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Electrostatic nonlinear waves in a dusty plasma with positive dust grains and two electron species

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Abstract. The existence of arbitrary amplitude nonlinear electrostatic waves is investigated for a plasma with positively charged dust in the presence of two-temperature electron species. It is shown that such a plasma supports both soliton and double layer structures.

1. Introduction

Nonlinear structures play an important role in many situations where they are associated with large amplitude waves in dusty plasmas (Farid et al. 2001). Such phenomena are present in space and cosmic environments as well as laboratory experiments. For example, the flux of energetic charged particles along auroral lines was explained by such a structure, which acts as an accelerating mechanism (Raadu 1989). Since the initial theoretical study by Rao et al. (1990) on the ultra low frequency dust acoustic (DA) waves in dusty plasmas and the experimental observations of these waves (Barkan et al. 1995; Pieper and Goree 1996), many aspects of the physics of dusty plasmas have been studied in the last decade. Most of the investigations focused on dusty plasmas with negatively charged dust grains described by the fluid equations. In this regard, nonlinear solitons and double layers in dusty plasmas have been investigated by several authors (Baboolal et al. 1988; Bharuthram and Shukla 1992; Vidya Lakshmi and Bharuthram 1994; Mamun et al. 1996). Recently, the properties of nonlinear DA waves were studied for dusty plasmas with negatively charged dust grains and two-temperature non-isothermal positive ion distributions (El-Labany et al. 2004).

The majority of studies in the literature incorporate negatively charged dust grains, due to the dust grains collecting an excess of the usually more mobile electrons. However, dust grains can be found to obtain net positive charge due to processes such as irradiation by UV light or thermionic emission (Verheest 2000; Shukla and Mamun 2002). Such a situation is common for laboratory plasmas with small grains and the Earth's atmosphere as is observed in the noctilucent clouds during polar summer mesopause (Shukla and Mamun 2002; Havnes 2002). Furthermore, such plasmas may have electron populations of different temperatures;

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for example, in the edge region of tokamak plasmas, where impurities created by sputtering constitute the dust component. Consequently, interest in dusty plasmas with positively charged dust grains has grown in recent times (Benilov and Shukla 2001; D'Angelo 2004; Mamun et al. 2004).

In this report we show that such dusty plasmas, with positive dust grains and two temperature electrons, support the existence of solitons and double layers. The paper is organized as follows. The basic theory is formulated in Sec. 2 and the numerical results are presented in Sec. 3. Finally, our results are summarized and discussed in Sec. 4.

2. Formulation

We consider a collisionless, unmagnetized plasma consisting of positively charged thermal dust grains of temperature T and two distinct groups of electrons, a hot component electrons (temperature T_{h}) and a cool component (temperature T_{c}). The density of the two electrons species is given by the Boltzmann distribution:

$$n_{\mathrm{eh}} = n_{\mathrm{oh}} \exp\{e\phi/T_{\mathrm{h}}\} \tag{1}$$

$$n_{\mathrm{ec}} = n_{\mathrm{oc}} \exp\{e\phi/T_{\mathrm{c}}\}, \tag{2}$$

where ϕ is the electrostatic potential and n_{oh}, n_{oc} are, respectively, the hot and cool electron densities at equilibrium.

The dynamics of the dust grains are described by the fluid equations, with the continuity and momentum equations given in one dimension, respectively, by

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial (n_{\mathrm{d}} v)}{\partial x} = 0 \tag{3}$$

and

$$m\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = -q\frac{\partial \phi}{\partial x} - \frac{T}{n_{\mathrm{d}}}\frac{\partial n_{\mathrm{d}}}{\partial x}, \tag{4}$$

where m and v are, respectively, the dust mass and velocity, and n_{d} is the dust density. The positively charged dust grains have a uniform charge $q = Z_{\mathrm{d}}e$ $(Z_{\mathrm{d}} > 0)$.

The set of equations is closed by Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_{\mathrm{ec}} + n_{\mathrm{eh}} - Z_{\mathrm{d}} n_{\mathrm{d}}). \tag{5}$$

Charge neutrality at equilibrium is given by

$$n_{\mathrm{oc}} + n_{\mathrm{oh}} = Z_{\mathrm{d}} n_{\mathrm{od}} = n_{\mathrm{o}}, \tag{6}$$

where n_{od} is the dust density at equilibrium.

The nonlinear analysis is carried out via the Sagdeev-potential method (Sagdeev 1966). For convenience we express the set of equations (1)–(6) in normalized form. Time is normalized by the inverse of the dust plasma frequency $\omega_{\mathrm{pd}}^{-1} = (4\pi n_{\mathrm{od}} Z_{\mathrm{d}}^2 e^2/m)^{-1/2}$, speed by the dust acoustic speed $C_{\mathrm{sd}} = (T_{\mathrm{eff}}/m)^{1/2}$, distance by $\lambda_{\mathrm{d}} = C_{\mathrm{sd}}/\omega_{\mathrm{pd}}$ and the densities by n_{o}. Here $T_{\mathrm{eff}} = n_{\mathrm{od}} Z_{\mathrm{d}}^2/(n_{\mathrm{oc}}/T_{\mathrm{c}} + n_{\mathrm{oh}}/T_{\mathrm{h}})$ and the normalized potential $\Phi = Z_{\mathrm{d}} e\phi/T_{\mathrm{eff}}$. Then transforming to the stationary frame $\xi = x - Mt$, where M is the Mach number, and following standard techniques (Bharuthram and Shukla 1992; Vidhya Lakshmi and Bharuthram 1994), from (3)

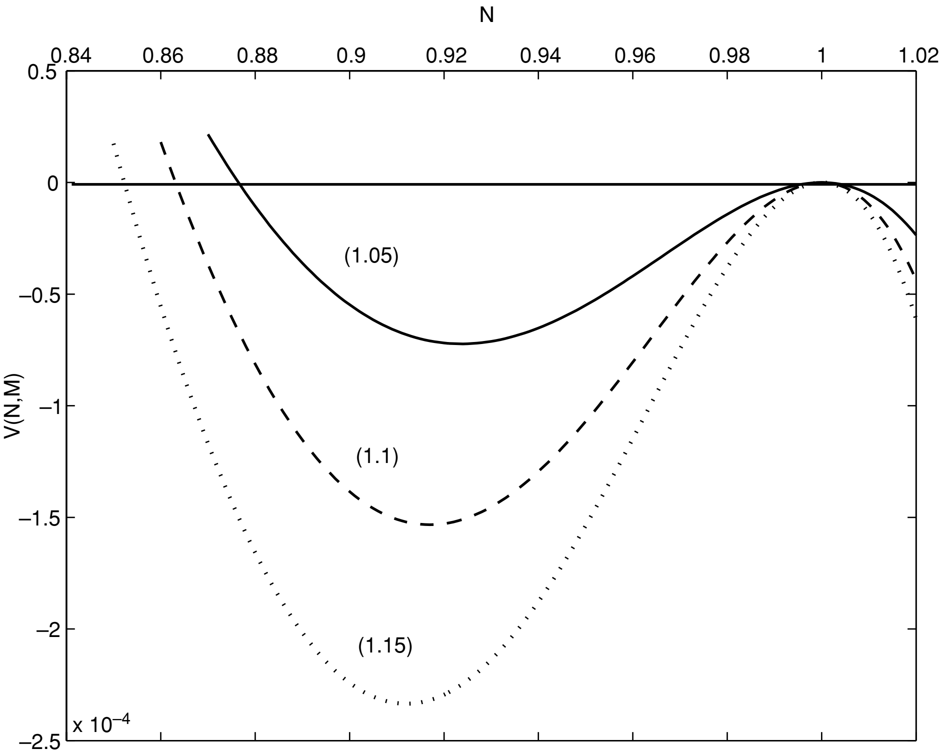

Figure 1. The Sagdeev potential $V(N, M)$ versus density N for $N_{\rm oc} = 0.1$; $N_{\rm oh} = 0.9$; $T_{\rm h}/T_{\rm c} = 20$ (—), 30(- -), 40(· · ·); $T_{\rm d}/T_{\rm h} = 0.01$; and $Z_{\rm d} = 1000$. The parameter labelling the curves is the Mach number, M.

one obtains

$$N = \frac{M}{(M - \tilde{v})}, \tag{7}$$

where the boundary conditions at $\xi \to \infty$ are $\tilde{v} \to 0$ and $N \to 1$ and $N(\tilde{v})$ is the normalized dust density (speed).

As a result of the pressure gradient term in (4), we cannot obtain the 'energy integral' for a pseudo particle with the Sagdeev potential in terms of Φ (Baboolal et al. 1988). However, following Singh and Rao (1997), we may use the normalized dust density N as the variable. Then from (4) one obtains

$$\Phi = \frac{M^2}{2}\left[1 - \frac{1}{N^2}\right] - \delta \ln(N). \tag{8}$$

By integrating Poisson's equation in the stationary frame we obtain

$$\frac{1}{2}\frac{\partial}{\partial \xi}\left[\frac{\partial \Phi}{\partial \xi}\right]^2 = N_{\rm oh} e^{\alpha_{\rm h}} \frac{\partial \Phi}{\partial \xi} + N_{\rm oc} e^{\alpha_{\rm c}} \frac{\partial \Phi}{\partial \xi} - N \frac{\partial \Phi}{\partial \xi}. \tag{9}$$

From (8) we have

$$\frac{\partial \Phi}{\partial N} = \frac{1}{N}\left[\frac{M^2}{N^2} - \delta\right]. \tag{10}$$

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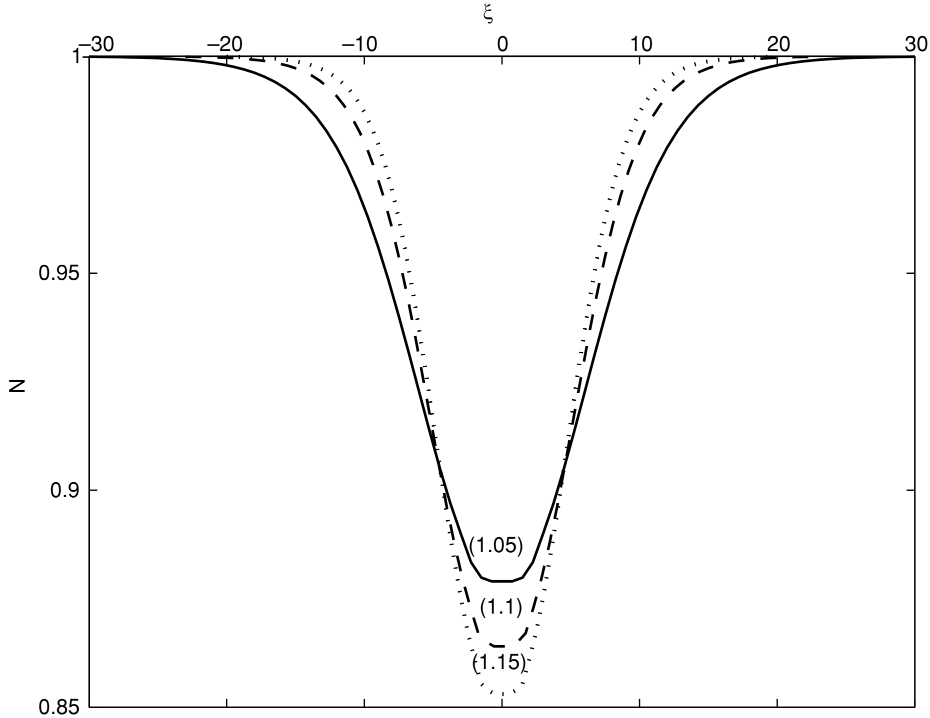

Figure 2. The soliton structures corresponding to the curves in Fig. 1.

Combining (9) and (10) yields

$$\frac{1}{2}\left(\frac{dN}{d\xi}\right)^2 + V(N, M) = 0 \tag{11}$$

where the Sagdeev potential is given by

$$V(N, M) = N^2\left\{\frac{N_{\rm oc}}{\alpha_{\rm c}}(1 - e^{\alpha_{\rm c}\Phi}) + \frac{N_{\rm oh}}{\alpha_{\rm h}}(1 - e^{\alpha_{\rm h}\Phi}) + M^2\left(1 - \frac{1}{N}\right)\right.$$
$$\left. + \delta(1 - N)\right\}\Big/\left[\frac{M^2}{N^2} - \delta\right]^2, \tag{12}$$

where $\alpha_{\rm h,\,c} = T_{\rm h,\,c}/T_{\rm eff}$ and $\delta = T/T_{\rm eff}$ and Φ is as defined by (8).

3. Numerical results

In this section (11) and (12) are numerically solved to show the existence of both soliton and double layer structures.

3.1. Soliton

For the existence of a soliton the standard requirements are (Sagdeev 1966):

(i) $V(N = 1, M) = V(N = N_{\rm m}, M) = 0$;

(ii) $\partial_{\rm N}V(N = 1, M) = 0$ and $\partial_{\rm N}V(N = N_{\rm m}, M) > 0$ (< 0) for $N_{\rm m} > 1 (< 1)$;

(iii) $V(N, M) < 0$ for $1 < N < N_{\rm m}$ $(N_{\rm m} < N < 1)$ when $N_{\rm m} > 1$ (< 1).

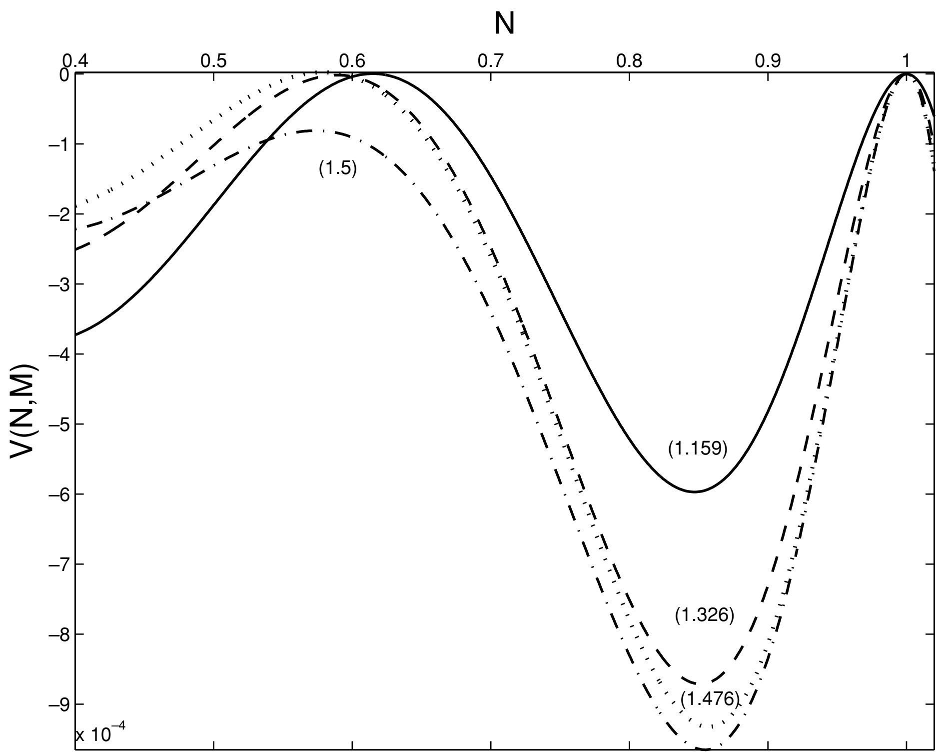

Figure 3. The Sagdeev potential $V(N,M)$ versus density N for $N_{\rm oc}=0.1$; $N_{\rm oh}=0.9$; $T_{\rm h}/T_{\rm c}=20$ (—), 30(- -), 40(· · ·), 40(- · -); $T_{\rm d}/T_{\rm h}=0.01$; and $Z_{\rm d}=1000$. The parameter labelling the curves is the Mach number, M.

Numerical investigation is carried out for dust grains with charge $Z_{\rm d} = 1000$. We concentrate on the structures that satisfy conditions (i)–(iii) above. In Fig. 1 we plot the Sagdeev potential versus the normalized density N using (12) for selected values of M and $T_{\rm h}/T_{\rm c}$. In this case the ratio between cool and hot electrons is set to $N_{\rm oc}/N_{\rm oh} = 0.11$. Our computation shows that for $T_{\rm h}/T_{\rm c} = 40$ the soliton solution is only possible for M values satisfying $1 < M \leq 1.15$, with the range of M values depending on the choice of $T_{\rm h}/T_{\rm c}$. The corresponding plot of N against ξ (by integrating (11)) is shown in Fig. 2. It is seen that the solitons are rarefactive in nature.

3.2. Double layer

The formation of double layer is subject to the following conditions (Sagdeev 1966):

(i) $V(N=1,M) = \partial_{\rm N} V(N=1,M) = 0$;

(ii) $V(N=N_{\rm m},M) = \partial_{\rm N} V(N=N_{\rm m},M) = 0$;

(iii) $V(N,M) < 0$ for $1 < N < N_{\rm m}$ $(N_{\rm m} < N < 1)$ when $N_{\rm m} > 1$ (< 1).

From (ii) we have from (12) that

$$\frac{N_{\rm oh}}{\alpha_{\rm h}}(1-e^{\alpha_{\rm h}\Phi_{\rm m}}) + \frac{N_{\rm oc}}{\alpha_{\rm c}}(1-e^{\alpha_{\rm c}\Phi_{\rm m}}) + M^2\left(1-\frac{1}{N_{\rm m}}\right) + \delta(1-N_{\rm m}) = 0 \qquad (13)$$

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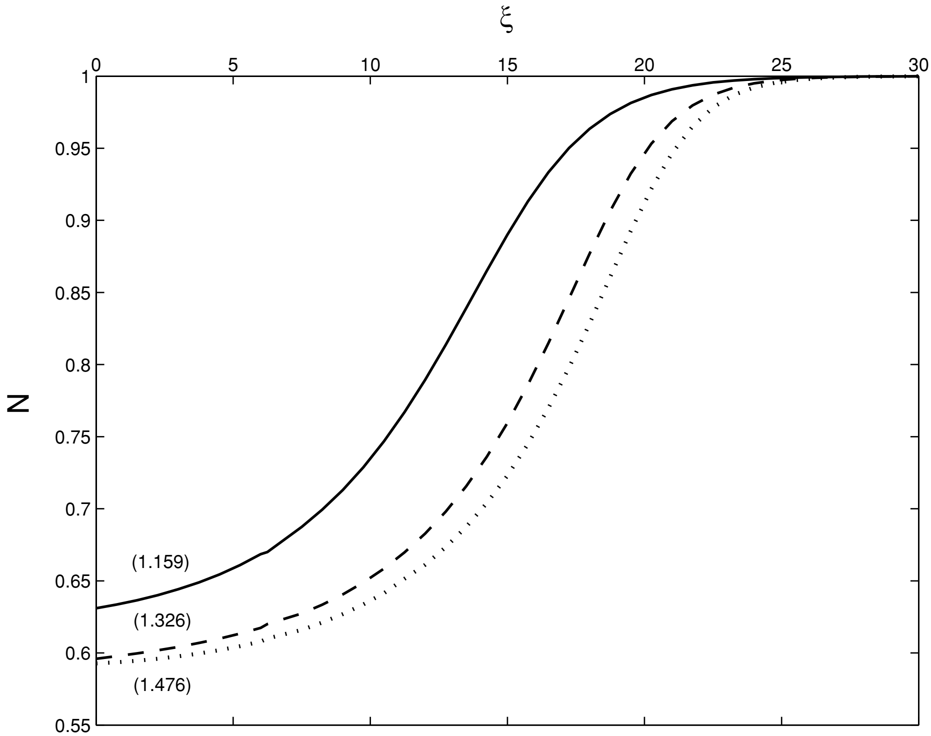

Figure 4. The double layer structures corresponding to the curves in Fig. 3.

and

$$N_{\rm m} - N_{\rm oh} e^{\alpha_{\rm h}\Phi_{\rm m}} - N_{\rm oc} e^{\alpha_{\rm c}\Phi_{\rm m}} = 0, \tag{14}$$

with (from (8))

$$\Phi_{\rm m} = \frac{M^2}{2}\left[1 - \frac{1}{N_{\rm m}^2}\right] - \delta \ln(N_{\rm m}). \tag{15}$$

We note also that from (12) the condition (i) is clearly satisfied for $N = 1$. We have to numerically solve the coupled equations (13) and (14) for M and $N_{\rm m}$. For the fixed parameters $Z_{\rm d} = 1000$, $N_{\rm oc}/N_{\rm oh} = 0.11$ and $T_{\rm h}/T_{\rm c} = 20, 30, 40$ the double-layer Sagdeev potential is shown in Fig. 3. It is seen from the figure that for the chosen fixed parameters and $T_{\rm h}/T_{\rm c} = 40$, the double layers exist for $M < 1.5$, since for the curve corresponding to $M = 1.5$ in Fig. 3, the double-layer condition (ii) is not satisfied. The corresponding double-layer structure is presented in Fig. 4. It is seen that the rarefactive double-layer height and width increase with $T_{\rm h}/T_{\rm c}$.

4. Conclusion

Using nonlinear theory we have shown that arbitrary amplitude rarefactive solitons and double layers may exist in a plasma composed of positive dust grains and two-electron species of different temperatures. Our numerical investigations show that the structures exist for a restricted range of Mach number values, and their amplitudes are functions of plasma parameters such as particle densities and temperatures. These nonlinear structures may contribute to the turbulence and associated anomalous particle and energy transports in such plasma found in laboratory experiments, as well as space and astrophysical environments.

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