

MEASURING SPECIFIC FREEDOM

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This paper is about the measurement of specific freedoms – the freedom of an agent to undertake some particular action. In a recent paper, Dowding and van Hees discuss the need for, and general form of, a “freedom function” that assigns a value between 0 and 1 to a freedom or right and that describes the expectation that a person may have about being in a position to exercise (“being free to perform”) that freedom or right. An examination of the literature shows that there is as yet no agreed framework for defining such a function. Based on the framework of a game form, I develop a very simple and natural measure of specific freedom as the “conditional probability of success.” It is also shown that in an important way “negative freedom is membership of powerful coalitions.”

1. INTRODUCTION

This paper is about the measurement of specific freedom – the freedom of an agent to undertake some particular action. In this regard its general subject matter is not new. In a recent paper, Dowding and van Hees (2003) discuss, for example, the need for, and general form of, a “freedom function” that assigns a value between 0 and 1 to a freedom or right and that describes the expectation that a person may have about being in a position to exercise (“being free to perform”) that freedom or right. The usefulness of such a function is that in principle it could be used to define threshold values for indicating whether or not a person has a particular freedom or right in a *material* (or effective) sense – as against a mere formal

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or legal sense – and therefore for making non-welfaristic judgments about social states or to design the assignment of rights related to government policy, public regulation, or legal rules.¹ In a nutshell, such a function would allow us to make comparative judgments about the extent to which a person can expect, to some degree or another, to exercise a freedom or have that freedom respected.²

Much light, however, still needs to be shed on the nature of such a function. In their contribution, Dowding and van Hees leave the matter more or less open, claiming only that the extent to which a person is free to perform a particular type of action or right depends only on the probabilities with which each of the relevant instances of the action or right will not be prevented. A straightforward example is that of determining our “freedom of expression.” According to Dowding and van Hees this is given by the probability that shouting “Down with the Government” at Whitehall at a given time and date and doing the same thing at Piccadilly Circus, etc. will go unprevented.

Dowding and van Hees refer to the recent and burgeoning literature on measuring freedom for a hint as to how such a function could variously be defined (Arrow 1995; Carter 1999; Dowding 1992; Pattanaik and Xu 1990; Pattanaik and Xu 1998; Sugden 1998; Rosenbaum 2000). A perusal of this literature indicates, however, that as yet there is no agreed upon framework for defining this function as the “probability of being unprevented.”³ The papers by Arrow, Dowding, Pattanaik and Xu, Rosenbaum and Sugden are all concerned with “freedom of choice” rather than with the “freedom to do something” *per se* (freedom *simpliciter*).⁴ Even Carter’s (1999) extensive analysis of measuring overall freedom as an aggregation of the probability of being unprevented to do something does not suggest an explicit model for determining the “input probabilities” into a freedom function. Instead, they enter into his measure as an exogenous variable.⁵ There is, therefore,

¹ This is an idea also discussed by Pogge (2002) in his analysis of the relationship between human rights and poverty.

² Dowding and van Hees (2003) discuss four modes of existence of a freedom (or right): (i) formal existence as a particular; (ii) formal existence as a universal; (iii) material existence as a particular; and (iv) material existence as a universal.

³ In a follow-up paper, Dowding and van Hees (2004: 306–7) admit that it is entirely unclear how this function should be specified.

⁴ For a discussion of the importance of maintaining the distinction between freedom *simpliciter* and freedom of choice, see Carter (2004) and Kramer (2003a). However, one can – as van Hees (1998) has done – interpret the concept of an *opportunity set*, which underpins the freedom of choice literature, as expressing the extent of a person’s specific freedom.

⁵ In his review of Carter’s measure of overall freedom, van Hees (2000) does not tackle this issue either. To the best of my knowledge, the only two papers that come anywhere close to hinting at a reasonable model of such a freedom function are Sugden (1978) and Bavetta (1999). Both implicitly assume a game form. I will not discuss these contributions here

still an open question about (i) the source of the input probabilities and by implication (ii) how to aggregate these probabilities into a value as suggested by Dowding and van Hees. This paper provides a partial and tentative framework for answering both questions.

Starting from a purely negative conception of freedom, I will construct a function that describes *i*'s freedom to perform an action as the "conditional probability of success." This model makes an agent's freedom to do something a function of the propensities of other agents to choose certain actions (strategies) and the "decision rule," which is a function that maps those actions into a unique outcome in which the agent can or cannot perform the specified action. The basic idea is that an agent is guaranteed to be unprevented to perform a specific action if, and only if, she belongs to a subset of agents (a coalition) that can guarantee the performance of that action. Roughly speaking, "negative freedom is membership of powerful coalitions," and a measure, in the sense of ascertaining the expectation that an agent can perform a specified action, is the probability of being a member of a such a coalition.⁶

In the process of constructing a freedom function I make two other contributions of general theoretical importance. First, I address the issue of how to measure freedom in a strategic rather than the parametric setting of social choice theory that developed since Sen's (1970) seminal contribution. Although a number of writers have, for some time, considered this to be a necessary step (Nozick 1974; Gärdenfors 1981; Sugden 1985; Gaertner *et al.* 1992; Pattanaik and Suzumura 1996; van Hees 2000), it is still a largely underdeveloped area (Deb 2004).

Second, I add to the nascent literature that attempts to develop formal models of freedom in an explicit philosophical framework (Steiner 1983; Carter 1999; Dowding and van Hees 2003; Bavetta 2004), rather than deriving such a model from a set of intuitive desiderata. In other words, I do not take for granted a particular notion of freedom, but instead base my measure on a philosophically grounded generic concept and syntax of freedom.

2. THE CONCEPTION AND SYNTAX OF SPECIFIC FREEDOM

When constructing a measure of freedom, it is essential to be clear from the outset about the *conception* or *type* of freedom that we want to deal with. Primarily, this requires us to distinguish between an "opportunity"

because they are in fact only very suggestive; neither actually defines a freedom function in a precise way.

⁶ This gives a twist to the meaning of 'success' which was independently introduced by Penrose (1946), Rae (1969), and Barry (1980a, 1980b) in the voting power literature.

and an “exercise” concept of freedom.⁷ As Carter (2004), who employs this distinction in his dissection of Pattanaik and Xu’s (1990) axioms of freedom of choice, puts it, “where freedom is treated as an opportunity concept, it means the *possibility* for an agent of performing some action or actions” (Carter 2004: 64), where “possibility” is understood as meaning a lack of constraints of various kinds. Taken in this sense, freedom is concerned with actions that *might* be performed, given the absence of constraints, at some moment subsequent (or identical) to that at which the agent possesses the freedom in question. On this view, freedom is a matter “of what we can do, of what it is open to us to do, whether or not we do anything to exercise these options” (Taylor 1979: 177).

In contrast, freedom as an “exercise concept” concerns the performance by an agent of some action or actions; it is “to *do* certain things or to achieve certain outcomes in a certain way” (Carter 2004: 64). On this view, freedom usually involves exercising control over one’s life, so that one is free to the “extent that one has effectively determined oneself and the shape of one’s life” (Taylor 1979: 177). Clearly the Hobbes–Bentham notion of negative freedom as simply the “absence of external physical or legal obstacles” (Taylor 1979: 176) is an opportunity concept, while the Rousseau–Marx notion of positive freedom as “self-realization” or “collective self-government” is an exercise concept.

The aim in this paper is to define a freedom function that is applicable to an opportunity concept of freedom. I make no apology for this restriction for the simple reason that I want such a function to also be applicable to the language of rights, and rights generally concern the “opportunity” to do things (voting, protesting, reading) and not “exercising”: if I have a right to read a certain book, then I have that right whether or not I ever read it; or whether or not I read it as a Marxian “species being” (and the same is true if it is merely a freedom to read that book, i.e. irrespective of the right to do so). Furthermore, in this connection I will basically assume a more or less *empirical* conception of freedom in that I will not explicitly concern myself with the thorny issues of the deontic permissibility of the freedoms in question (the *moral freedom* to do something, such as entering your house, irrespective of the actual or empirical freedom to do so). The normative conception of negative freedom will take a back seat in this analysis, although, as it will become evident, it is not excluded from the freedom function that is derived. For similar reasons I will also exclude

⁷ The opportunity–exercise distinction originates with a classic essay by Taylor (1979) in which he studies Berlin’s (1969) distinction between *negative* and *positive* liberty. Taylor argues that the gamut of views of negative liberty fall into either opportunity or exercise concepts but positive views are only ever exercise concepts. That is, opportunity and exercise concepts are generic categories into which any concept of freedom can be classified.

the issue of the eligibility of the freedoms in terms of the reasonableness or rationality of a particular option (e.g. Sen's (1991:24) famous example, "being beheaded at dawn").

Now, in order to specify a freedom function in a systematic manner, the conception of freedom must be further fleshed out with a syntax of freedom. MacCallum's (1967) now canonical analysis is the obvious candidate. MacCallum formulates a freedom ascription as a triadic relation between agents, constraints (preventing conditions), and possible actions: "of something (an agent or agents), from something, to do, not do, become, or not become something . . ." (p. 314). This relation can be summarized in the format of:

"*i* is (is not) free from *j* to do (not do, become, not become) φ ,"

where *i* ranges over agents, *j* ranges over "preventing conditions" such as constraints, restrictions, interferences, and barriers, and φ ranges over actions ("doings") or conditions of character (Marxian "self-fulfillment" or "realization of one's true nature") or circumstance ("becoming angry").

To define a freedom function that can be used to analyze the extent of my *material* freedoms and rights along the lines that Dowding and van Hees (2003) suggest, MacCallum's rather opened-ended syntax must be honed down. In particular, the range variables have to be restricted to capture only those opportunities that are *social*:

- *i* covers only natural or juridical persons (or groups);
- *j* covers only preventing conditions that are inflicted by the *actions* of other such *agents* or *groups of agents*;⁸
- φ covers only *possible actions* or 'doings'.⁹

⁸ For reasons of tractability I ignore here the valid philosophical problems of determining the *range* of afflictions or misfortunes imposed on *i* that qualify as brought about by the *actions* of other agents. Essentially this issue amounts to providing justification for the elements of the "strategy sets" that are alluded to in section 4 and discussed more fully in section 5. Note that this restriction excludes preventing conditions of natural origin as well as "internal" psychological or neurobiological states of mind so that it will not be applicable to assessing the freedom of the mountaineer who has become physically stuck in a crevasse or the person who is hindered from performing an action because of a morbid fear or phobia, depression, or lack of awareness, etc. no matter how figuratively correct it may be to speak of their conditions in terms of freedom or lack of it.

⁹ It can of course be contested that even for a purely negative conception of freedom the restriction to "doings" is arbitrary (Kramer 2003b: 156–69). However, as the main contribution of this paper actually hinges on the *j* variable I will ignore this issue because it takes us into the Byzantine intricacies of the philosophy of action without adding anything to the derivation of a freedom function.

3. THE COALITIONAL NATURE OF SPECIFIC FREEDOM ASCRIPTIONS

Having set out a conception and syntax of specific freedom, the next natural step is to adopt a formal framework from which we can derive a freedom function. Prior to doing this, however, we have to first sort out a logical conundrum that afflicts canonical definitions of negative specific freedom. The solution to this conundrum is a mainstay of the contribution of this paper.

The usual method, which Carter (1999: 27), Kramer (2003b: 3), and Steiner (1994: 8) employ, is to define prevention in terms of the action(s) of natural or juridical *individual* agents. Indeed, a review of the literature on specific freedom indicates a preoccupation with dyadic relations: *j* preventing or not preventing *i* from φ -ing. This, it is easy to show, is not true in general: prevention arises not from the action of an *individual* agent, but from the combined actions of a non-empty *set* of individual agents. Rather, prevention is a function of individual agents who “belong together” by dint of a common action (or omission), coordinated or otherwise, that is inimical to (“is against”), but not necessarily an action that can alone “prevent,” some other agent performing φ . In *n*-person game theory, these sets are referred to as *coalitions* (we can regard a coalition (a group) as an index for a certain collection of actions by individuals that can be mapped into the same possible outcome).¹⁰

Establishing this thesis is straightforward. Suppose, for instance, a society consists of four members, denoted by the set $N = \{a, b, c, d\}$. Suppose further that there is some action φ that for *a* to perform it, at least two others must not perform some action inimical to *a* performing φ . Now, if we apply an individual agent-based definition of specific freedom and unfreedom we find that there is a configuration of agents who together prevent *a* from performing φ but none of these agents can be said to be doing the “preventing” as such. This is the case when all other agents, i.e. the subset $\{b, c, d\}$, perform actions inimical to *a*-ing φ .

To prove this proposition, let us, without loss of generality, take Carter’s (1999: 27) definitions as our point of departure. (I do not take Steiner’s because he only formally defines unfreedom and I do not take Kramer’s because he includes a condition for an agent’s personal ability to do something which in this context only complicates the issue without adding anything.) Carter says that an agent is free to φ ,

¹⁰ Note that the literature that makes use of *n*-person game theory implicitly accounts for coalitions in our understanding of freedom, although this is not explicitly marked by the authors as being the *generic* formulation of a freedom ascription. See, among others, Gärdenfors (1981), Deb (1994), Peleg (1998), van Hees (1995, 2000). In a different context, Pettit (1996, 1997: 52) allows for “collective agents” such as coalitions in his definition of freedom, but he neither discusses the necessity for doing so, the relationship between individual agents and the “collectivity,” nor the implications that follow.

“if every other agent refrains from preventing her doing it”;

and she is unfree to φ ,

“if some other agent prevents her doing it.”

Take the configuration $\{b, c, d\}$, the members of which have chosen, either jointly or severally, to perform, consciously or otherwise, some action that is inimical to a performing φ . Obviously a cannot be free to φ because, assuming that by “refrains from preventing” Carter means the same as “not preventing,” it is not the case that “every other agent does not prevent a from φ -ing”; they clearly are.¹¹ But by Carter’s account, a cannot be said to be unfree to φ either because this would require that “some other agent” (at least one) is preventing a from φ -ing, which is not the case. This can be shown as follows. If some other agent is preventing a from φ -ing, it means that there is at least one agent who, if, ceteris paribus, she were to decide otherwise, she would see to it that a was free to φ . Otherwise this agent could not be said to be doing the preventing. As is usual when assessing counterfactuals we have to take some state of the world as given. In this case we hold the decisions of c and d constant and ask whether a would be free to φ if b were not to oppose her. The answer is “no,” because at least two agents must do so. So it cannot be said that b is doing the preventing. Applying the same reasoning to c and d we find that in both cases the answer is also “no.” Hence no member of $\{b, c, d\}$ is doing the preventing as such. Thus, while the non-fulfillment of Carter’s conditions for a specific unfreedom (it is not the case that some agent j, k, \dots, n is preventing i) logically entails the fulfillment of his conditions for a specific freedom (all other agents j, k, \dots, n are not preventing i) this entailment does not necessarily imply that an agent will possess a specific freedom, as our example unambiguously demonstrates.¹²

There are two important points to observe here. The first is that there can be no dispute that the *logical* relationship between Carter’s two

¹¹ The assumption that “refrains from preventing” means the same as ‘not preventing’ is crucial, otherwise the fulfillment of Carter’s condition for specific freedom is not straightforward. In a personal communication, Carter indicated that “not preventing,” is what he had in mind because he was not assuming anything about the opportunities or potential to prevent, i.e. it is not to be thought that this definition is referring to some action or strategy called ‘refraining from prevention’. Note, therefore, that in the event of b and c not opposing a – i.e. the coalition $\{b, c\}$ is in favor – performing φ , this fulfils Carter’s condition for the ascription of a specific freedom, even though not all other agents perform an action that does not interfere: a is free to φ because by performing the requisite actions b and c are ‘not preventing’ and nor is d , who, despite being opposed to a performing φ , cannot prevent it.

¹² Obviously if by “agent” Carter – or for that matter, any other theorist working within a similar framework – would include “collective agents” such as coalitions within the meaning of “agent,” then this criticism would not hold. But a close reading of Carter, and the work of others, suggests that by “agent” he, and others, mean a natural person.

definitions is correct: his use of the universal quantifier for freedom and existential quantifier for unfreedom assures this. What is disputed is the *acceptability* of *both* his definitions together because they fail to pick up cases where there is absence of “agental prevention.”

Secondly, there is an important conceptual issue that is thrown up. The source of the difficulty with the individual agent-based definition of freedom is that it is based on a very strong assumption about the nature of *power* relations that govern prevention: that they are individualistic. The assumption is that every social state can be forced by *some* (at least one) individual agent (either *i* can see to it that she performs φ or there is some j, k, \dots, n who can see to it that she does not perform φ). This is neither logically nor empirically true. To belabor the point, if “agental prevention” exists this is simply the special case of the singleton set. In the case of $\{b, c, d\}$ preventing *a* from φ -ing there is no such “agental prevention,” because neither $\{b\}$ nor $\{c\}$ nor $\{d\}$ can see to it that *a* is free to perform φ ; but there is if $\{c, d\}$ is the preventing coalition, because either $\{c\}$ or $\{d\}$ can see to it that she is free to do so. In a social context, then, power is a property to be ascribed to coalitions and not to individuals. Tempting as it may be, *i* is not to be confused with $\{i\}$ (Holler and Widgrén 1999). To confuse the two is to commit what is best called the “individualistic fallacy.” Once we drop the requirement of agental prevention a solution to the problem is obvious: preventing conditions must be formulated in terms of sets of agents (coalitions) because a “preventing set” can *always* be identified.

4. FORMAL FRAMEWORK

We can now formalize a generic definitional framework for ascribing a specific freedom or specific unfreedom:

Definition 4.1 (*Specific freedom*) “*i* is free to φ ”: if *i* were to attempt to φ , then no non-empty set of agents prevent *i* from φ -ing.

Definition 4.2 (*Specific unfreedom*) “*i* is unfree to φ ”: if *i* were to attempt to φ , then some non-empty set of agents prevent *i* from φ -ing.

There are three observations to be made here. Firstly, the reference to hypothetical actions in the form of what an agent attempts to do means that in an important sense the definitions concern an “impure” conception of negative freedom. In line with Gray (1980) and Flathman (198), what counts for freedom is not merely unimpeded or impeded movement or “behavior” but action.

The restriction to hypothetical actions instead of merely hypothetical behavior (or “movement”) is not innocent. In its absence, the definition of a specific freedom would merely say that *i* is free to φ “if no non-empty set of agents prevent *i* from φ -ing.” Without any further specification of

the meaning of φ -ing,¹³ an implication of the simpler formulation is that it ascribes i the freedom to φ even if i were compelled by the physical *force* (not simply the coercion or threats) of others to do so (which i could not resist). In the now proverbial case of a 's freedom to φ , this would be if $\{b, c, d\}$ or any of its two player subsets could not only see to it that a performs φ if a were to attempt to do so, but also see to it that a do so even if a were not to make such an attempt. The "pathology" – if one can call it that – of the simpler formulation is that it ascribes a the freedom to φ even if $\{a\}$ cannot prevent the outcome in which a performs φ . More concretely, the simpler formulation counts as an instance of my freedom to smoke, the case where I do not attempt to smoke but two of my friends hold me down, put a cigarette in my mouth, hold my mouth closed, and block my nose so that by mere reflex I inhale the smoke as I breath. In contrast, Definitions 4.1 and 4.2 require that I would perform some action that could be denoted as "attempting to smoke" before it can be said that I am free or unfree to smoke. In the case of my friends holding me down and blocking my nose, etc. while I am clearly "smoking," I am not really "doing" anything. It is a mere undertaking.

The definitions do not, however, exclude as an instance of my freedom to smoke a scenario where I attempt to smoke and then my friends force me to do so in the above manner the moment I reach for the cigarette packet, because what counts is that my attempt goes unprevented. My friends have merely made me unfree to *not* smoke. Technically, the restriction to hypothetical actions says that the potential coalitions that we inspect to determine a 's freedom *must* include a , i.e. $\{a, b, c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a\}$. This is the basis of the slogan introduced at the beginning: "freedom is membership of powerful coalitions" – because in the game theoretic jargon we will use later, the coalitions that are necessary and sufficient for an agent to perform a particular action are denoted as "powerful."

The second observation is that Definitions 4.1 and 4.2 do not require that the freedom to φ logically entails the freedom to not- φ . Although linguistic intuitions might suggest this tack, such a requirement is too strong as there are many things that I am free to do but unfree to not do. My attempt to smoke may go unprevented as my friends stand behind me making sure that I smoke (they do not physically force me to do so, however) and therefore I am free to do so; but at the same time I may be unfree to not smoke (they would hold me down, block my nose, etc. if I attempted not to smoke).

¹³ One could of course say that this is just an imprecise rendition of Definition 4.1, on the grounds that the element of action is contained in the meaning of φ -ing. It is true that this could be stipulated, but for reasons that will become clear in section 5, Definition 4.1 makes "action" more explicit. In section 5, there is a separation between strategies (attempting to φ) and outcomes (φ -ing).

Thirdly, the definitions make no direct reference to i 's power to φ or the power of j, k, \dots, n with respect to i φ -ing. All that is required under this conception of freedom is that potential constraints are inoperative; not that they do not exist. As above, a is free to φ in $\{a, b, c, d\}$ although any two member subset of $\{b, c, d\}$ could, but does not, prevent a from φ -ing. That is, the definitional framework ascribes i the freedom to φ even if constraints *could* have been operative had i attempted to φ , but *would not have been*, because the set of agents that *could* have made the constraints operative would not have done so (because, for example, the members of such a set had no common intention to do so). An implication of this account of freedom is that we ascribe i the freedom to φ even if it is at the grace and favour of some set of agents who could, at will and with impunity, make i unfree to φ . Freedom here is not absence of potential "domination."

Although this hiatus between power and freedom would be troublesome for some theorists such as Pettit (1996, 1997), it is not really something that we need to worry about here given that we have assumed a negative conception of freedom. Isaiah Berlin (1969: 130) famously remarked, "Liberty is principally concerned with the area of control, not with its source"; and, he continued, "The answer to the question 'Who governs me?' is logically distinct from the question 'How far does government interfere with me?'."

5. A GAME THEORETIC MEASURE

5.1 Types and tokens

Having specified (i) a conception of specific freedom (unfreedom) and (ii) the conditions for ascribing a specific freedom (unfreedom), we can now turn our attention to the problem of constructing a function that describes i 's expectation that she is free (unfree) to φ , that is, a measure for making judgments about the degree to which an individual can enjoy the freedom to φ . Following Dowding and van Hees' (2003) proposal, we want this expectation to reflect the different instantiations r_1, \dots, r_n , called *act-tokens*, of performing a particular type of action R , called an *act-type*, given by:

$$(5.1) \Gamma_i(R) = \Gamma(p(r_1), \dots, p(r_n))$$

where $p(r_i)$ is the probability that an act-token r_i will not be prevented (the agent is free to perform an instance of the act-type or right, R).¹⁴ The basic idea is that while the formal existence of a class of acts (an *act-type*),

¹⁴ Note that most of the recent philosophical literature on the measurement of freedom discusses types and tokens in some detail. See Steiner (1994), Carter (1999), van Hees (2000), and Kramer (2003b).

R , is given by the possibility that at least one of its instantiations, the *tokens* r_1, \dots, r_n , are *possible*, we want to determine how *probable* each of these tokens or instantiations are and from this derive a probabilistic judgment about the extent to which R can be said to *materially* (or *effectively*) and not just *formally* exist given the probability of each of these instantiations.

In the language that I have been using, an action φ can be taken as either an act-type, R , or an act-token, r_i , because a “specific freedom” can be more or less “specific” (Carter 1999; Steiner 1994; van Hees 2000). To use the example of “freedom of expression” again, this is an act-type R that can be instantiated in the different ways we have said: r_1 is shouting “Down with the government” at Whitehall at a particular time and date, r_2 is doing so at Piccadilly Circus, and so on. Each of these tokens can be specified further as act-types themselves: shouting “Down with the government” at Whitehall alone or doing so with others, etc. An action to which there is a unique corresponding particular event is an act-token; it is an action in which all spatiotemporal and physical components are specified. Thus in the example of a performing φ , each of the coalitions $\{a, b, c, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$ are the instantiations (tokens) r_1, \dots, r_n of a performing the act-type φ . Hence, given our definitional framework, we arrive at the central idea of this paper: the natural way to define $\Gamma(\cdot)$ is on the domain of possible coalitions.

5.2 Game forms

To define $\Gamma(\cdot)$ on the domain of coalitions in a systematic manner, we have to skip through some game theoretic preliminaries. The basic concept that we need is that of a *game form* (all of which has been implicit in our example of a 's freedom to φ). A game form is a specification of a finite set of outcomes X , a finite set of individuals (or players) $N = \{1, \dots, n\}$, a finite set of feasible actions or strategies A_i for each $i \in N$,¹⁵ and an outcome function π (or decision rule) that yields some single outcome $x \in X$ for any given n -tuple $[a_i]$ of strategies, one strategy $a_i \in A_i$ for each i , i.e. $g = (N, \{A_i\}_{i \in N}, \pi)$. A game form can be said, therefore, to specify the “rules of the game.”

For our purposes, we are interested in a particular game form in which the outcome set, X , has two elements, either i can perform φ (φ_i) or cannot perform φ ($\neg\varphi_i$), i.e. $X = \{\varphi_i, \neg\varphi_i\}$ and in which each player (including i) has two possible strategies: to either “agree” (do not act in a manner inimical to i performing φ) that i should be free to φ or not (attempt to “resist”), which we designate as $A_i = \{yes, no\}$. To be clear, by “strategy”

¹⁵ If we would be interested in freedom under legal rules, then the set of feasible strategies should be restricted to those that are admissible. See Fleurbaey and Gaertner (1996) and Fleurbaey and van Hees (2000).

is not necessarily meant a particular action as such, but rather a “bundle of actions”; they should be seen as courses of action. Depending upon the context of the specific freedom, the act of agreeing to or hindering i 's freedom to φ may involve different things. It could be as minor as a nod or a wink or providing a signature; or it could involve moving a heavy object; or it could even be an “omission” in the sense of not doing something that is required, either consciously or unconsciously. In any case, what are involved are many actions (to provide a signature I must pick up a pen, put the pen to paper, hand over the signed form, etc.), each of which I must be free to perform.¹⁶ Note, then, that under this construction a specific freedom or unfreedom presupposes other prior specific freedoms and that the specific freedom or unfreedom in question is the outcome of a combination of such bundles of actions as determined by a “decision rule,” π .

Now, in accord with our definitional framework, π defines the subsets of agents, $S \subseteq N$, called coalitions, that can force an outcome in X . That is, we are looking at a game form with a very sharp distribution of power: a coalition S , which is a collection of members of N who have made the same strategy choice, has either *full power* (is “winning”) or *zero power* (is “losing”). Thus, as before, a has the support of b and c in $\{a, b, c\}$ and this coalition has the power to see to it that a can φ , while its complement, $\{d\}$, is powerless (cannot prevent a from φ -ing); while a only has the support of b in $\{a, b\}$, which because it is not enough is therefore powerless to see to it that a can φ , while its complement $\{c, d\}$ is powerful (can prevent a from φ -ing).

Such a game form is also called a *simple game* and can be represented by a non-empty set $W \subseteq 2^N$ consisting of the winning (“powerful”) coalitions. We assume, as is usual, that W satisfies three basic conditions: (i) $\emptyset \notin W$, otherwise all coalitions would be winning and no player could prevent anything; (ii) $N \in W$, i.e. the grand coalition is powerful; and (iii) if $S \in W$ and $S \subseteq T$, then $T \in W$, i.e. if a coalition is winning then additional support will not alter the outcome. Note that the first condition, $\emptyset \notin W$, guarantees the freedom game to be non-trivial because if the empty set is winning then every set would be winning because every subset includes the empty set, which would imply that both φ_i and $\neg\varphi_i$ would be the outcome. Note also that the non-emptiness of W implies that the specific freedom formally exists because it means that at least one act-token, given by a coalition, is possible.

¹⁶ This point is discussed in detail in Braham and Holler (2005a, 2005b): an element of A_i by definition presupposes that a player is free to perform that strategy; otherwise it would not be in A_i and not part of the game form.

5.3 The conditional probability of success

Once we have identified the set of winning coalitions for a given specific freedom φ , it can be said that we have a freedom game form, $W(\varphi)$. We need make no other assumption as regards the decision rule π (in particular, we need not specify which or how many players constitute a winning coalition); it can simply be assumed to be “natural” in the sense that no social law or convention need be contained in it.¹⁷

With this apparatus at hand, Definitions 4.1 and 4.2 reincarnate as:

Definition 5.1 (*Specific freedom*) “ i is free to φ ” if, for some $S, i \in S \in W(\varphi)$ ($N \setminus S$ does not prevent i from performing φ , i.e. it is not a blocking coalition).

Definition 5.2 (*Specific unfreedom*) “ i is unfree to φ ” if, for some $S, i \in S \notin W(\varphi)$ ($N \setminus S$ prevent i from performing φ , i.e. it is a blocking coalition).

Note: (i) In accord with definitions 4.1 and 4.2, $i \in S \in W(\varphi)$ implies that no set of agent prevents i from φ -ing because $S \setminus \{i\}$ is not preventing and neither is $N \setminus S$ because it is powerless; and if $i \in S \notin W(\varphi)$ then at least one set of agents is preventing because $N \setminus S$ has the power to do so. (ii) The requirement that we restrict our attention to attempted actions in the definition of specific freedom and unfreedom is captured by the conditions $i \in S \in W(\varphi)$ and $i \in S \notin W(\varphi)$. The two conditions also capture the subjunctive element in Definitions 4.1 and 4.2 because i 's necessarily being a member of a winning coalition guarantees that i 's φ -ing is preceded by i 's attempting to φ . Without these two restrictions we would have the much weaker $S \in W(\varphi)$ and $S \notin W(\varphi)$ respectively and would yield a different measure. (iii) To avoid indeterminacy of the freedom ascription it is assumed that (a) S and $N \setminus S$ form and (b) if $S \in W(\varphi)$ then i is free to φ , i.e. for each $S \in W(\varphi)$ we exclude the possibility that any decisive subset of S will not in fact permit i to φ ;¹⁸ if this would be the case then these members by definition belong to $N \setminus S$ (it is also assumed that those members of N not in S are in $N \setminus S$). (iv) As discussed above, there is no reference to i 's power; this would require a further restriction such that $i \in S \in W$ but $S \setminus \{i\} \notin W$.

Thus to speak of i 's freedom to φ in a freedom game form $W(\varphi)$ is to speak of membership of a powerful coalition. Following the idea that $\Gamma(\cdot)$

¹⁷ Note: $W(\phi)$ can be a weighted game, i.e. a game in which there are non-negative weights (w_1, \dots, w_n) attached to the players and a quota $0 < q \leq \sum_{i \in N} w_i$ such that $S \in W$ iff $\sum_{i \in N} w_i > q$. The weights can be taken to represent resources such as money, social status, or authority; or even as a way of operationalizing “capability” and “material wherewithal” views of freedom associated with the work of Amartya Sen and Philippe van Parijs. Obviously this is a subject of future research.

¹⁸ This includes the case of individuals being decisive, i.e. those who could make S losing by unilaterally changing their decision, $j \in S \in W$ but $S \setminus \{j\} \notin W$.

is to be an aggregation of the probabilities of act-tokens r_i of an act-type R as in (5.1), then $\Gamma(\cdot)$ is precisely the probability of such an $i \in S \in W(\varphi)$.¹⁹

To define $\Gamma(\cdot)$ in a more precise fashion we need some additional structure and notation because calculating the probability of an $i \in S \in W(\varphi)$ requires a probability model for S . This means incorporating a minimal, but necessary, amount of behavioral information. That is, for any coalition S that may arise we may either know, be able to estimate, or be able to make a reasonable *a priori* judgment as to the probability $p(S)$ that the players in N will choose an element of their strategy set such that S occurs.²⁰ In other words, $\Gamma(\cdot)$ is made up of two components, the 2^N elementary events denoted by each $S \subseteq N$ and a probability distribution $p : 2^N \rightarrow \mathbb{R}$ that associates each S with its probability of occurrence $p(S)$. That is, $p(S)$ gives the probability that players in S consent to i performing φ (by choosing “yes” from their strategy set A_i) and those in $N \setminus S$ will not (by choosing “no” from their strategy set A_i). (As is usual, $0 \leq p(S) \leq 1$ for any $S \subseteq N$, and $\sum_{S \subseteq N} p(S) = 1$.) Our freedom function $\Gamma(R)$ is, then, specified by the pair $(W(\varphi), p)$.²¹

With this basic set-up, the natural specification of $\Gamma(\cdot)$ is given by a conditional probability that describes the *expectation* that i can perform φ , based on all hypothetical instances of φ -ing. For a given $W(\varphi)$ and p :

$$(5.2) \quad \Gamma_i(W(\varphi), p) =_{def} \text{Prob}\{\text{outcome is } \varphi_i \mid i \text{ chooses } \varphi_i\} = \frac{\sum_{S: i \in S \in W(\varphi)} p(S)}{\sum_{S: i \in S} p(S)}$$

To summarize, a measure of the expectation that an agent has a specific freedom is none other than a conditional variant of the notion of “success” used in the voting power literature that was independently introduced by Penrose (1946) and Rae (1969) and more fully discussed by Barry (1980a, 1980b).

¹⁹ Note that it may be more reasonable to restrict S to the set of *minimal winning coalitions* (MWC), W^m , where $S \in W^m$ if $S \in W$, but $T \notin S \in W$. It is questionable if coalitions that are in excess of their minimal size add to the freedom of i to perform φ . In our example, if $\{a, b, c\}$ is sufficient for a to φ , in what way does $\{a, b, c, d\}$ contribute to a 's freedom? This is a question that can not be answered here. Is this another instantiation of φ -ing or is merely double counting what $\{a, b, c\}$ can achieve? This does not, however, substantially affect the form of the measure that is derived.

²⁰ The probability $p(S)$ does not, however, necessarily contain information about intentions or preferences. The precise meaning of $p(S)$ is an open question. On the one hand, it can be taken to reflect preferences; on the other, it can be taken to reflect social structure and conventions.

²¹ The notation is borrowed from Laruelle and Valenciano (2004).

6. CONCLUDING REMARKS

Although the formalization of a measure of specific freedom as $\Gamma_i(W(\varphi), p)$ has, except in very special circumstances, limited practical applicability,²² it does have a fair amount of conceptual and programmatic value.

For a start, $\Gamma_i(W(\varphi), p)$ is a partial and tentative answer to an open question in the literature. This is an important question because if we are serious about making comparative judgments of the extent to which a person will be in a position to perform some freedom or have a freedom respected, rather than just saying that a person is free or unfree merely on the grounds that an instance of that freedom (act-token) exists or does not exist, then we need a measure that permits us to distinguish between different grades of freedom.

Next, the fine-grained process of constructing $\Gamma_i(W(\varphi), p)$ from a conception and syntax of specific freedom has churned up a significant conceptual finding: an individualistic definition of specific freedom taken in a broadly negative sense appears to be logically unsustainable. An agent derives her freedom from membership of collectivities (coalitions); or following Dowding and van Hees' analysis (2003), coalitions are a necessary component in the construction of freedom and rights. This clearly has conceptual implications that extend well beyond the narrow scope of this paper. One such implication is that my material freedom to perform some action is increased by the creation of institutions that "simulate" the coalitions necessary for me to perform that action; or by giving me additional means that reduce my reliance on others and therefore increase the number of possible winning coalitions of which I am a member. If we have a concern for expanding a person's specific freedom, we must look at the game form (the natural or formal procedural rules) that governs this activity.

Finally, the game theoretic formalization allows us to characterize different conceptions of freedom in a single framework. Consider, for instance, Pettit's (1997) "republican" conception of freedom that was briefly touched upon above.²³ As stated, this account of freedom is based on a criterion of "non-domination" or what Pettit (1996) also calls "antipower." In precise terms, Pettit would ascribe *i* the freedom to φ if, and only if, *i* cannot be prevented from φ -ing by *any* set of agents, viz. is immune from *any* interference. This idea says that a person can only be ascribed a freedom if they are not only free in the *actual* world where there

²² Bureaucracies are an obvious case for such an area of application. Here we generally find clear permission structures and decision rules. $\Gamma_i(W(\phi), p)$ could also be used to give conceptual and empirical content to the management science literature on empowerment, which is often taken as meaning the "freedom to do something" in an organization. See, for example, Conger (1988), Gal-Or and Raphael (1998), Spreitzer (1995, 1996), Pfeffer (1992).

²³ This is also true for Sen's (1970) conception of minimal liberty. See List (2004).

is no *effective* resistance (where no set of agents *can* prevent *i* from φ -ing), but also *no* such resistance in *all nearby possible* worlds (List 2004). In the model introduced here, the set of possible worlds is the set of act-tokens which is denoted by the set of possible coalitions. Non-domination merely says that the set of preventing coalitions must be empty, which means that none of the act-tokens can be denied to the agent. It may be trivial, but this is just another way of saying that with respect to φ -ing, *i* must be *dictator*, i.e. $\{i\}$ is a winning coalition and any coalition without *i* is losing. To summarize, we can discriminate accounts of freedom on the domain of coalitions because the key properties of power and robustness will single out which coalitions are considered as providing us with the freedom to do something. The measure we have defined informs us of the likelihood of these coalitions.

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