

# Transient Modeling in Simulation of Hospital Operations for Emergency Response

Jomon Aliyas Paul, PhD; Santhosh K. George, MS; Pengfei Yi, PhD; Li Lin, PhD

Department of Industrial Engineering,  
University at Buffalo, Buffalo, New York USA

## Correspondence:

Dr. Li Lin  
438 Bell Hall  
Department of Industrial Engineering  
University at Buffalo  
Buffalo, New York 14260 USA  
E-mail lilin@eng.buffalo.edu

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## Abbreviations:

AFOSR = Air Force Office of Scientific Research  
ED = emergency department  
EOC = emergency operations center  
FEMA = Federal Emergency Management Agency  
ICU = intensive care unit  
LOS = length-of-stay  
OR = operating room

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## Abstract

Rapid estimates of hospital capacity after an event that may cause a disaster can assist disaster-relief efforts. Due to the dynamics of hospitals, following such an event, it is necessary to accurately model the behavior of the system. A transient modeling approach using simulation and exponential functions is presented, along with its applications in an earthquake situation. The parameters of the exponential model are regressed using outputs from designed simulation experiments. The developed model is capable of representing transient, patient waiting times during a disaster. Most importantly, the modeling approach allows real-time capacity estimation of hospitals of various sizes and capabilities. Further, this research is an analysis of the effects of priority-based routing of patients within the hospital and the effects on patient waiting times determined using various patient mixes. The model guides the patients based on the severity of injuries and queues the patients requiring critical care depending on their remaining survivability time. The model also accounts the impact of prehospital transport time on patient waiting time.

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## Introduction

Hospitals and emergency departments (EDs) constitute an important part of the healthcare system. During a disaster, their role becomes even more critical. It is vital to provide timely treatment to patients injured in the disaster in order to minimize the fatalities. When a disaster occurs, the number of patients who require treatment in EDs may increase 3–5 times the normal volume,<sup>1</sup> which easily could overwhelm the hospitals' resources. Emergency preparedness helps hospitals cope with this sudden surge of patients by making temporary room and utilizing resources from other hospital departments. Well-coordinated, disaster relief efforts are needed to ensure timely treatment of all injured victims while continuing to treat those patients who normally would present to the ED and demand immediate attendance.

Hospital capacity estimates can assist disaster mitigation efforts to provide timely treatment to the injured and ill. The hospital capacity information can be used to make patient/ambulance routing decisions and resource allocations. However, obtaining hospital capacity estimates is not a trivial task due to many factors. First, a postulated, analytical model is of little value due to the underlying complexity of hospital operations and the dynamic nature of the problem. Each hospital is different in terms of its resources, capabilities, and operational efficiency. In addition, capacity estimates are required from *all* hospitals in the disaster region, since hospitals in the vicinity of the disaster provide critical medical support. The problem is complicated further by the fact that capacity estimates must be made in real-time so that they are useful for disaster management.

The complexity of the modeling task justifies the use of discrete-event simulation, which is a widely used modeling tool for evaluation of system performance. The performance of the hospital is simulated using historical, rep-

representative data. However, the hospital model in disaster management differs from the model of normal operations in the characteristics of patient arrivals, which renders the system essentially transient in nature. Although simulation is used in hospital studies, there were no reported applications in the literature on the estimation of transient hospital capacity, real-time or otherwise.<sup>2-5</sup>

#### *Simulation of Hospital Operations for Regional Disaster Relief*

In order to provide informational support for coordinated disaster relief efforts, the hospital model should be able to represent multiple hospitals in the disaster region. That is, a generic model must represent various hospitals in the disaster region. A discrete-event simulation model of hospitals with variable parameters was developed for this purpose. However, conventional simulation is not feasible for real-time applications due to the requirements on replications to establish statistical confidence intervals and the necessity to model each of the hospitals individually. Therefore, outputs from off-line simulation runs were used to obtain a simulation meta-model, which can represent hospitals of various sizes and capabilities and could be used to estimate hospital capacity during a disaster.<sup>6</sup>

#### *Significance of Transient Modeling*

The hospital modeling approaches differ in their purpose in modeling normal operations and modeling in a disaster situation. In normal situations, hospital modeling generally aims at improving performance. However, in disaster situations, modeling is useful to estimate hospital capacity in real-time. Such estimates assist the Emergency Operations Center (EOC) to efficiently coordinate disaster relief operations.

During a disaster, patient arrivals to a hospital are dynamic. After an event caused by natural hazards, such as an earthquake, it is common to experience a sudden increase of patients 3–5 times the normal numbers in the ED.<sup>1</sup> Although this sudden surge of patients lasts only for a short period of time, e.g., a few days, it affects patient flow time, resulting in serious delays in treating the patients.<sup>2</sup> The system is likely to follow a transient period under the high arrival rates. Consequently, steady-state performance measures of normal operations are inadequate in disaster modeling. Thus, transient modeling has presented a unique challenge to hospital operations modeling during a disaster. This challenge is compounded further when the transient behavior must be an integral part of the aforementioned generic model for hospitals of various sizes and capabilities in the disaster region.

#### *Needs for Modeling Injury Severity and Prehospital Transport Time*

Patients with different severities of injuries have different survivability times. Therefore, patients must be classified according to the severity of injuries/illnesses, and priorities must be assigned to the patients with lower survivability time so as to render care to the right patient at the right time.

A hospital's capacity to treat injured/ill patients within a short period of time, i.e., the so-called "golden hour",

depends on the number of patients, severity of the injuries, and the hospital's ability to render the medical services. Thus, the effects of patient mix of various severities must be studied so that an EOC can best use the information to direct the ambulances based on which hospitals can accept the victims and how many patients each can receive.

During a disaster, patients might be in a situation in which the ambulance transportation to the hospital takes an extended period of time. If the transport time is too long, this might change the severity of the patients' injuries, thereby reducing the available survivability time.

#### *Research Objectives*

To meet the needs and research challenges described above, this research has the following objectives:

1. To develop a set of meta-models to represent the transient, in-hospital operations for use in disaster relief efforts;
2. To study the effects of injury/illness severities and a priority-based, routing system of patients on their waiting times;
3. To develop a relationship between the change of patient mix and patient waiting time;
4. To study the effects of prehospital transport time on patient waiting time and mortality; and
5. To demonstrate the developed meta-models for real-time estimation of hospital capacity.

This research focuses on dealing with the initial surge of trauma injuries and other patients in the early stages of a sudden-onset disaster. The subsequent services, such as intensive care and inpatient care, are not within the scope of this paper. Although the results reported here might be useful for design or improvement of hospital facilities for disaster planning, follow-up research would be required to address these issues.

#### *Literature Review*

##### *Modeling Hospital Operations*

A variety of modeling methods used for hospital operations modeling are found in the literature. These include: (1) linear programming;<sup>7</sup> (2) dynamic programming;<sup>8</sup> (3) queuing models;<sup>9</sup> (4) system dynamics;<sup>10</sup> and (5) discrete-event simulation models.<sup>2-5</sup> Boyd used *linear programming* to optimize resource allocation in hospitals when alternative methods of treating a patient are available.<sup>7</sup> The *dynamic linear programming resource allocation model*, described by Zon and Kommer, captured the expected improvement in the patients' future healthcare needs as a result of the healthcare interventions and improvement of resource allocation.<sup>8</sup> Although linear and dynamic programming allow quantitative analysis, these deterministic, mathematical models are unable to deal with problems of random nature. In that regard, *queuing models* are capable of representing stochastic processes. For example, Bretthauer and Shetty used a Jackson network of queues (Poisson external arrivals and exponential service times at the single server nodes) in healthcare capacity planning problems.<sup>9</sup> Their model tried to minimize capacity costs while satisfying a constraint on the expected length-of-stay (LOS) of patients. However,

queuing models only capture long-term system performance. On the other hand, system dynamics models have applications in a solution-oriented model of a simple system and in macroscopic system modeling. An example of a solution-oriented application for cancer screening has been described by Royston *et al.* In a *macroscopic system*, it is useful to understand the relationship between various elements in the model for developing emergency health and social care.<sup>10</sup>

#### *Discrete-Event Simulation Modeling*

As quantitative models lack the capability of modeling complex systems, such as hospital operations and obtaining capacity estimates in a dynamic environment, discrete-event simulation is a useful method capable for modeling detailed functioning of hospitals.

In a survey on the use of discrete-event simulation in healthcare industries, Jun *et al* found that the main effort is concentrated on patient flow and resource allocation.<sup>11</sup> For example, Cote developed a simulation model for an outpatient clinic and studied the influence of examination room capacity and patient flow on four performance measures: (1) room utilization; (2) room queue length; (3) examination rooms' occupancy; and (4) patient flow time.<sup>2</sup> The model only was affected by the arrival rate. Simulation modeling also was used to find optimal parameters in the simulation study of a clinic by Weng and Houshmand.<sup>5</sup> In addition to standard performance measures such as throughput, time in the system, queue times, and queue lengths, they also measured the total cash flow.

While the above simulation studies were conducted for specific hospitals, Lowery designed a simulation model of a hospital's critical care units to represent a variety of different hospitals.<sup>3</sup> The results suggest the possibility of designing a generic critical care model, which could be used to represent a specified range of hospitals with various capacity and operational characteristics.

By using factorial design in combination with simulation modeling, it is possible to find the best operating parameters. Although the examples in the literature are case-specific and only study the steady-state behavior of hospital systems, they demonstrate the capability of simulation in hospital operations modeling.<sup>11</sup> It is a promising approach to create a generic hospital model,<sup>3</sup> followed by meta-modeling using experimental design methods.<sup>4</sup>

Simulation modeling requires multiple replications for the results to be acceptable statistically. Since real-time capacity estimation is required for disaster relief, multiple replications of all of the hospitals in the disaster region require prohibitively considerable computing effort. Therefore, the direct use of simulation in real-time applications for disaster relief is impractical.

#### *Regression and Parametric Models*

In a systems study, it is beneficial to establish quantitative relationships between the input and output variables. Giraldo *et al* developed a parametric model that relates the number of admission requests and available time at the operating theaters (surgical suites) to the number of

patients on a waiting list. This offered useful insight into the system's operating characteristics.<sup>12</sup>

Chang combined data envelopment analysis with regression analysis to evaluate hospital efficiency, as a function, to a number of hospital operating characteristics.<sup>13</sup>

It is possible to create a generic hospital model by specifying the hospital characteristics, although hospitals vary in size and capability.<sup>2</sup> Groenendaal and Kleijnen designed experiments for regression meta-modeling in a simulation study with uncertain inputs and parameters.<sup>14</sup>

#### **Transient Modeling Approaches**

##### *Transient Modeling by Discrete-Event Simulation*

Widely used for their flexibility, discrete-event simulation models inherently are capable of modeling the transient behavior.<sup>15</sup> In any simulation model, which starts from an empty and idle state, the system passes through a transient stage before reaching a steady state. This transient modeling capability of simulation is demonstrated by many applications in the literature. For example, a real-time scheduling algorithm was developed to select a dispatch rule dynamically in Flexible Manufacturing Systems using the transient modeling by discrete-event simulation.<sup>16</sup>

By combining simulation and meta-models to study system transient behavior, Cochran and Lin developed compound, dynamic event meta-models to approximate the transient by an exponential process in a manufacturing application.<sup>17</sup> This suggests potential applications of meta-models for modeling transient dynamics in hospital operations.

##### *Transient Modeling by Control-Theoretic Models*

Ortega designed a control-theoretical model to represent the transient behavior of a manufacturing system.<sup>18</sup> He also used simulation results to estimate the parameters of the control-theoretic model. However, it is not realistic to calibrate each individual control-theoretical model for all possible hospitals, as is required for the generic applicability in regional disaster relief efforts.

##### *Significance of Severity Separation/Triaging*

In the ED, the use of triage first diagnoses the severity of patients' injuries in order to identify those patients needing immediate treatment. This diagnosis is based on urgency of treatment, likelihood of survival, and available resources.<sup>19,20</sup> As earlier research did not consider any priority assignment to patients depending on the likelihood of survival, patients were served on a first-come, first-serve basis. Thus, using available resources to treat patients, based on the severity of their injuries, was not possible with the previous model. In this regard, the current model attempts to consider these factors properly as it is vital for the mass-casualty emergency preparedness during any disaster.

Saunders *et al* developed a simulation model of the emergency department by incorporating priority for patients with various levels of needs.<sup>21</sup> Severely injured patients wait less time to see a physician than do those with less severe injuries, which is desirable in a disaster situation. Garcia *et al* conducted a simulation to study the effects of patient priority on the waiting time of low priority patients

and the impact of a fast-track lane.<sup>22</sup> As expected, the results indicate that the higher priority patients experience shorter waiting time, while the lowest priority patients have the longest waiting time.

#### *Significance of Patient Mix of Various Severities*

Patient mix could affect the use of available resources, and thus, could affect patient waiting time. Patient mix effect on a clinic<sup>23</sup> has been studied using simulation.<sup>3</sup> The results indicated that clinical environments are highly sensitive to small changes in patient mix and patient scheduling rules. The changes of patient mix greatly impacts physician utilization and consequently the average, daily clinic overtime costs. Similarly, intensive care unit (ICU) performance is considerably different for various case mixes, in terms of standardized mortality ratio.<sup>23</sup> *Standardized mortality ratio* can be defined as the number of actual deaths in a given year as a percentage of those expected. Expected deaths depend on the standard mortality of the reference period, adjusted for age, gender, etc.

#### *Effect of Prehospital Transport Time*

Patients during a disaster are classified into several severity types (levels). Long prehospital transport times can lead to the development of a more serious severity type. This would result in a change of patient mix received by the hospital, which might affect patient waiting time and hospital resource utilization. Therefore, prehospital transport time is identified as a factor affecting the mortality from traumatic injuries.<sup>24,25</sup> In a study on the effect of the prehospital transport time on the waiting time, the transport time was assumed to follow a normal distribution.<sup>24</sup>

### Methods

#### *Overview of the Methodology*

By combining the design of simulation experiments using factorials and off-line simulation runs, a generic simulation model can be developed that can represent any hospital with various ED patient volumes, hospital size, and operating efficiency. Using ANOVA in the off-line simulation, a response surface is obtained. This relates the dependent variable of hospital performance to hospital characteristics, which are the independent variables.

The parametric response surface model thus represents the steady-state behavior of the system. By further combining the transient behavior with this generic hospital model, a meta-model is prepared to capture the temporal behavior of hospitals during a disaster.

In this context, hospital capacity was defined as the number of emergency patients who can be treated within a certain time period in a timely manner, as demanded by the necessary medical procedures for various types of injuries. In this research endeavor, only the initial time for treatment of patients was modeled. However, after the initial treatment is over, there still could be bed surge capacity issues. The current research does not address these issues, but a number of strategies exist that could be used to solve this problem, for example the work by Davis *et al.*<sup>26</sup> In planning disaster relief, the [US] Federal Emergency Management

Agency (FEMA) uses software systems, such as HAZUS (NIBS, Washington, DC), to predict the number and severity of injuries resulting from an earthquake. This should enable hospitals to prepare for the expected number and types of patients.<sup>27</sup> Such software tools can be used with this hospital model for the improvement of disaster relief coordination.

#### *Design of Experiment for the Generic Hospital Model*

Since the capability to represent any generic hospital is needed, parametric modeling of the hospitals is required. The number of beds, number of operating rooms (OR), and OR efficiency (average number of surgeries per OR per year) can be used to adequately model a generic hospital for disaster mitigation.<sup>6</sup>

In the experimental design, three levels for each of the factors are used. They are:

1. Number of Beds: 100, 300, or 500;
2. Number of ORs: 5, 10, or 15
3. *Operating Room Efficiency Index*: 600, 900, or 1,200 (surgeries/OR/year)

This 3 x 3 x 3 factorial design leads to 27 combinations. However, since it is unlikely for a large hospital with 500 beds to have only five ORs or a 100-bed small hospital to have 15 ORs, such non-feasible combinations were removed from the experiment. This results in 21 combinations to be simulated. The scope of this design is sufficient to support the development of a generic hospital model, as national statistics of all hospitals (for the year 1999) indicate that hospitals with >100 beds comprise 85% of the total number of beds in the US.<sup>28</sup> For any hospital whose factors' values are within the range of this design, their performance can be obtained by interpolation, by using the parametric response surface model.

#### *Modeling Steady-State Operations of Hospitals*

As discussed, although discrete-event simulation is a valuable tool for hospital modeling, real-time simulation runs for estimating hospital capacity are not feasible for the following reasons:

1. It is not possible to build individual simulation models for all of the available hospitals in the disaster area, which may vary in size *after* the event occurs; and
2. Even when simulation models of all the hospitals were available, time-consuming multiple runs still are required for the results to be acceptable statistically due to the random nature of simulation experiments. This does not support real-time applications.

To overcome these weaknesses, the modeling power offered by simulation should be used, while avoiding time-consuming runs by using off-line simulation and statistically generalized simulation results. Therefore, a meta-model generalized from the simulation results offers a sound solution method.

Patient waiting time is the response variable of the simulation meta-model, as it indicates how busy the hospital is, and directly impact survivability. In other words, the hospital capacity during a disaster is indicated by how quickly it can treat the injured patients. *Survivability*, defined as the maximum allowable time before the patient is treated to



| Important Parameters | Linear Regression | Non-linear Regression |
|----------------------|-------------------|-----------------------|
| $T_O$                | 82%               | 94%                   |
| $T_L$                | 72%               | 90%                   |
| $\lambda_U$          | 90.5%             | 96.5%                 |
| $T_U$                | 74%               | 85%                   |

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**Table 1**— $R^2$  values for linear and non-linear regression (With Severity Separation)

Where

$T_O$  = pre-earthquake steady-state waiting time;

$T_L$  = post-earthquake steady-state waiting time corresponding to the lower bound on arrival rate ("base case" waiting time);

$\lambda_U$  = post-earthquake upper bound of patient arrival rate that begins to saturates the system ("critical case" arrival rate); and

$T_U$  = post-earthquake steady-state waiting time corresponding to the upper bound on arrival rate ("critical case" waiting time).

avoid fatality, depends on the type and severity of injuries. Survivability times specific to hospital operations, determined only when a patient reaches the ED, were used. Therefore, by requiring the waiting time to be shorter than the survivability time, the hospital capacity is estimated.

In this model, average daily patient arrival rates prior to the earthquake are calculated from national statistics.<sup>29</sup> Since post-event arrival rates are unknown, a set of various arrival rates are simulated. This will result either in a steady state following a certain transient period or a congested system due to a patient volume that exceeds hospital capacity. Thus, for each of the arrival rates that does not congest the system, the corresponding waiting times are collected. Regression equations then are obtained for both pre-earthquake and post-earthquake steady-state waiting times.

#### *Transient Modeling of Hospital Operations*

Immediately after the earthquake, the system passes through a transient period. The transient state is of utmost importance. Shifting from a pre-earthquake waiting time to the post-earthquake waiting time, the transient state may have a similar exponential behavior to the dynamic operations in manufacturing systems as demonstrated by Cochran and Lin.<sup>17</sup> Nonetheless, the exact characteristics depend on the hospital parameters and patient arrival rates after the earthquake.

The form of the transient is approximated using a single exponential function.<sup>17</sup> This form is used to capture the transient when there is a sudden increase of patient volume.

The off-line simulations are completed with a constant arrival rate to obtain the steady-state hospital performance on patient waiting times. However, in a real-time application that uses simulation, the arrival rates are dynamic and unknown. Therefore, patient arrival rates are estimated based on actual

arrivals to the hospital. In this work, the average number of arrivals in a moving time window is used as the arrival rate. This rate is updated at fixed, small time intervals, resulting in a step function. A time window of 30 minutes and increments of five minutes were suitable for this application.

The general distributions of inter-arrival times do not have a considerable effect on the waiting times when a constant arrival rate is simulated. This observation supports the use of pseudo-dynamic rates. However, since the arrival rates change continuously over time during a disaster event, a continuously varying arrival rate should be used to model the patient arrivals with improved fidelity. Estimation of dynamic arrivals is addressed in ongoing work.

#### *Severity of Injuries and Priority Assignment*

Three different severities are considered in the capacity estimation model. Severity 1 class patients are patients with minor injuries, e.g., lacerations, cuts, wounds, minor respiratory problems, fractures, who do not require surgery. Severity 2 patients are patients who arrive with comparatively more severe problems, and might not require surgery. Severity 3 patients are the highest severity patients, arriving with major issues, e.g., burns, head injuries, fractures, which require surgery. Since Severity 1 patients do not require surgery, only Severity 2 and Severity 3 patients are considered in the queue before entering the OR. In addition, inpatients also utilize the OR. Their surgery can be postponed due to the large number of emergency patients requiring surgery. Therefore, they have the lowest priority for using the ORs. Only when there are no Severity 2 and Severity 3 patients are waiting for an OR, can these inpatients be operated on.

Each patient is assigned a survivability time corresponding to his/her severity of injury. The survivability time is the maximum time that a patient can wait before the treatment in an OR (for Severity 2 and Severity 3 patients) or ED (for Severity 1 patients). From interviews with hospital staff, the following survivability times are established:

Inpatients: Infinitely long period of time

(for the purpose of modeling priority queue)

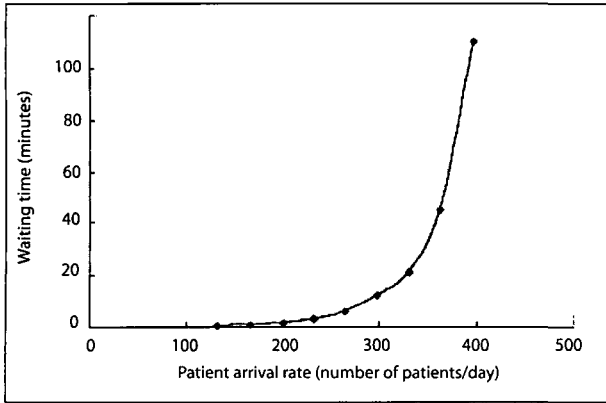
Severity 1: 390 minutes

Severity 2: 270 minutes

Severity 3: 80 minutes

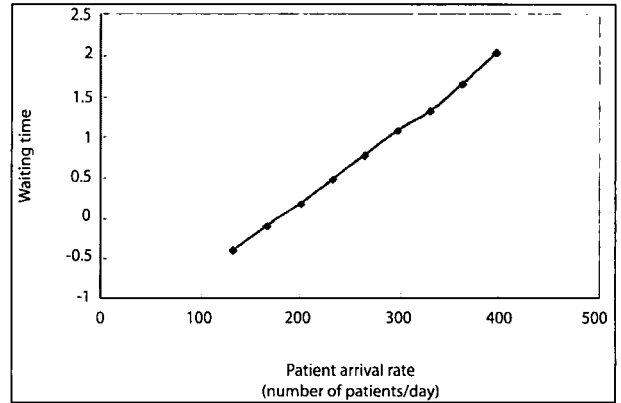
Severity of injury alone is not enough to determine the priority of a patient. It also is necessary to consider the remaining allowable waiting time, which is defined as the difference between a patient's survivability time and the current waiting time. The highest priority is given to those patients with the least remaining allowable waiting time.

Based on the above priorities, a simulation is run for each of the 21 generic hospitals. Both steady state and transient state waiting times are collected for Severity 2 patients and Severity 3 patients separately. Here, the simulation model still considers an earthquake as a event likely to result in disaster, and it does not incorporate the damage to the facility. Whenever an OR becomes available, the remaining allowable waiting times are updated for all of the patients waiting in the OR queue. The patient with the least allowable wait-



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**Figure 1a**—Relationship between steady-state patient waiting time and patient arrival rate; steady-state waiting time vs. patient arrival rate (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index)



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**Figure 1b**—Relationship between steady-state patient waiting time and patient arrival rate; logarithmically-scaled steady-state waiting time vs. patient arrival rate (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index)

ing time remaining obtains the highest priority and is sent to the OR.

*Modeling Change in Patient Mix*

In the previous research, the patient mix was fixed based on the statistical analysis of historical data.<sup>6</sup> While a fixed patient mix captures the general injury types, it is not flexible to deal with a dynamic patient mix. In this research, the effect of change in patient mix has been studied. Patient mix can be of two types: (1) the ratio of the number of OR patients/total number of patients, denoted by  $\alpha$ ; and (2) the ratio of Severity 2 (OR)/(Severity 3 + Severity 2 (OR)), denoted by  $\beta$ . An ANOVA test determined the significance of  $\alpha$  and  $\beta$  and their interactions with the other factors on the patient waiting times.

*Modeling Effect of Prehospital Transport Time*

The patient severity, as identified before transport, may not remain the same upon arrival to the hospital. The survivability time also must be updated based on the transport time. Here, two possibilities can occur: (1) the patient severity type remained the same as it was before transport, but the survivability time has decreased; or (2) the patient severity type has changed and the survivability time has decreased. This can have two outcomes: (1) the patient waiting times may be affected; or (2) the available capacity of the hospital might change. The input to the simulation model is the updated survivability time, which is obtained after deducting the transport time. The transport time is assumed to follow a normal distribution with a mean,  $\mu$ , and variance,  $\sigma^2$ .

**Simulation Meta-Model for Steady-State Conditions**

*Regression of Steady-State Operations*

During a disaster, the types of injuries differ from those during normal times. Therefore, the experiments are made using historical data compiled from the statistics of five earthquakes in California from the late 1970s to mid-

| Important Parameters | Linear Regression | Non-linear Regression |
|----------------------|-------------------|-----------------------|
| $T_{O2}$             | 64.2%             | 94.6%                 |
| $T_{L2}$             | 77.4%             | 96.5%                 |
| $\lambda_{U2}$       | 92.7%             | 98.3%                 |
| $T_{U2}$             | 64.2%             | 88.1%                 |

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**Table 2**— $R^2$  values for linear and non-linear regression (Severity 2)

Where:

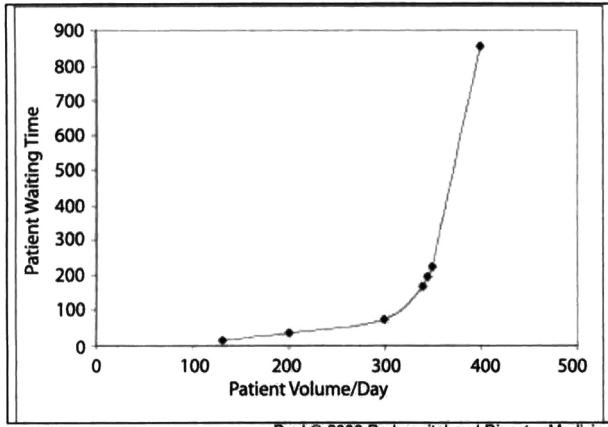
- $T_{O2}$  = pre-earthquake steady-state waiting time for severity two type patient;
- $T_{L2}$  = post-earthquake steady-state waiting time corresponding to the lower bound on arrival rate (base case waiting time) for Severity 2 patient;
- $\lambda_{U2}$  = post-earthquake upper bound of patient arrival rate that begins to saturates the system (critical case arrival rate) for severity two patient; and
- $T_{U2}$  = post-earthquake steady-state waiting time corresponding to the upper bound on arrival rate (“critical case” waiting time) for a Severity 2 patient

1990s.<sup>23–27</sup> To capture the steady-state operational behavior of the generic hospital, the following general parametric form of quadratic regression was used:

$$Z = C_0 + C_1B + C_2O + C_3E + C_4B^2 + C_5O^2 + C_6E^2 + C_7BO + C_8BE + C_9OE \quad (\text{Equation 1})$$

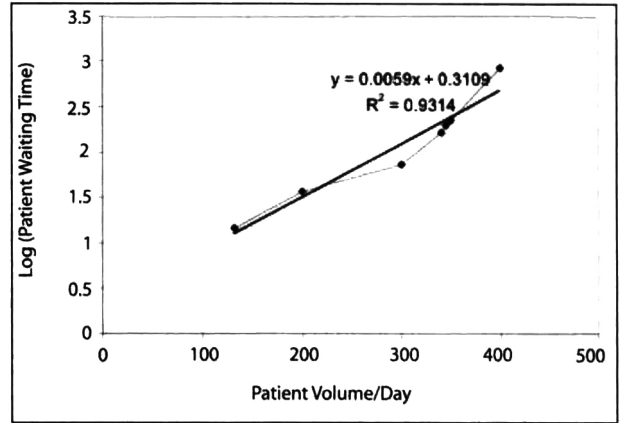
Where:

- Z = patient waiting time (before treatment) on which we are regressing;
- B = number of beds in the hospital;
- O = number of ORs;
- E = OR efficiency index; and
- $C_0, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$  are coefficients.



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**Figure 2a**—Relationship between steady-state patient waiting time and patient arrival rate; Steady-state waiting time vs. patient arrival rate (For a hospital with 500 beds, 10 ORs, 1,200 efficiency index)



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**Figure 2b**—Relationship between steady-state patient waiting time and patient arrival rate; Logarithmic scaled steady-state waiting time vs. patient arrival rate (For a hospital with 500 beds, 10 ORs, 1,200 efficiency index)

For the post-earthquake arrival rate, there are two bounds. The first is a hypothetical arrival rate without emergency patients injured from the event, and it is equal to the pre-earthquake arrival rate. This is the minimal arrival rate called the “base case”. The highest arrival rate is found from simulation runs during which the system will become over-capacitated with any additional volume. This corresponds to the maximum number of patients that the hospital can treat. This scenario is called the “critical case”. The parametric form of patient arrival rate is the same as in Equation 1.

To capture the steady-state operational behavior of the generic hospital, the following general parametric form of linear regression was used:

Case 1: Without severity separation: The  $R^2$  values for linear and non-linear regressions are shown in Table 1. The relationship between steady-state patient waiting time and patient arrival rate is shown in Figures 1a and 1b.

Case 2: With severity separation:

Severity 2: The  $R^2$  values for linear and non-linear regression are shown in Table 2. The relationship between steady-state patient waiting time and patient arrival rate is shown in Figures 2a and 2b. Similar results have been obtained for Severity 3 patients.

In order to find the waiting time corresponding to an arrival rate between the base case and the critical case, intermediate patient arrival rates were simulated. The simulation results show that the waiting time grows nearly exponentially with the increase in arrival rates (Figures 1a, 1b, 2a, 2b, and 3). Therefore, a regression of logarithmically transformed waiting time is made to obtain a good linear fit:

$$\ln(T_S) = a + b\lambda \tag{Equation 2}$$

Where:

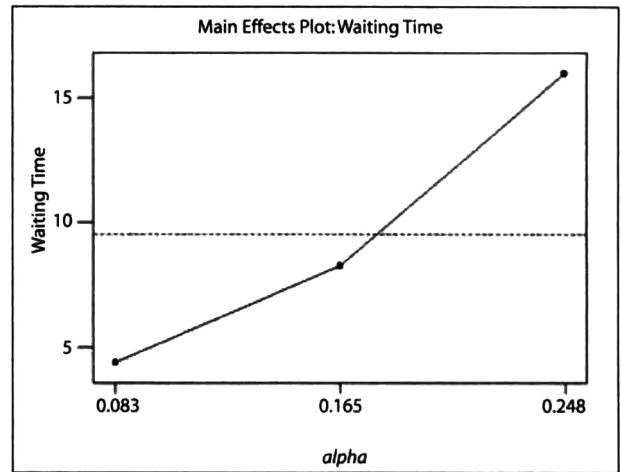
$\lambda$  = any patient arrival rate between  $I_L$  and  $I_U$ ;

$T_S$  = the steady-state waiting time corresponding to  $\lambda$ ;

and

$a$  and  $b$  = constants for a given hospital configuration.

Equation (2) allows estimation of the steady-state wait-



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**Figure 3**—Effect of change of  $\alpha$  on the waiting time

ing times corresponding to an intermediate patient arrival rate for any given hospital, in terms of the constants  $a$  and  $b$ , which are found by using the data from the base case and critical case simulations.

*Incorporation of the Effect of Change in Patient Mix*

Simulations were completed using different values for  $\alpha$  and  $\beta$ . When the patient mix is fixed, let the default value for  $\alpha$  and  $\beta$  be  $\alpha_d$  and  $\beta_d$ . To see the effects of  $\alpha$  and  $\beta$ , an experiment was designed where the  $\alpha$  value was  $0.5\alpha_d$ ,  $\alpha_d$ , and  $1.5\alpha_d$  and where the  $b$  value was  $0.5\beta_d$ ,  $\beta_d$ , and  $1.5\beta_d$ . The dynamic patient mix would not deviate drastically from the default mix; so this design should cover most realistic patient mixes. An ANOVA was performed on these simulation results obtained for the Severity 2 and Severity 3 patients. This showed that  $\alpha$  is a major factor while  $\beta$  is not. Thus, from the simulation results, it can be deduced that the patient waiting is a time function of number of beds, OR, Efficiency, and  $\alpha$ .

|      |                 |            |                | alpha        |         |         |
|------|-----------------|------------|----------------|--------------|---------|---------|
|      |                 |            |                | 0.083        | 0.165   | 0.248   |
| Beds | Operating Rooms | Efficiency | Patient Volume | Waiting Time |         |         |
| 100  | 5               | 600        | 31             | 6.2446       | 12.0523 | 22.0135 |
| 100  | 5               | 900        | 31             | 9.5883       | 19.3844 | 28.4813 |
| 100  | 5               | 1,200      | 31             | 15.6535      | 22.0973 | 29.2383 |
| 100  | 10              | 600        | 31             | 2.4919       | 3.5381  | 3.8979  |
| 100  | 10              | 900        | 31             | 7.1758       | 9.139   | 9.26    |
| 100  | 10              | 1,200      | 31             | 7.1758       | 9.139   | 10.2292 |
| 300  | 5               | 600        | 82             | 2.5607       | 23.7189 | 20.3459 |
| 300  | 5               | 900        | 82             | 2.1543       | 4.5374  | 24.665  |
| 300  | 5               | 1,200      | 82             | 13.3070      | 22.46   | 27.478  |
| 300  | 10              | 600        | 82             | 1.5356       | 3.5381  | 9.2704  |
| 300  | 10              | 900        | 82             | 3.5082       | 5.5468  | 9.6877  |
| 300  | 10              | 1,200      | 82             | 5.3805       | 10.2418 | 12.988  |
| 300  | 15              | 600        | 82             | 1.2577       | 1.5354  | 3.0138  |
| 300  | 15              | 900        | 82             | 2.1543       | 4.5374  | 7.6443  |
| 300  | 15              | 1,200      | 82             | 3.4669       | 5.8204  | 9.1064  |
| 500  | 10              | 600        | 132            | 1.1358       | 4.9027  | 16.316  |
| 500  | 10              | 900        | 132            | 2.8552       | 11.4487 | 19.211  |
| 500  | 10              | 1,200      | 132            | 5.0166       | 14.517  | 22.105  |
| 500  | 15              | 600        | 132            | 1.0484       | 1.8339  | 5.8515  |
| 500  | 15              | 900        | 132            | 1.8838       | 4.539   | 11.5558 |
| 500  | 15              | 1,200      | 132            | 3.0753       | 7.0073  | 15.66   |

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Table 3—Effect of alpha for different hospital configurations (Severity 2)

The relationship between the waiting time and these factors and their various interactions was found by performing simulations for different combinations of hospitals for different levels of  $\alpha$  (0.083, 0.165, 0.248) while holding  $\beta$  constant at 0.503 (generic case).

This is a reasonable result since  $\alpha$  determines the proportion of OR patients. The larger the  $\alpha$  value, the more patients who need surgical services, and the heavier the burden on OR resources, the longer the waiting time. While  $\beta$  determines the patient mix within the operating room patients, the requirement for surgical services remains relatively constant by varying  $\beta$  under a fixed  $\alpha$  value.

The general parametric form of the equation for quadratic regression is:

$$Z = C_0 + C_1B + C_2O + C_3E + C_4A + C_5B^2 + C_6O^2 + C_7E^2 + C_8A^2 + C_9BO + C_{10}BE + C_{11}BA + C_{12}OE + C_{13}OA + C_{14}EA$$

*Relationship between  $\alpha$  and Critical Volume*

From the simulation results, the following relationship was found between the critical volumes at  $\alpha = 0.165$ , the fixed value as taken in, and the critical volumes at the other  $\alpha$  values (0.083, 0.248) considered in the study.<sup>6</sup>

$$\text{Log (Cr_Vol at } \alpha \text{ value)} = \text{Log (Cr_Vol at } \alpha = 0.165 \text{ value)} + (0.212 + 0.608(\alpha - 0.083)/0.165) \times \text{Log (0.165}/\alpha)$$

Here, the Cr\_Vol is the total patient volume. But this equation holds true for critical volume of specifically both Severity 2 and 3 type patients.

*Significance of  $\beta$  Considering Percentage of Patients Served within Survivability Time*

Simulations were run for the 21 hospital combinations for the different values of  $\beta$ . An ANOVA was performed using the results and  $\beta$  was found to be insignificant. This result is not surprising because the patient mix within the patients requiring the surgical services change, but the total number of patients requiring these services remains the same. An ANOVA also was performed to check if  $\beta$  was significant when the percentage of the patients served within the survivability time is considered and was found to be insignificant.

*Sensitivity Analysis*

A sensitivity analysis was performed to determine the effect of changing  $\alpha$ , while other factors on the waiting time of a patient holding constant. All possible hospital combinations with beds from 100–500, ORs ranging from 5–15 were analyzed.

The sensitivity of the waiting time with respect to  $\alpha$  is given by the following equation:

$$\text{Waiting time} = \exp(((\lambda_U \log(T_U) - \lambda_O \log(T_O)) / (\log(T_U) - \log(T_O))) \lambda) / (\lambda_U - \lambda_O)$$



|      |                 |            |                | alpha                  |         |         |
|------|-----------------|------------|----------------|------------------------|---------|---------|
|      |                 |            |                | 0.083                  | 0.165   | 0.248   |
| Beds | Operating Rooms | Efficiency | Patient Volume | Waiting Time (minutes) |         |         |
| 100  | 5               | 600        | 31             | 4.8531                 | 10.8146 | 16.5517 |
| 100  | 5               | 900        | 31             | 10.5023                | 15.4255 | 23.5067 |
| 100  | 5               | 1,200      | 31             | 14.2524                | 19.8112 | 24.7213 |
| 100  | 10              | 600        | 31             | 2.1967                 | 3.0947  | 4.074   |
| 100  | 10              | 900        | 31             | 6.4086                 | 8.4955  | 8.8829  |
| 100  | 10              | 1,200      | 31             | 6.4086                 | 8.4955  | 10.2198 |
| 300  | 5               | 600        | 82             | 2.3212                 | 13.9292 | 32.9342 |
| 300  | 5               | 900        | 82             | 2.1284                 | 3.952   | 34.435  |
| 300  | 5               | 1,200      | 82             | 11.5771                | 26.4176 | 47.074  |
| 300  | 10              | 600        | 82             | 1.2986                 | 3.1649  | 7.7487  |
| 300  | 10              | 900        | 82             | 3.3908                 | 5.5468  | 9.6877  |
| 300  | 10              | 1,200      | 82             | 5.2393                 | 8.7313  | 12.988  |
| 300  | 15              | 600        | 82             | 1.2795                 | 1.3984  | 2.4558  |
| 300  | 15              | 900        | 82             | 2.1284                 | 3.952   | 6.5724  |
| 300  | 15              | 1,200      | 82             | 3.2383                 | 5.3244  | 7.9636  |
| 500  | 10              | 600        | 132            | 1.0961                 | 3.4816  | 15.1162 |
| 500  | 10              | 900        | 132            | 2.5929                 | 8.2064  | 23.0929 |
| 500  | 10              | 1,200      | 132            | 4.5248                 | 11.4507 | 22.5292 |
| 500  | 15              | 600        | 132            | 1.0408                 | 1.7086  | 4.2834  |

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Table 4—Effect of alpha for different hospital configurations (Severity 3)

Where:

- $T_0$  = pre-earthquake steady-state waiting time;
- $T_L$  = post-earthquake steady-state waiting time corresponding to the lower bound on arrival rate (base case waiting time);
- $\lambda_U$  = post-earthquake upper bound of patient arrival rate that begins to saturates the system (critical case arrival rate);
- $l_U^i$  = post-earthquake steady-state waiting time corresponding to the upper bound on arrival rate (critical case waiting time);
- $\lambda_0$  = the base patient arrival rate; and
- $\lambda \rightarrow$  ranging from  $l_0$  to  $l_U$ .

The sensitivities for Severity 2 and Severity 3 patients were evaluated. The following equation validated that the waiting time is sensitive to the change in  $\alpha$ :

$$\text{Sensitivity} > \Delta (\text{Waiting time}) / \Delta (\alpha) \text{ (All other factors remaining constant).}$$

The waiting times for the Severity 2 and Severity 3 type patients are shown in the Tables 3 and 4 for the different values of  $\alpha$ .

**Transient Modeling for Meta-Models**  
*Models Using Single Exponential Function*

The steady-state waiting time regression model represents the pre-earthquake performance of the hospitals and the

long-term, post-earthquake performance under a constant patient arrival rate. The steady-state model is used as a starting point in transient modeling. The shape of the dynamic responses was examined during the simulation runs. From the output, an exponential function appears to be appropriate to describe the transient behavior.

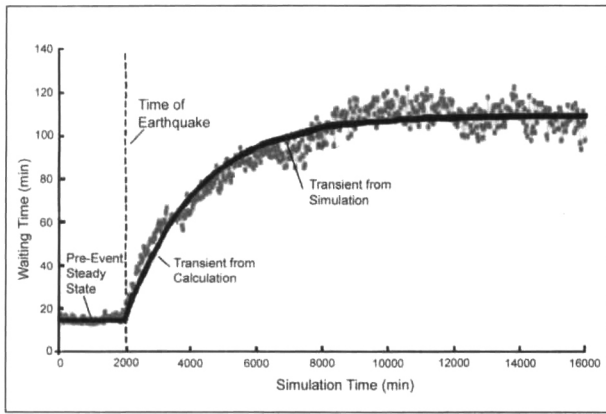
The transient behavior of waiting time shown in Figure 4 is for a hospital with 500 beds, 15 ORs, and a 1,200 efficiency index. The transient is the result of a change in patient arrival rate from 132 patients per day before the earthquake, to 396 per day after the earthquake, which strikes at 2,000 minutes in the simulation.

Using an exponential function, the transient is represented as:

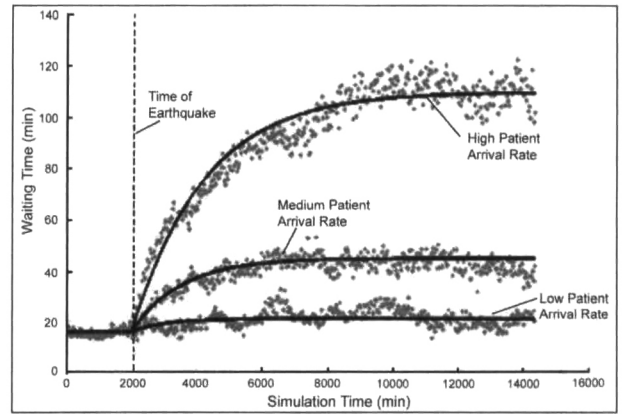
$$T_r(t) = T_i + (T_f - T_i) \cdot (1 - e^{-\frac{t_{eq}-t}{\tau}}) \tag{Equation 3}$$

Where:

- $T_r(t)$  = the transient waiting time at (clock) time  $t$ ;
- $T_i$  = the initial pre-earthquake steady-state waiting time;
- $T_f$  = the final steady-state waiting time corresponding to the post-earthquake patient arrival rate;
- $t_{eq}$  = the time when earthquake strikes; and
- $\tau$  = a time constant, which depends on the time it takes for the system to reach steady state.



**Figure 4**—Simulated vs. calculated waiting time from single exponential function (For a hospital with 500 beds, 15 ORs, 1,200 efficiency index)



**Figure 5**—Temporal waiting time corresponding to different patient arrival rates (For a hospital with 500 beds, 15 ORs, and a 1,200 efficiency index)

**Double Exponential Function**

Although the single exponential form can represent the waiting times for a given hospital adequately, no common, underlying function that can represent a relationship between arrival rates and the time constant,  $\tau$  for all of the hospital combinations was found. Therefore, instead of using a single exponential function, a double exponential function that allows generic modeling of transient waiting time was considered.

The transient waiting-time behavior of a hospital with 500 beds, 15 ORs, and a 1,200 efficiency index under various patient arrival rates is shown in Figure 5. The dots are patient waiting time observed from simulations. To see the trend of the waiting time more clearly, the solid curves were drawn using the moving average approach. By 2,000 minutes (the designated time of the earthquake), the hospital had reached a steady state, and the waiting time has stabilized at around 16 minutes. The earthquake strikes at 2,000 minutes, after which a constant patient arrival rate is assumed. Three curves are shown in the Figure 5, each corresponding to a certain arrival rate, with the highest curve corresponding to the highest rate. The higher the arrival rate, the longer it takes for the hospital to reach a steady state after the earthquake. Therefore, under base case, the hospital will take the shortest time to reach steady state, while under critical case, the hospital will take the longest time to be steady, and any other case will be between these two cases.

This suggests these two cases can be combined to represent any transient behavior in between. The form of double exponential function is:

$$T_i + (1 - p)(T_f - T_i)(1 - e^{-\frac{t_{eq}-t}{\tau_1}}) + p(T_f - T_i)(1 - e^{-\frac{t_{eq}-t}{\tau_2}})$$

(Equation 4)

Where:

- $\tau_1$  and  $\tau_2$  = the time constants associated with the “base case” and the “critical case” respectively; and
- $p$  = weighing factor between 0 and 1. In the “base case”,  $p = 0$ , and in the critical case,  $p = 1$ .

Let  $Tr_1(t)$  and  $Tr_2(t)$  represent the transient waiting time for base case and critical case from Equation 3. That is:

$$T_{r1}(t) = T_i + (T_f - T_i) \cdot (1 - e^{-\frac{t_{eq}-t}{\tau_1}})$$

(Equation 5)

$$T_{r2}(t) = T_i + (T_f - T_i) \cdot (1 - e^{-\frac{t_{eq}-t}{\tau_2}})$$

(Equation 6)

Then Equation 4 can be seen as a weighted sum of  $Tr_1(t)$  and  $Tr_2(t)$ :

$$T_r(t) = 1 - \alpha T_{r1}(t) + \alpha T_{r2}(t)$$

(Equation 7)

For the base case ( $p = 0$ ) and the critical case ( $p = 1$ ), Equation 7 reduces to the form of Equation 3. For an arrival rate close to base case, the value of  $p$  is close to 0; while for an arrival rate close to critical case,  $p$  will be close to 1.

For dynamic post-earthquake arrival rates, a similar relationship is established by assuming that the arrival rate is constant within a period of time. That is, if at time  $t_1$ , the waiting time is  $T_1$ , the patient arrival rate  $\lambda$ , is a constant during the time period from  $t_1$  to  $t_2$ , the steady-state waiting time  $T_2$  can be calculated corresponding to  $\lambda$  from Equation 2 as  $T_2 = e^a + b + \lambda$ . Then the transient waiting time  $T_r(t)$  at time  $t$  is:

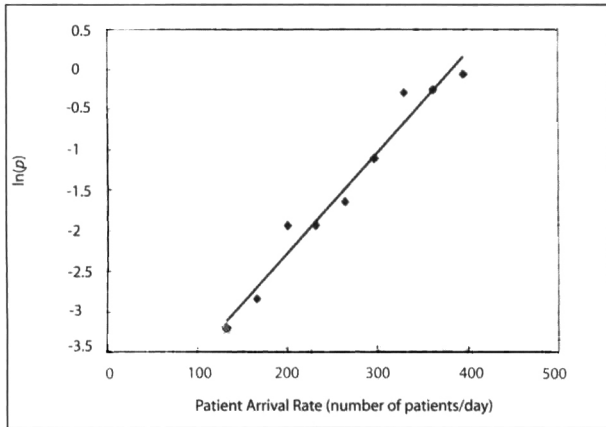
$$T_r(t) = T_1 + (1 - p)(T_2 - T_1)(1 - e^{-\frac{t-t_1}{\tau_1}}) + p(T_2 - T_1)(1 - e^{-\frac{t-t_1}{\tau_2}})$$

(Equation 8)

for  $t_1 \leq t < t_2$   
where  $\tau_1$ ,  $\tau_2$ , and  $p$  are the same as before.

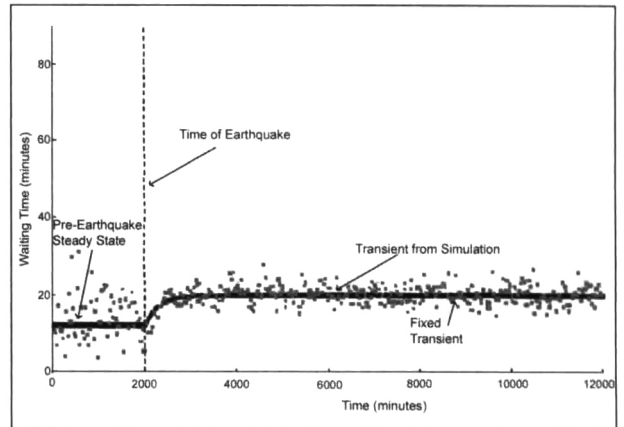
**Regression of Parameters of the Double Exponential Function Model**

To apply Equations 4 and 8, time constants  $\tau_1$  and  $\tau_2$  must be estimated first. For each of the generic hospital configurations, Equation 5 is used to obtain constant  $t_1$  by mini-



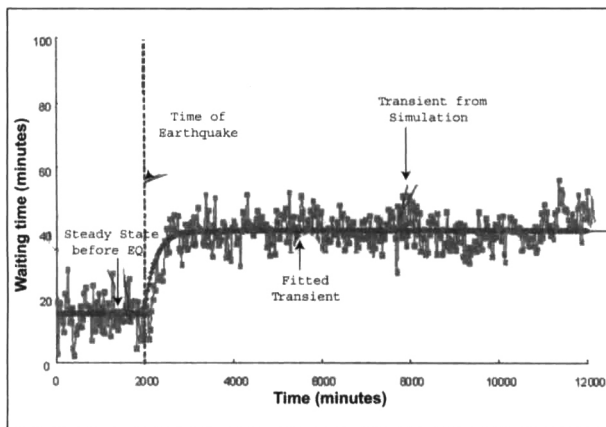
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Figure 6—Logarithmic scaled  $p$  vs. patient arrival rate



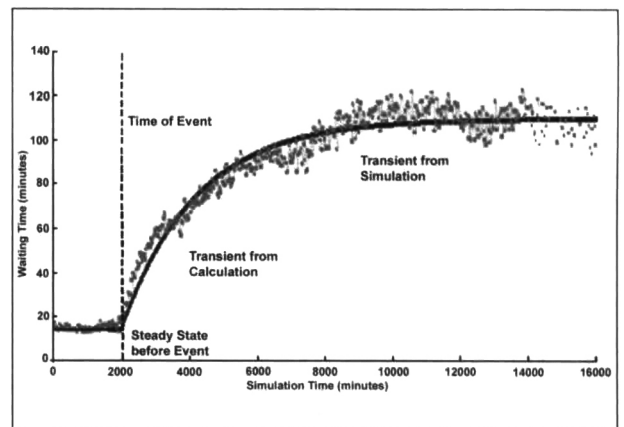
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Figure 8—Transient behavior for Severity 3 patients (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index)



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Figure 7—Transient behavior for Severity 2 patients (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index)



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Figure 9—Transient behavior without considering patients' severity

mizing the sum of squared deviations from the simulation results as follows:

Find  $\tau_1$  that minimizes the objective function  $\Sigma(W_{ti} - Tw_i(\tau_1))^2$   
Where:

$W_{ti}$  = the  $i$ th observation of waiting time during transient period, from simulation for “base case”;

$Tw_i$  = the  $i$ th value obtained from Equation 5, which is a function of  $\tau_1$

In case of  $\tau_2$ , Equation 6 is applied instead of Equation 5. Using optimization software GAMS 31 (GAMS Development Corporation, Washington, DC), the values for  $\tau_1$  and  $\tau_2$  are obtained for each of the hospital configurations. The non-linear regression equations of these two constants are

$$\tau_1 = 2291.16 + 4.85B - 2.819O - 2.71E + 0.002B^2 + 10.79O^2 + 0.002E^2 - 0.17BO + 0.004BE = 0.07OE$$

$$\tau_2 = 1531.42 - 6.73B + 526.06O - 5.9E - 0.009B^2 - 0.89O^2 + 0.01E^2 + 0.59BO + 0.01BE - 0.81OE$$

The  $R^2$  values of the regression are 96% and 79% respectively.

*Estimation of p*

To determine the value for  $p$ , several simulations with different post-earthquake patient arrival rates for selected

hospital configurations were performed. Similar to the approach used to estimate  $\tau_1$  and  $\tau_2$ , the  $p$  values obtained under each of the simulated patient arrival rates by minimizing the squared deviations.

The relationship between patient arrival rate and  $p$  is shown in Figure 6 for a hospital with 500 beds, 15 ORs, and a 1,200 efficiency index. The higher the patient arrival rate, the larger the value of  $p$ . The logarithmically scaled  $p$ -value is proportional to patient arrival rate with  $R^2$  value equal to 96%:

$$\ln(p) = c + d\lambda \tag{Equation 9}$$

Where:

$c$  and  $d$  are constants for a particular hospital; and  $\lambda$  = patient arrival rate.

Hospital-specific coefficients  $c$  and  $d$  can be determined using the “base case” ( $p = 0$  and  $\lambda = \lambda_L$ ) and “critical case” values ( $p = 1$  and  $\lambda = \lambda_U$ ), for any hospital. Notice that when  $p = 0$ ,  $\ln(p)$  does not exist. However, as seen in Figure 7, as  $p$  approaches zero,  $\ln(p)$  approaches  $-3.2$ . Therefore,  $\ln(0) = -3.2$  is used as an approximation in the calculations.

To compare the accuracy of double exponential function and single exponential function, the time constants were

| Parameters                | R <sup>2</sup> (Linear Regression) | R <sup>2</sup> (Quadratic Regression) |
|---------------------------|------------------------------------|---------------------------------------|
| Severity 2 τ <sub>1</sub> | 59.7%                              | 82.0%                                 |
| Severity 2 τ <sub>2</sub> | 53.2%                              | 75.6%                                 |
| Severity 3 τ <sub>1</sub> | 59.2%                              | 75.6%                                 |
| Severity 3 τ <sub>2</sub> | 53.3%                              | 83.8%                                 |

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**Table 5**—Regression on time constants

estimated independently for a set of arrival rates for single exponential function. The *p*-values for the same set of arrival rates for double exponential function also are calculated using Equation 9. The transient waiting time is calculated using Equations 3 and 4; the errors are comparable in both models. However, the double exponential model offers a distinct advantage with an improved functionality, since the time constants are obtained without the need for simulation runs for any patient arrival rate. Simulation only is necessary for the base case and the critical case.

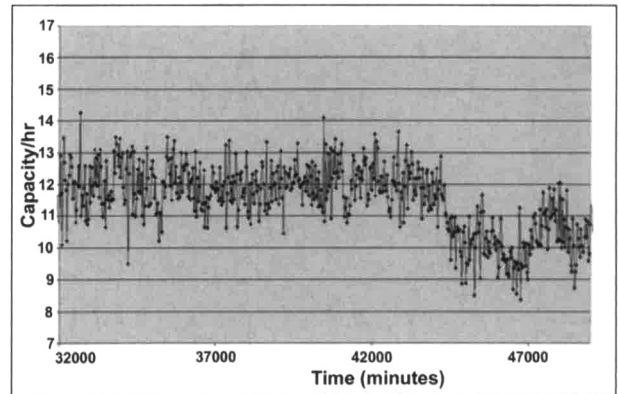
*Transient Behavior*

Assuming a constant patient arrival rate during the disaster, the patient waiting time will experience a transient period until it stabilizes at the steady state. This transient behavior is obtained from simulation for both Severity 2 and 3 patients. The transient waiting time for Severity 2, Severity 3, and all patients under the same overall patient arrival rate for the same hospital are illustrated in Figures 7, 8, and 9. As expected, the transient waiting time for Severity 3 patients is less than is that for the Severity 2 patients, since the former has a higher priority in general. The reason that the overall patient waiting time is higher than the separated waiting time is that there are more inpatients surgeries performed in the former case. In the previous stage of the simulation (no severity separation), 50% of the scheduled pre-event inpatients' surgeries still are performed during the disaster. In current stage of the simulation (with severity separation), those pre-disaster scheduled inpatients' surgeries are performed only when there are no earthquake patients waiting for the OR.

Similar to the approach used to obtain the transient behavior, a double exponential equation can be used to fit the transient waiting time for Severity 2 and 3 patients, with different time constants τ<sub>1</sub> and τ<sub>2</sub>. The regression equations of the time constants for Severity 2 and 3 patients have the same parametric form as the waiting time equations noted before. The R<sup>2</sup> values for these regressions are provided in Table 5. The quadratic fittings are satisfactory.

**Capacity Estimation of the Hospital**

For both Severity 2 and Severity 3 patients, the model given by Equation 8 can be used in combination with Equation 2 to estimate hospital capacity. Assuming the maximum allowable waiting time for Severity 2 patients is T<sub>m2</sub>, then from Equation 2, in steady state, this waiting time corresponds to



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**Figure 10**—Capacity (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index) without travel time alpha changes at 44,000 minutes from 0.165 to 0.248

a maximum patient arrival rate (λ<sub>m2</sub>) given by λ<sub>m2</sub> = (ln(T<sub>m2</sub>) - a/b). Assuming the current waiting time T<sub>r2</sub>(t) to be a steady-state waiting time for a certain patient arrival rate λ<sub>s2</sub>, this arrival rate can be calculated as λ<sub>s2</sub> = (λ<sub>m2</sub>(T<sub>r2</sub>) - a/b). The available capacity is equal to the difference between the maximum capacity and the used capacity:

$$C = \lambda_{m2} - \lambda_{s2} \times \Delta t \tag{Equation 10}$$

Where:

C = the available capacity; and

Δt = the length of time

The same approach can be used to obtain capacity for Severity 3 patients.

*Significance of Prehospital Transport Time*

As far as the waiting time for the patient is concerned, the prehospital time was found to be inconsequential. If the prehospital transport time was so high as to change the patient from a Severity 1 to Severity 2 or 3, it would affect the waiting time considerably, since α is an important factor. The change of a patient from Severity 2 to 3 does not have any effect. Transport times so high as to change the patient from Severity 1 to Severity 2 or 3 occurs rarely because the survivability time of Type 1 is 390 minutes while Type 2 and Type 3 is 270 and 80 minutes respectively. The transport time does not affect the waiting time. The survivability times are updated and are no longer are 390, 270, or 80 minutes, as assumed earlier and the maximum waiting time available for a patient is less. Therefore, the capacity available at the hospital is lower when the effect of the patients' travel time from the point they are picked up to the point they are brought into the hospital is considered.

The new capacity could be found out using the following equation:

$$C = (\lambda_{m2new} - \lambda_{s2}) \times \Delta t \tag{11}$$

Where:

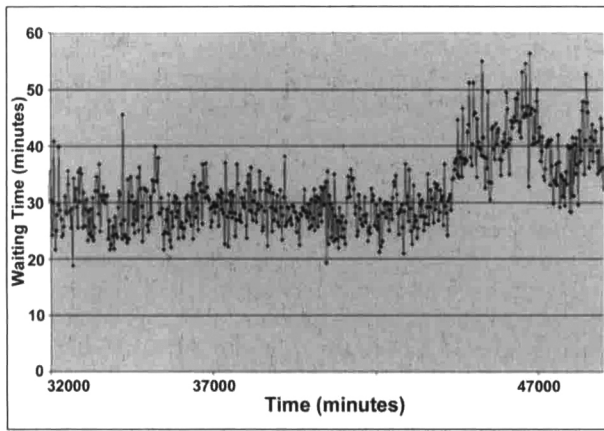
λ<sub>m2new</sub> = (ln(T<sub>m2</sub> - μ) - a)/b; and

μ = mean of prehospital transport time.

*An Illustrative Example*

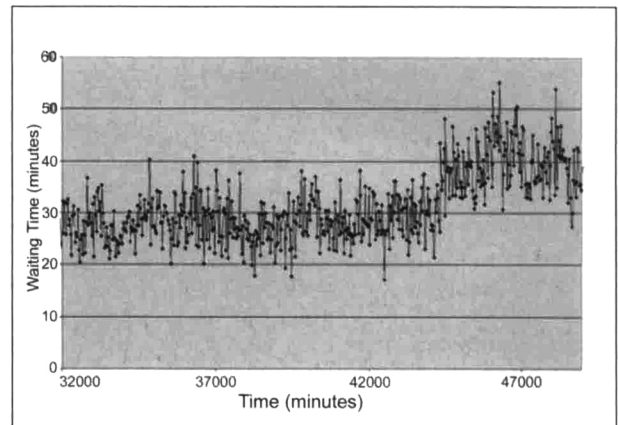
Consider a hospital with 100 beds, five ORs, and 1,200 efficiency. Suppose the patient mix in a time transient period (t1) beginning from the point the earthquake strikes





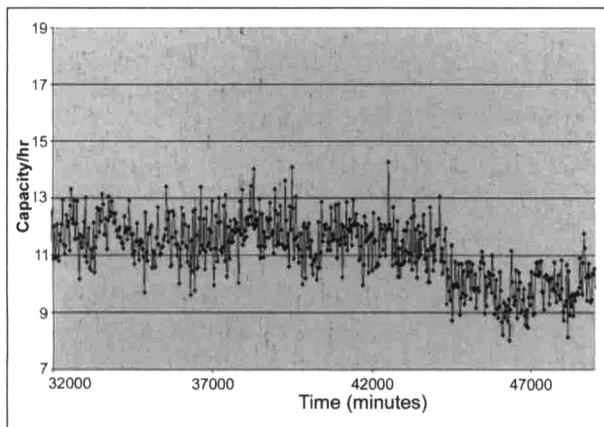
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**Figure 11**—Waiting time (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index) without travel time alpha changes at 44,000 minutes from 0.165 to 0.248



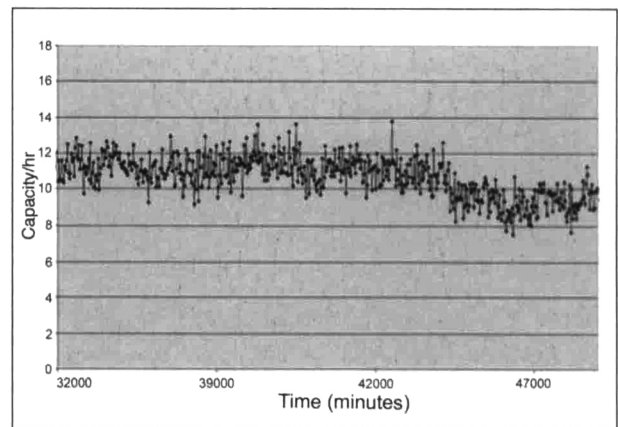
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**Figure 13**—Waiting time (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index) with travel time 30 min, alpha changes at 44,000 minutes from 0.165 to 0.248



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**Figure 12**—Capacity (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index) with travel time 30 min, alpha changes at 44,000 minutes from 0.165 to 0.248



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**Figure 14**—Capacity (for a hospital with 500 beds, 15 ORs, 1,200 efficiency index) with travel time 50 min, alpha changes at 44,000 minutes from 0.165 to 0.248

(30,240 minutes) to 44,400 minutes is 0.165, and for the rest of transient period, the patient mix is 0.248. This scenario has been simulated twice. Once with a travel time distribution of  $N(30,10)$  and once without the consideration of travel time to study the effect of both patient mix and travel time. Assuming a normal distribution, the travel time data generated by simulation which is  $\leq 0$  or extremely large, unrealistic values were not considered. The results showed that due to the change in the patient mix there was considerable change in waiting time and capacity of the patients as illustrated in Figures 10, 11, and 12. The travel time does not have an effect on the waiting time of the patients, but it affects the capacity that is available with the hospital. This is reinforced further by considering another scenario with travel distribution of  $N(50,10)$ . This is due primarily to the fact that the maximum available survivability times with the patients are less because travel time is now updated including the effect of travel time. The survivability times now available are not 270 and 80 minutes for Severity 2 and

Severity 3 patients thereby the capacities available with hospital is reduced per Equation 11 (Figures 13 and 14).

### Conclusion

In this research, a high-fidelity hospital operations transient model was developed by using a generic simulation model approach with a set of meta-models. The results show that a double exponential model with parameters estimated by regression gives satisfactory representation of the transient operations in the hospitals. This work demonstrated the following significance:

1. The meta-models for hospital transient operations are capable of representing any hospital in a disaster situation, indicating the validity of the proposed generic approach to hospital modeling;
2. The model gives greater priority to severely injured patients thereby the total patient waiting time considering the patient survivability times reduced and effective and more efficient use of hospital resources can be accomplished;

3. The model can predict the patient waiting times for any patient mix, and thereby, the routing of patients can be done effectively in order to receive medical attention as soon as possible; and
4. The method allows real-time capacity estimation for all of the hospitals in the disaster region, with minimal computational requirements. Since simulation runs are made off-line, this has overcome the weakness of conventional simulation for which long executions prohibit real-time applications.

In the development of double exponential functions, Equation 9 shows that  $\ln(a)$  is directly proportional to the arrival rates. This clearly indicates that any errors in the estimation of patient arrival rate have a considerable effect

on the estimated capacity. However, the estimation of arrival rate is not straightforward since the rate keeps changing in a disaster event. Therefore, estimation of arrival rates in a real-time model must be addressed in future work. The effect of facility damage on the hospital functioning also should be addressed in the future work.

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