frac{P_t(m)}{P_t}\right]^{-\varepsilon-1}, \quad \textbf{(C.2)}$$

$$y_t(m) = \left[\frac{P_t(m)}{P_t}\right]^{-\varepsilon}\frac{N_t}{M_t}x_t + \phi(1+n)y_{t-1}(m). \quad \textbf{(C.3)}$$

Let

$$\eta_t(m) \equiv \frac{P_t(m) - P_t w_t}{P_t(m)} \quad \textbf{(C.4)}$$

denote the relative markup charged by firm m. Let us define $\epsilon_t(m)$ as the absolute value of price elasticity of demand:

$$\epsilon_t(m) \equiv \varepsilon\left[1 - \phi\,(1+n)\,\frac{y_{t-1}(m)}{y_t(m)}\right].$$

Equation (C.2) becomes

$$\lambda_t(m) = \frac{P_t(m)}{\epsilon_t(m)P_t}$$

and equation (C.1) becomes

$$\eta_t(m) = \frac{P_t(m)}{\epsilon_t(m)P_t} - \phi(1+n)E_t R_{t,t+1}^{-1}\frac{P_{t+1}(m)}{\epsilon_{t+1}(m)P_{t+1}}. \quad \textbf{(C.5)}$$

APPENDIX D: STEADY STATE

The aim of this Appendix is to prove the existence and uniqueness of the steady state equilibrium. The steady state system consists of the following main equations:

$$\Re(y) = \beta^{-1} + \zeta\frac{\bar{b}}{a}, \quad \textbf{(D.1)}$$

$$\Upsilon\,(R) = \left\{\left[1 - \frac{1-\frac{\tilde{\phi}}{R}}{\varepsilon(1-\tilde{\phi})}\right]\alpha^{-1}\right\}^{\sigma}, \quad \textbf{(D.2)}$$

with

$$a = (1-\tilde{\phi})y - \alpha\frac{y^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}},$$

where $\tilde{\phi} = \phi(1+n)$ and $\zeta = n(\beta^{-1}-1)$. First, a must be positive, because otherwise preferences are undefined. This implies the following necessary condition:

$$0 < y < \overline{y} \equiv \left[\frac{(1-\tilde{\phi})\left(1+\frac{1}{\sigma}\right)}{\alpha}\right]^{\sigma}. \quad \textbf{(D.3)}$$

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Second, we notice from (D.2) that y cannot be less than $\underline{y}$, defined by

$$\underline{y} \equiv \left[\frac{1 - \frac{\varepsilon^{-1}}{(1-\tilde{\phi})}}{\alpha} \right]^{\sigma} = \lim_{R \to +\infty} \Upsilon(R). \quad \textbf{(D.4)}$$

According to (D.4), (D.3) becomes

$$\underline{y} < \overline{y}; \quad \textbf{(D.5)}$$

that is

$$\Phi(\tilde{\phi}) \equiv d\tilde{\phi}^2 + (1 - 2d)\,\tilde{\phi} + d + \varepsilon^{-1} - 1 > 0, \quad \textbf{(D.6)}$$

with $d = 1 + \sigma^{-1}$.

First, $\Phi(\tilde{\phi})$ is always positive for $0 < \sigma < 4(\varepsilon - 4)^{-1}$. In fact, the discriminant of $\Phi(\tilde{\phi})$ is negative, implying the positiveness of $\Phi(\tilde{\phi})$, because $\Phi(0) = \sigma^{-1} + \varepsilon^{-1} > 0$.

Second, for $\sigma > 4(\varepsilon - 4)^{-1}$, the discriminant of $\Phi(\tilde{\phi})$ is positive. $\Phi(\tilde{\phi})$ is positive only for $\tilde{\phi} \in [0, \tilde{\phi}_1) \cup (\tilde{\phi}_2, 1]$, where $\tilde{\phi}_1$ and $\tilde{\phi}_2$ denote the roots of $\Phi(\tilde{\phi})$. That is,

$$\tilde{\phi}_1, \tilde{\phi}_2 = \frac{2d - 1}{2d} \mp \sqrt{\frac{\varepsilon - 4d}{4\varepsilon d^2}}. \quad \textbf{(D.7)}$$

Now we have to check under which conditions the curves corresponding to (D.1) and (D.2), respectively, intersect in the yR plane. So let us analyze $\Upsilon(\cdot)$ and $\Re(\cdot)$. We observe that the inverse of the function, $\Upsilon(\cdot)$, is strictly decreasing, as its derivative is strictly negative in $(\underline{y}, +\infty)$. On the other hand, $\Re(y)$ is decreasing in $(\underline{y}, y_{\min}]$ and increasing in $[y_{\min}, \overline{y})$. In fact, its derivative, i.e.,

$$\Re_y(y) = \zeta \bar{b} \frac{y^{\frac{1}{\sigma}} - (1 - \tilde{\phi})}{\left[(1 - \tilde{\phi})y - \frac{y^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} \right]^2}, \quad \textbf{(D.8)}$$

vanishes for

$$y_{\min} = (1 - \tilde{\phi})^{\sigma}$$

and is negative when $y < y_{\min}$ and positive when $y > y_{\min}$. Moreover, when y goes to zero, $\Re(y)$ goes to infinity. In other words, $\Re(y)$ admits a vertical asymptote for $y = 0$. We deduce that, if condition (D.5) is satisfied, it is sufficient that $\underline{y}$ is positive, so that the two curves intersect once. In other words, the necessary and sufficient condition for the existence and uniqueness of the steady state equilibrium is

$$0 < \underline{y} < \bar{y},$$

which can be rewritten as

$$\tilde{\phi} < \tilde{\phi}_{\max} \equiv (1 - \varepsilon^{-1}) \quad \textbf{(D.9)}$$

for $\sigma < 4(\varepsilon - 4)^{-1}$ and

$$\tilde{\phi} \in [0, \tilde{\phi}_1) \cup (\tilde{\phi}_2, \tilde{\phi}_{\max}) \quad \textbf{(D.10)}$$

for $\sigma > 4(\varepsilon - 4)^{-1}$.