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# REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the Journal in 1936 until the end of 1999. The Association moved the reviews to this Bulletin, beginning in 2000.

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## Two papers by Ulrich Felgner on the history of mathematics

Ulrich Felgner. *Das Induktions-Prinzip*. ***Jahresbericht der Deutschen Mathematiker-Vereinigung***, vol. 114, no. 1 (2012), pp. 23–45.

Ulrich Felgner. *Hilbert's "Grundlagen der Geometrie" und ihre Stellung in der Geschichte der Grundlagendiskussion*. ***Jahresbericht der Deutschen Mathematiker-Vereinigung***, vol. 115, no. 3-4 (2014), pp. 185–206.

The two papers under review discuss aspects of the history of logic from antiquity to the twentieth century. The first paper deals with the history of induction from Aristotle and Euclid to the axiomatization of number theory by Dedekind and Peano and subsequent applications in proof theory. The second paper covers the history of the axiomatization from Euclid to Hilbert's "Grundlagen der Mathematik". Both papers are in German; however, a complete understanding of especially the first paper requires of the reader in addition a nontrivial knowledge not only of English and French but also of Latin and Italian! (However, both are written for a general mathematical audience with no special training in logic.) Given the decline of foreign language skills among logicians, it is therefore a pity that these two papers are not accessible to the reader comfortable with the English language only; and this reviewer very much hopes to see both papers in translation very soon, possibly even as expository papers in the current journal!

Let me now explain in some more detail why I found the two papers so fascinating.

In the first paper, after explaining the difference between simple induction and the $\omega$-rule (concluding $\forall x\, \Phi(x)$ from the infinitely many hypotheses $\Phi(n)$ for all $n \in \omega$), the author first explains the role of induction in the general history of scientific inquiry from Aristotle and Cicero to French and German eighteenth century philosophers and scientists, and highlights the difference between "complete induction" (in the usual mathematical sense) and "incomplete induction" (as a method of scientific inquiry, generalizing from specific cases). In the third and main section of the paper, the author starts in earnest by analyzing various forms of the complete induction principle and the rigor with which it was applied from Euclid to Fermat, Pascal and finally Dedekind, whose use of it in his number theory lectures led to the general acceptance of complete induction as a valid mathematical principle. In the fourth section, the author then traces induction and its role in mathematical logic from Frege to Hilbert, Peano and Gödel and connects it with well-orderings. Felgner concludes with a brief reference to transfinite induction and its use in proof theory.


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The second paper starts out by highlighting geometry as the area of mathematics which was first axiomatized successfully, by today's standards of mathematical rigor, namely, in Hilbert's ground-breaking "Grundlagen der Geometrie". In a number of sections, the author then outlines the various steps in geometry which led to this culmination: Hippasos of Metapont (around 425 B.C.) first proved that the ratio of the diagonal over the side of a square is irrational, leading to an early "foundational crisis" of what geometry is all about and whether the objects it is considering "have reality". Euclid (around 300 B.C.) summarized the achievements of Greek geometry in his "Elements", starting his proofs from axioms (or, rather, postulates, as opposed to the "self-evident" axioms), including his famous Parallel Postulate. There was then a long hiatus until the sixteenth and seventeenth century, when Descartes, Pascal, and Hobbes began considering geometry again from a more philosophical point of view, trying to "define" the objects of geometry, whereas Tschirnhaus and Wolff tried to combine the definitions of the objects of geometry with the axiomatic method of deductive reasoning. The author then jumps directly to the late 1890's, when Hilbert completed what Felgner calls "cutting the umbilical cord of geometry from (physical) reality" and formulated a system of axioms which describe geometric objects as idealized rather than physical objects. The rest of the paper describes Hilbert's method and way of thinking in fair detail, in particular the so-called "Completeness Axiom", which ensures that each line in space is order-isomorphic to the real line, and comments further on Hilbert's "structuralist" point of view, according to which the objects of geometry (points, lines, etc.) can only be described but not defined.

In summary, the two papers make for a captivating read on how two fundamental topics in mathematics were viewed over the course of more than two millennia, with many direct quotes from thinkers along the way, part of which accounts for the linguistic challenge in reading the two papers.

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Nicholas J. J. Smith. ***Logic: The laws of truth***. Princeton University Press, Princeton NJ, 2012, xiv + 528 pp.

Nicholas J. J. Smith's stated aim in this book is to provide students of logic with both "the how and the why of logic" (p. xi). The result is an introductory text notable for both its breadth and its depth.

The book is divided into three parts. Parts I and II provide clear and comprehensive introductions to Propositional and Predicate Logic respectively, using trees. Part III contains extension material not typically found in introductory texts, including a chapter introducing the main alternatives to trees: axiomatic proofs; natural deduction and sequent calculus. Although comparatively brief, the presentations of these systems give students some familiarity with their methods, and some appreciation of the relationships between systems. There is also a well-paced chapter on the metatheory of the systems introduced in Parts I and II, and a chapter on set theory, which provides a general introduction to essential material in the area, as well as functioning as an appendix to further elucidate topics introduced earlier in the text. Exercises are provided throughout the chapters to allow for the scaffolding of skills, with solutions available through an associated website.

The book is written in a way that balances an accessible and readable style with formal precision. Important discussions are relatively slow-paced to avoid common confusions (I would note, for example, the pace of the introduction of the languages of Monadic and General Predicate Logic, which students often find difficult), but at the same time Smith avoids glossing over difficult issues, and uses model-theoretic and set-theoretic terminology and concepts in a way that most introductory texts do not, providing invaluable preparation for students continuing with logic beyond the introductory level. Endnotes provide a more