JOINT OPTIMIZATION OF TRANSITION RULES AND THE PREMIUM SCALE IN A BONUS-MALUS SYSTEM

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ABSTRACT

Bonus-malus systems (BMSs) are widely used in actuarial sciences. These systems are applied by insurance companies to distinguish the policyholders by their risks. The most known application of BMS is in automobile third-party liability insurance. In BMS, there are several classes, and the premium of a policyholder depends on the class he/she is assigned to. The classification of policyholders over the periods of the insurance depends on the transition rules. In general, optimization of these systems involves the calculation of an appropriate premium scale considering the number of classes and transition rules as external parameters. Usually, the stationary distribution is used in the optimization process. In this article, we present a mixed integer linear programming (MILP) formulation for determining the premium scale and the transition rules. We present two versions of the model, one with the calculation of stationary probabilities and another with the consideration of multiple periods of the insurance. Furthermore, numerical examples will also be given to demonstrate that the MILP technique is suitable for handling existing BMSs.

KEYWORDS

Adverse selection, bonus-malus system, integer programming.

JEL codes: C44; C61; D82; D86; G22

1. INTRODUCTION

Bonus-malus system (BMS) is a risk managing method whose best known application is in automobile third-party liability insurance. It will be assumed that there are some unobservable parameters which influence each policyholder's personal risk. The estimation of these parameters is difficult with

Astin Bulletin 50(3), 743-776. doi:10.1017/asb.2020.27

statistical methods, though with multi-period contracts the insurance company can estimate the overall risk more accurate for each policyholder.

In other words, policyholders can be categorized by their risks, although the insurance company is unable to determine precisely with observable parameters (such as age, sex, education, etc.) which risk-group a particular policyholder belongs to. These risk-groups are usually called 'types' in the literature. Therefore, this classification is prone to error, that is, there is an underlying unobservable parameter that explains the risks of the policyholders. On the other hand, policyholders know their true types, a typical situation of asymmetric information. Asymmetric information causes welfare loss whose magnitude can be reduced, for example, by the application of a BMS.

In a BMS, there are finitely many classes, each having a different premium. At the start of the contract, each policyholder is assigned to the 'initial class.' Subsequently, if the policyholder has a claim in the following period, then he/she moves to a worse class, so the payment of the policyholder may increase in the following period. If he/she does not have a claim in a particular period, then he/she moves to a better class; therefore, his/her payment may become less in the subsequent period.

Without asymmetric information, each policyholders' premium (for each risk type) would be equal to the expected claim (in each class). Hence, the problem is to set the premiums to approximate the 'ideal situation' (i.e. the case without asymmetric information) as closely as possible (this is not the same as to adjusting expected premium levels to expected claims). A perfect match is impossible in real situations. Thus, a natural goal is to minimize the difference from the 'ideal' solution. To achieve this, we strive at setting a 'good' premium scale and 'good' transition rules. The first possibility is widely studied in the actuarial literature; however, there is less emphasis on the second one. In this article, we use mixed integer linear programming (MILP) to jointly optimize the premium scale and the transition rules in a BMS.

In Section 2, we overview the relevant literature on BMS with particular emphasis on optimization models. In Section 3, we present an LP model to optimize the premiums and introduce a MILP model for the optimization of the transition rules. We also construct another model for the joint optimization of the transition rules and premiums. In Section 4, the previously introduced models are modified: instead of the stationary distribution multi-period optimization is applied. In Section 5, results of some numerical experiments are discussed. In Section 6, we discuss computational issues related to the MILP model.

2. Literature overview

In the context of insurance mathematics, asymmetric information was first studied by Rothschild and Stiglitz (1976). They found that adverse selection causes social welfare loss and equilibrium of the market does not always exist. Social welfare loss caused by adverse selection can be reduced. Cooper and

Hayes (1987) investigated multi-period contracts and found that if there is a contract where the premiums (and indemnities) depend on claim history of the previous year(s), social welfare loss can considerably be reduced.

In general, in real-life situations, adverse selection is accompanied by moral hazard. In the models of moral hazard, the probability of claims also depends on the effort of the policyholder. The insurance company, however, does not know the exact effort the policyholder makes to reduce the risk of the damage. The insurer can merely estimate it from the claim amount (Shavell, 1979). With the use of a BMS, the insurer motivates the policyholders to reduce risk. This is so because if someone has a claim, then the following period he/she will be assigned to a worse class (if there is any). Hence, his/her premium will increase.

The typical appearance of moral hazard in the automobile third-party liability insurances is the so-called 'bonus-hunger' meaning that policyholders choose self-financing the damage rather than making a claim to the insurance company. If the claim amount is lower than the premium increase of the following periods, then it is not worth reporting a claim to the insurer (De Prill, 1979; Sundt, 1989).

Adverse selection and moral hazard are usually present at the same time (as in the case of BMS). Contract theory investigates these kinds of problems (such as Bolton and Dewatripont, 2005). In the literature, there is less emphasis on the empirical tests of adverse selection and moral hazard. The empirical findings are not completely straightforward, see, for example, Dahlby (1983) and Puelz and Snow (1994) who argue for the existence of adverse selection, while Chiappori and Salanié (2000) are against it. The existence of moral hazard in automobile third-party liability insurance has been empirically studied by Lee and Kim (2016), Dionne *et al.* (2013), Vukina and Nestić (2015), and Abbring *et al.* (2008).

The efficiency of a BMS is typically measured by an indicator called 'elasticity' introduced in Loimaranta (1972). Elasticity shows how expected payment will increase if risk increases by 1%. Elasticity is a helpful indicator for optimizing a system where it is required that higher-risk policyholders have a higher expected payment, although it does not reflect the risk aversion of the policyholders. In any BMS, this means that the variance of the premiums should not be too high.

There are a lot of articles where the expected premiums and expected claims are matched (Lemaire, 1995; Heras *et al.*, 2004; Denuit *et al.*, 2007). However, the aspect that the policyholder may be satisfied with a bit higher expected premium on the condition that the variation of premiums is decreased significantly is somewhat disregarded (or partially regarded). Nonetheless, in Loimaranta (1972) there is a model which minimizes the variance of the premium scale assuming a fixed level of elasticity.

The transition rules define the classification of a BMS. Transition rules tell us how many classes should the policyholders go down in the following period if they cause claims. Additionally, there should be a claim-free transition rule which sends the policyholder one or more classes up in the subsequent period.

Designing a BMS requires choosing the transition rules between the classes and determines the number of classes, the scale of premiums and the initial class.

There are many papers about the optimization of BMSs (e.g. Cooper and Hayes, 1987; Lemaire, 1995; Denuit and Dhaene, 2001; Brouhns *et al.*, 2003; Heras *et al.*, 2004; Mert and Saykan, 2005; Denuit *et al.*, 2007; Najafabadi and Sakizadeh, 2017). Typically, in these works, the number of classes, the transition rules and the initial class are fixed while the scale of premiums is determined in the optimization process. Recently, Tan *et al.* (2015) have incorporated in their model the effect of the transition rule changes. Gyetvai and Ágoston (2018) gave an optimization model for choosing the best transition rules, while the premium scale is fixed. In the present article, we extend this model to optimize the premium scale and transition rules jointly.

3. A MILP MODEL FOR OPTIMIZATION OF THE TRANSITION RULES

In this section, we construct a MILP model for joint optimization of the premium scale and transition rules. In the literature of BMSs, using LP (or MILP) technique is not so common. Only one known LP model exists, Heras *et al.* (2004). We adopted the assumptions made in that article.

3.1. Preliminaries

We assume that there are I different risk-groups (types) among the policyholders. Each type has different risks that do not change over time. In the practice of BMS, transition rules are based only on claim numbers, and the claim amount is ignored. This is reasonable since the risk-groups can be distinguished more accurately by the number of claims than by the (conditional) claim amount. We use the same assumption in the MILP model. Therefore, we only consider the number of claims. For the sake of simplicity, we assume that the claim amount is the same for each type of policyholders (in every model, we assume it is one for each risk-group).

Let $M > 0$ be the highest number of possible claims in a period and let λ^i_m be the probability of the occurrence of m claims for the policyholders of type i (for estimation of claim numbers, see, for instant, Arató and Martinek, 2014) ($i = 1, \ldots, I$, $\sum_{m=0}^{M} \lambda^i_m = 1$). We denote the risk-parameters (expected claim amount) for risk-group i with λ^i ($\lambda^i = \sum_{m=0}^{M} m\lambda^i_m$). To keep notation simple, the types are indexed in an increasing risk order; the expected claim amount is the least for type 1 and the highest for type I. Let ϕ^i be the proportion of the type i policyholders among all of the policyholders ($\sum_{i=1}^{I} \phi^i = 1$). In BMS, there are $K + 1$ classes indexed from 0 to K. The premium of class k is denoted by π_k. In a BMS, the premiums should be monotonic; hence, we assume that $\pi_{k-1} \geq \pi_k$ ($k = 1, \ldots, K$).

The transition rules give the way the policyholders will be reclassified after a certain number of claims occur. Hence, there is a transition rule for all m, and we can write the rules in matrices T^m. Each T^m is a binary matrix that means if the element of row k_1 and column k_2 equals to one ($T^m_{k_1,k_2} = 1$), then a policyholder with m claims and currently in the class k_1 will be reclassified into the class k_2 in the next period. Let X^i_t be the classification of a type i policyholder in period t.

Definition 1. *A discrete-time stochastic process is called a Markov chain if*

$$\begin{aligned} &\mathbb{P}(X_{t+1} = k_{t+1} | X_t = k_t, X_{t-1} = k_{t-1}, \ldots, X_1 = k_1, X_0 = k_0) \\ &\quad = \mathbb{P}(X_{t+1} = k_{t+1} | X_t = k_t) \end{aligned} \tag{3.1}$$

holds for every period t.

Since transition rules are based only on the previous period's claim history and classification, condition (3.1) holds for the process of classification X^i_t. The transition probability matrix (P^i) contains the probabilities of transitions between periods for type i policyholders:

$$P^i_{k_1,k_2} = \mathbb{P}(X^i_{t+1} = k_2 | X^i_t = k_1) = \sum_{m=0}^{M} \lambda^i_m T^m_{k_1,k_2}. \tag{3.2}$$

Let $c^i_{k,t}$ be the probability that the policyholder in period t is classified into class k. In other words, the process X^i_t takes value k, that is, $\mathbb{P}(X^i_t = k) = c^i_{k,t}$. We denote C^i_t the vector form of the $c^i_{k,t}$ variables. For each period for the probabilities $c^i_{k,t}$, the following equation should hold (Lemaire, 1995):

$${C^i_t}^\top = {C^i_0}^\top (P^i)^t, \tag{3.3}$$

where $\top$ stands for transposition, and in the expression $(P^i)^t$ the t is an exponent denoting the time period.

If a Markov chain is aperiodic and irreducible, then probabilities $c^i_{k,t}$ tend to a unique stationary distribution c^i_k. This distribution can be calculated by solving the following system of equations:

$$c^i_k = \sum_{j=0}^{K} c^i_j \mathbb{P}(X^i_{t+1} = k | X^i_t = j) \quad k = 0, \ldots, K \tag{3.4}$$

$$\sum_{k=0}^{K} c^i_k = 1. \tag{3.5}$$

Substituting (3.2) into (3.4), the first part of the system of equations can be written as:

$$c^i_k = \sum_{j=0}^{K} \sum_{m=0}^{M} c^i_j \lambda^i_m T^m_{j,k} \quad k = 0, \ldots, K. \tag{3.6}$$

Definition 2. *A Markov chain is said to be irreducible, if for every class k, j and period t there exists a period s, such that*

$$\mathbb{P}(X^i_{t+s} = k | X^i_t = j) > 0. \tag{3.7}$$

Not every transition rule results in an irreducible Markov chain. In the optimization of the transition rules, we can ensure irreducibility with constraints that exclude the non-irreducible transition rules. If the policyholder is in the highest class, then it is not possible to classify him/her into a higher class. In addition, from the lowest class the policyholders cannot be classified lower. Therefore, the finite number of classes and the considered transition rules ensure the aperiodicity of the irreducible Markov chain.

Optimizing a BMS means looking for an appropriate premium scale and transition rules that minimize the difference between the expected claim amount and the premiums in some norm:

$$\min_{(\pi_0,\pi_1,\ldots,\pi_K,T^0_{0,0},T^1_{0,0},\ldots,T^m_{k_1,k_2},\ldots,T^M_{K,K})} \sum_{k=0}^{K} \sum_{i=1}^{I} \phi^i c^i_k (T^0_{0,k}, T^1_{0,k}, \ldots, T^M_{K,k}, \lambda^i_0, \lambda^i_1, \ldots, \lambda^i_M) d(\pi_k, \lambda^i)$$

subject to

constraints on the decision variables,

where $d(.,.)$ is usually the ℓ^2 or ℓ^1 norm (see, Norberg, 1976; Heras *et al.*, 2002). Most used constraints are the profitability constraint and constraints on the premium scale: for example, the difference between the premiums of two consecutive classes cannot be more than 20%. Certainly, besides these most used constraints we can give other constraints for special purposes. The above minimization problem is nonlinear, in Sections 3.2–3.4, we describe how the problem can be linearized. Optimization of a BMS is usually based on stationary probabilities (see, e.g., Lemaire, 1995; Heras *et al.*, 2002). Initially, we accept this assumption and construct our MILP model accordingly.

In practice, however, every policyholder starts the contract in the initial class. Reaching the stationary distribution would require several periods. Therefore, the early probabilities $c^i_{k,t}$ differ significantly from the stationary probabilities. For this reason, we construct an alternative MILP model as well where the stochastic process is modeled only for the first fixed number of periods.

3.2. Optimizing the premium scale when the transition rules are fixed

Optimizing the premium scale means that we seek the appropriate premiums for a BMS with fixed transition rules. Since transition rules are external parameters, the stationary probabilities are parameters as well. The premiums can be obtained by the following LP:

$$\min \sum_{i=1}^{I} \sum_{k=0}^{K} \phi^i g_k^i \tag{LP1.obj}$$

Subject to

$$\pi_k c_k^i + g_k^i \geq \lambda^i c_k^i \qquad \forall i, k \tag{LP1.1}$$
$$\pi_k c_k^i - g_k^i \leq \lambda^i c_k^i \qquad \forall i, k \tag{LP1.2}$$
$$\pi_{k-1} \geq \pi_k \qquad k = 1, \ldots, K \tag{LP1.3}$$
$$\pi_k \geq 0 \qquad \forall k$$
$$g_k^i \geq 0 \geq 0 \qquad \forall k, i$$

Because the transition rules are fixed, the stationary probabilities can be calculated. Thus, for each class (k) and group (i), the c_k^i are known parameters. Besides, the expected number of claims of each group (λ^i) and their ratios (ϕ^i) are also outer parameters.

The variables of the model are the premiums of the classes (π_k). The objective is to find a premium scale where each groups' expected payment is as close to their expected number of claims as possible. In this LP model, we minimize the absolute deviation. Hence, we introduce g_k^i auxiliary decision variables that denote the absolute deviations of the group i in class k. Constraints (LP1.1) and (LP1.2) define the deviation of group i and class k. Constraints (LP1.3) set the premium scale to be monotonic. In the objective function of the model, the absolute deviation variables (g_k^i) are weighted by the ratio of the groups (ϕ^i).

Moreover, it is worth to remark that an approximation of a quadratic (and many other) loss functions can be used in an LP model, instead of the absolute deviation.

This kind of LP problem first appeared in Heras *et al.* (2002), but the above LP is different. In the original model, the difference between the *expected* premium and expected claim is minimized, so the constraints (LP1.1) and (LP1.2) looked as:

$$\sum_{k=0}^{K} \pi_k c_k^i + g_k \geq \lambda^i \qquad \forall i \tag{3.8}$$

and

$$\sum_{k=0}^{K} \pi_k c_k^i - g_k \leq \lambda^i \qquad \forall i \tag{3.9}$$

and the objective function was $\sum_{i=1}^{I} \phi^i g_k$.

If the number of types is less than the number of BMS classes, the optimal objective function value will be 0 in this model. In other words, the expected premium equals the expected claim for all risk-groups. Merely considering the overall expected deviation would result in high dispersion among the premiums of the classes. In some numerical experiments, we encountered cases where the highest premium was more than 1 million times higher than the lowest one. Such a substantial difference between premiums is undoubtedly not adequate in an insurance contract because it would not reduce the risk for the policyholder. The most standard way to handle this problem is the manual limitation of the dispersion of the premiums. We applied a different approach for managing the risk aversion of policyholders.

In our objective function (LP1.obj), we minimize the absolute deviations of each type's expected payment from the expected claims weighted with the proportion of the types (this expression appears in many studies Norberg, 1976; Tan *et al.*, 2015 with the difference that ℓ^1 norm is used instead of ℓ^2). The zero value for the objective function in this model would mean that each risk-group's premium is constant (i.e. does not change from class to class) and it is equal to the expected claim for each type. In real circumstances, this is definitely impossible. Heras *et al.* (2002) set other constraints to limit the Lomaintra-efficiency. These constraints can be inserted in our models, but we think that a smaller objective value would be preferable to the policyholder than a higher objective value with a better efficiency measure.

Theorem 1. *There is an optimal solution of LP1, where for all k there is a risk-group i, where* $\pi_k = \lambda^i$.

Proof. Assume on the contrary that there is a class k where the premium differs from each type's expected claim ($\pi'_k \neq \lambda^i, \forall i$).

Set $\mathcal{K}_0$ contains classes where premium equals to π'_k ($\mathcal{K}_0 := \{k|\pi_k = \pi'_k\}$). Furthermore, set $\mathcal{I}_p$ and $\mathcal{I}_n$ contain risk-groups where the expected claims are greater/less than π'_k:

$$\mathcal{I}_p := \{i|\lambda^i > \pi'_k\}; \qquad \mathcal{I}_n := \{i|\lambda^i < \pi'_k\}.$$

If we start to increase π'_k with a value ε, then the objective of the model will change with εg where

$$g = \left(\sum_{i \in \mathcal{I}_n} \sum_{k \in \mathcal{K}_0} \phi^i c^i_k - \sum_{i \in \mathcal{I}_p} \sum_{k \in \mathcal{K}_0} \phi^i c^i_k \right).$$

If $\pi'_k < \lambda^1$, then g is negative which means that with the increase of the π'_k, the value of the objective function can be better. The situation is similar if $\pi'_k > \lambda^I$. In this case, g is positive, which means that decreasing π'_k leads to a smaller objective function value.

For the case when $\lambda^1 < \pi'_k < \lambda^I$ notice that the value of g depends on the values of $\mathcal{I}_n$ and $\mathcal{I}_p$. This means that if there is a premium $\pi'_k > \lambda^i$ and we increase this premium with ε, then g changes only if $\pi'_k + \varepsilon > \lambda^{i+1}$. Put differently, if g of the $\pi'_k + \varepsilon$ is zero, then λ^{i+1} should also be optimal.

Since this holds for each k, there should be an optimal premium scale where the premiums are all equal to a λ. □

Theorem 1 suggests a quite unusual solution. The remarks below are meant to explain the motivation.

- If there is only one risk-group, then each class premiums are equal ($\pi_k = \pi$, $\forall k$); this fact corresponds to the statement of theorem 1 for $I = 1$. In this context, it is just a generalization that when there are two types of policyholders, then there are two premium values. For further motivation, see Example 3.1.
- If we have only a few types of policyholders, in many classes the premium can be the same. At first sight, it seems that we can get the same results with a less spread BM system. However, the stationary probabilities of a larger BM system may differ from a smaller one's. For instance, in Example 3.1, a policyholder with 0.9 expected claim pays the high premium with a probability 0.99998 and the low premium with probability 0.00002. These probabilities cannot be reproduced in a two-class BM system.
- If there are many risk-groups, then every class may have separated premium value.
- If the designer of the BM system prefers to have distinct premium values in each class, she/he can easily prescribe it with additional constraints.

Example 3.1. *Let us consider a BM system with the most straightforward transition rule: in the event of any claims, the policyholder moves downward, otherwise upward a class. Given this transition rule, there is a relatively easy relation amongst the stationary probabilities:*

$$c_k^i = \left(\frac{1-\lambda^i}{\lambda^i}\right)^k c_0^i, \tag{3.10}$$

where the k outside the parenthesis is power and not an index. After applying the expression for the sum of this geometric sequence:

$$c_0^i = \frac{\frac{1-\lambda^i}{\lambda^i} - 1}{\left(\frac{1-\lambda^i}{\lambda^i}\right)^{K+1} - 1}. \tag{3.11}$$

Let $K = 2h + 1$, then:

$$\sum_{k=0}^{h} c_k^i = \frac{\frac{1-\lambda^i}{\lambda^i} - 1}{\left(\frac{1-\lambda^i}{\lambda^i}\right)^{2h+2} - 1} \frac{\left(\frac{1-\lambda^i}{\lambda^i}\right)^{h+1} - 1}{\frac{1-\lambda^i}{\lambda^i} - 1} = \frac{\left(\frac{1-\lambda^i}{\lambda^i}\right)^{h+1} - 1}{\left(\frac{1-\lambda^i}{\lambda^i}\right)^{2h+2} - 1}. \tag{3.12}$$

If $\lambda^i > 0.5$, then expression (3.11) tends to 1 (as h tends to infinity), otherwise it tends to 0. Hence, a policyholder with claim probability higher than 50% *will*

TABLE 1

STATIONARY PROBABILITIES OF A 10 CLASS BMS.

Class	$\lambda^i = 0.1$	$\lambda^i = 0.9$
9	0.8889	2.29×10^{-9}
8	0.0988	2.06×10^{-8}
7	0.0110	1.86×10^{-7}
6	0.0012	1.67×10^{-6}
5	0.0001	1.51×10^{-5}
4	1.51×10^{-5}	0.0001
3	1.67×10^{-6}	0.0012
2	1.86×10^{-7}	0.0110
1	2.07×10^{-8}	0.0988
0	2.29×10^{-9}	0.8889

be almost surely in the lower half of the BM system. Similarly, any policyholder with less than 50% *claim probability would be most likely in the upper half.*

Let us assume that there are two types of policyholders: $\lambda^1 < 0.5$ *and* $\lambda^2 > 0.5$. *The premium is* λ^2 *for classes* $0, \ldots, h$ *and* λ^1 *for the other classes. Asymptotically both types will pay the same amount as their risks. This statement holds for quadratic (and any other meaningful) loss function. If the premium increases gradually, we cannot get the same result. In certain cases, quite small h is enough to approximate the asymptotic result: let* $\lambda^1 = 0.1, \lambda_2 = 0.9$ *and* $h = 4$. *Table 1 shows both risk-groups' stationary probabilities. If the premium is* 0.1 *for classes 5–9 and* 0.9 *for classes 0–4, then both types of policyholders would pay their fair premium with* 0.99998 *probability.*

3.2.1. *Profit constraint.*

A crucial question is the financial balance of the BMS. In the long run, it is not worth to design an unprofitable BMS. In the model LP1, if the objective value is as close to zero, as it is financially balanced. Besides that, each model that operates with the Lomaintra-efficiency ensures some kind of balance.

In the relevant literature, there are studies where profitability is explicitly prescribed (Coene and Doray, 1996) and articles where it is not (Heras *et al.*, 2004; Tan *et al.*, 2015). With our notation, the profit constraint takes the form

$$\sum_{i=1}^{I} \left(\phi^i \sum_{k=0}^{K} \left(\pi_k c_k^i \right) \right) \geq \sum_{i=1}^{I} \phi^i \lambda^i. \tag{3.13}$$

The financial balance of the BMS is an important requirement; however, in this case, Theorem 1 does not hold any more.

Theorem 2. *There is an optimal solution of LP1 with constraint* (3.13)*, where there is only one type of premium that is unequal to any risk-group's expected claim.*

https://doi.org/10.1017/asb.2020.27 Published online by Cambridge University Press

Proof. By way of contradiction, let us assume that the optimal solution involves two premium values ($\pi_{k_1} < \pi_{k_2}$) that differ from any type's expected claim. Sets $\mathcal{K}_1$ and $\mathcal{K}_2$ contain classes where the premium equals to π_{k_1} and π_{k_2} ($\mathcal{K}_1 := \{\kappa | \pi_\kappa = \pi_{k_1}\}$, $\mathcal{K}_2 := \{\kappa | \pi_\kappa = \pi_{k_2}\}$). Furthermore, sets $\mathcal{I}_{p_1}$, $\mathcal{I}_{p_2}$ and $\mathcal{I}_{n_1}$, $\mathcal{I}_{n_2}$ are defined as

$$\mathcal{I}_{p_1} := \left\{i | \lambda^i > \pi_{k_1}\right\}; \qquad \mathcal{I}_{p_2} := \left\{i | \lambda^i > \pi_{k_2}\right\};$$
$$\mathcal{I}_{n_1} := \left\{i | \lambda^i < \pi_{k_1}\right\}; \qquad \mathcal{I}_{n_2} := \left\{i | \lambda^i < \pi_{k_2}\right\}.$$

If $\pi_{k_1} < \lambda^1$, then the premium in classes $k \in \mathcal{K}_1$ shall be increased by ε. This change does not violate the profit constraint (3.13) but reduces the objective function by $g\varepsilon$;

$$g = \sum_{i \in \mathcal{I}_{p_1}} \phi^i \sum_{k \in \mathcal{K}_1} c_k^i,$$

which means the premium scale cannot be optimal.

If $\lambda^1 < \pi_{k_1}$, then decreasing premiums in classes $k \in \mathcal{K}_1$ by ε produce premiums that are equal to the increase $\delta(\varepsilon)$ in classes $k \in \mathcal{K}_2$. To preserve the financial balance of the system, we must have

$$\delta(\varepsilon) = \varepsilon \frac{\sum_{i=1}^{I} \sum_{k \in \mathcal{K}_1} c_k^i}{\sum_{i=1}^{I} \sum_{k \in \mathcal{K}_2} c_k^i}.$$

Decreasing premium in classes $k \in \mathcal{K}_1$ by ε (and increasing it by $\delta(\varepsilon)$ in classes $k \in \mathcal{K}_2$) will change the objective function by $g\varepsilon$;

$$g = \sum_{i \in \mathcal{I}_{p_1}} \phi^i \sum_{k \in \mathcal{K}_1} c_k^i - \sum_{i \in \mathcal{I}_{n_1}} \phi^i \sum_{k \in \mathcal{K}_1} c_k^i + \frac{\sum_{i=1}^{I} \sum_{k \in \mathcal{K}_1} c_k^i}{\sum_{i=1}^{I} \sum_{k \in \mathcal{K}_2} c_k^i} \left(\sum_{i \in \mathcal{I}_{n_2}} \phi^i \sum_{k \in \mathcal{K}_2} c_k^i - \sum_{i \in \mathcal{I}_{p_2}} \phi^i \sum_{k \in \mathcal{K}_2} c_k^i \right).$$

If g is negative, then the increase of the premium in classes $k \in \mathcal{K}_1$ will result in a better value for the objective function; if it is positive, then the value of the objective function will be worse.

Let $\pi_{k_1} > \lambda^{i_1}$ for $k_1 \in \mathcal{K}_1$ and $\pi_{k_2} < \lambda^{i_2}$ for $k_2 \in \mathcal{K}_2$. If the premium decreases in classes $k_1 \in \mathcal{K}_1$, then the g changes only if $\pi_{k_1} - \varepsilon < \lambda^{i_1}$ for $k_1 \in \mathcal{K}_1$ or $\pi_{k_2} + \delta(\varepsilon) > \lambda^{i_2}$ for $k_2 \in \mathcal{K}_2$. This means that if $g = 0$, then at least one premium can be replaced with a λ which is the assertion of the theorem. □

3.3. Optimizing transition rules when the premium scale is fixed

Transition rules are typically defined by transition matrices as in Section 3.1. In order to build a MILP model, we introduce binary variables $T_{j,m,k}$ for each entry of the transition matrices. If $T_{j,m,k} = 1$, then the policyholders with m claims are moved from class k, j classes forward (backward if $j < 0$) in the following period. Denote the domain of j by $J_k = [\underline{J_k} : \overline{J_k}]$ for class k where $-k = \underline{J_k} < 0$ and $K - k = \overline{J_k} > 0$ are the two extremes. If a binary variable $T_{j,m,k} = 1$ and index j is positive, then the policyholders with m claims are put move upward in the system. Put differently, they move to a class with a lower premium if it is possible. In the case of $j < 0$, the policyholders move downward if they have m claims. Index j can be 0 as well, which means that they stay in the same class in the subsequent period.

The aim of the model is to find the best transition rule that separates the expected payment of the risk-groups most evenly, that is, we want to minimize the deviation of the payment and the number of the expected claims (in some norm) of each class.

First, we present a model for the optimization of the transition rules with a fixed premium scale. The stationary distribution depends on the transition rules, as we defined in Section 3.1. Therefore, the stationary probabilities are also variables. The model looks now

$$\min \sum_{i=1}^{I} \sum_{k=0}^{K} \phi^i g_k^i \qquad \text{(MILP1.obj)}$$

Subject to

$$\sum_{j=\underline{J_k}}^{\overline{J_k}} T_{j,m,k} = 1, \qquad \forall m, k \qquad \text{(MILP1.1)}$$

$$\sum_{j=\min(\overline{J_k},1)}^{\overline{J_k}} T_{j,0,k} = 1, \qquad \forall k \qquad \text{(MILP1.2)}$$

$$\sum_{j=\underline{J_k}}^{\max(\underline{J_k},-1)} T_{j,M,k} = 1, \qquad \forall k \qquad \text{(MILP1.3)}$$

$$\sum_{\ell=j}^{\overline{J_k}} T_{\ell,m,k} \geq T_{j,m+1,k} \qquad \forall j, k,\ m = 0, \ldots, M-1 \qquad \text{(MILP1.4)}$$

$$\sum_{k=0}^{K} c_k^i = 1 \qquad \forall i \qquad \text{(MILP1.5)}$$

$$d_{k,j,m}^i \geq \lambda_m^i c_k^i - (1 - T_{j,m,k}) \qquad \forall i, j, k, m \qquad \text{(MILP1.6)}$$

$$c_k^i = \sum_{j=-(K-k)}^{k} \sum_{m=0}^{M} d_{k-j,j,m}^i \qquad \forall i, k \qquad \text{(MILP1.7)}$$

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$$\sum_{i=1}^{I} c_k^i \geq \tau \qquad \forall k \tag{MILP1.8}$$

$$\pi_k c_k^i + g_k^i \geq \lambda^i c_k^i \qquad \forall i, k \tag{MILP1.9}$$

$$\pi_k c_k^i - g_k^i \leq \lambda^i c_k^i \qquad \forall i, k \tag{MILP1.10}$$

$$T_{j,m,k} \in (0, 1) \qquad \forall j, m, k$$

$$g_k^i \geq 0; \; c_k^i \geq 0 \qquad \forall k, i$$

$$d_{k,j,m}^i \geq 0 \qquad \forall k, j, m, i.$$

We briefly explain the constraints in the model.

3.3.1. *Defining reasonable transition rules.*
The constraints (MILP1.1) ensure a transition rule for each possible claim and classes. Constraints (MILP1.2) ensure that the policyholder without claims should move upward and in class K, he/she should stay in the class. Constraint (MILP1.3) ensures that there should be at least one case (the highest possible number of claims) where there is a downward classification. Constraint (MILP1.4) guarantees the transition rule to be stricter if the number of claims gets higher.

3.3.2. *Obtaining the stationary distribution.*
Equations (MILP1.5) states that each type of policyholders should be in one of the classes. We also have to take care of connecting the stationary probabilities to the transition rules (see (3.6)). The following quadratic constraints accomplish this:

$$c_k^i = \sum_{j=-(K-k)}^{k} \sum_{m=0}^{M} \lambda_m^i T_{j,m,k} c_{k-j}^i \quad k = 1, \ldots, K-1, \forall i. \tag{3.14}$$

To make these constraints linear, we introduce the variables $d_{k,j,m}^i$, as the probabilities that an individual from group i and from class k moves to class $k+j$ in the next period. We define these variables with the constraints (MILP1.6). Constraints (MILP1.7) are meant to linearize the quadratic constraints. To make the linearization complete, we would need additional constraints besides those in (MILP1.6), namely

$$d_{k,j,m}^i \leq \lambda_m^i c_k^i \qquad \forall i, j, k, m \tag{3.15}$$

and

$$d_{k,j,m}^i \leq T_{j,m,k} \qquad \forall i, j, k, m. \tag{3.16}$$

In fact, we do not need them explicitly stated as is shown in the next theorem.

Theorem 3. *If* $T_{j,m,k} = 1$*, then* $d^i_{k,j,m} = \lambda^i_m c^i_k$*, otherwise* $d^i_{k,j,m} = 0$*, provided that* (MILP1.1)*,* (MILP1.5)*,* (MILP1.6) *and* (MILP1.7) *hold.*

Proof.

$$1 = \sum_{k=0}^{K} c^i_k \geq \sum_{k=0}^{K} \sum_{m=0}^{M} \sum_{(j,m)|T_{j,m,k}=1} d^i_{k,j,m} \geq \sum_{k=0}^{K} \left(\sum_{m=0}^{M} \lambda^i_m c^i_k \right) = \sum_{k=0}^{K} c^i_k = 1,$$

The first equality holds because of constraints (MILP1.5), the second inequality holds because of (MILP1.7). The third inequality comes from (MILP1.6) and the last equality is valid since the sum of variables λ^i_m equals 1 for each type. This means that all relations are equalities implying that variables d that are not present in $\sum_{k=0}^{K} \sum_{m=0}^{M} \sum_{(j,m)|T_{j,m,k}=1} d^i_{k,j,m}$ have to be 0 while all other d's have to be equal to $\lambda^i_m c^i_k$. □

Because of Theorem 3, the constraints (3.15) and (3.16) can be omitted from the *MILP1* model.

Often in practice, the transition rules do not differ from class to class which means that there is a unified transition rule for each claim m. This means that instead of binary variables $T_{j,m,k}$ we can simply use binary variables $T_{j,m}$. In this case, J_k is the same for all k; therefore, it is sufficient to set only one upper ($\overline{J} = K$) and lower limit ($\underline{J} = -K$). Also, the constraints (MILP1.7) should be different because we have to omit reclassifications leading to non-existent classes.

$$c^i_k = \sum_{j=\underline{J_k}}^{0} \sum_{\ell=j}^{0} \sum_{m=0}^{M} d^i_{k-\ell,j,m} \qquad k = 0, \forall i$$

$$c^i_k = \sum_{j=\max(\underline{J_k}, -(K-k))}^{\min(\overline{J_k}, k)} \sum_{m=0}^{M} d^i_{k-j,j,m} \qquad k = 1, \ldots, K-1, \forall i \qquad \text{(MILP1.7')}$$

$$c^i_k = \sum_{j=0}^{\overline{J_k}} \sum_{\ell=0}^{j} \sum_{m=0}^{M} d^i_{k-\ell,j,m} \qquad k = K, \forall i$$

3.3.3. *Ensuring irreducibility of Markov chains.*
The constraints (MILP1.8) are needed for the irreducibility condition. Because in the optimization models there are stationary probabilities, it is sufficient to assume that each stationary probability (for each k) be positive. In MILP models, we cannot use strict inequalities but with a parameter $\tau > 0$ and $\tau \approx 0$ we can prescribe that each stationary probabilities be positive. This is an eligible condition for an irreducible Markov chain, but if τ is unnecessarily high, we may exclude some transition rules that give irreducible Markov chains. There are alternative solutions for the irreducibility constraint as well.

For example, we can set the transition rule of the claim-free case to exactly one upward (as it is ordinarily done in practice). Moreover, there is a more precise solution if we exclude one by one those transition rules that would not lead to an irreducible Markov chain. The question of the objective function is discussed in Section 3.2 and the profit constraint (3.13) can be inserted into the model according to wish.

3.4. Joint optimization of transition rules and premiums

In this section, we introduce a modification of the model in Section 3.3. In this modification, we can jointly optimize the transition rules and premiums. In this case, if we use π_k ($\forall k$) as nonnegative variables, then we get a quadratic constraint problem (MIQCP). Because solving a MILP usually needs less computational time than the corresponding MIQCP, we linearize the quadratic constraints. First, we consider the model without a profit constraint. Due to Theorem 1, it is sufficient if we allow only finitely many possibilities for the premiums.

To this end, we start with default premiums for each class that we can increase if needed. We set each default premium to the expected claims of types with the lowest risk $\pi_k = \lambda^1$, $\forall k$. We introduce ε as a value for changing the default premium. We also consider various layers of these modifications. Denote by ε^ℓ how much the premium changes in layer ℓ compared to the default premium.

By Theorem 1, it is sufficient if we set the values of the changes to $\varepsilon^\ell = \lambda^\ell - \lambda^1$, $\ell = 2, \ldots, L$, and $L = I - 1$. Binary variable O_k^ℓ indicates whether we increase the premium in class k by ε^ℓ. That is, if $O_k^\ell = 1$, then the final premium of the class k is $\lambda^1 + \varepsilon^\ell = \lambda^\ell$. The final premiums should be monotonously decreasing:

$$\pi_k + \sum_{\ell=1}^{L} \varepsilon^\ell O_k^\ell \geq \pi_{k+1} + \sum_{\ell=1}^{L} \varepsilon^\ell O_{k+1}^\ell \qquad k = 0, \ldots, K. \tag{MILP1.13}$$

Only one change should be active in each class:

$$\sum_{\ell=1}^{L} O_k^\ell \leq 1 \qquad \forall k. \tag{MILP1.14}$$

In addition, the premium changes should be considered in the constraints (MILP1.9) and (MILP1.10). This would, however, change these linear constraints into quadratic ones. We can linearize these constraints with continuous nonnegative variables $o_k^{\ell,i}$. With the following constraints, we can prescribe that if $O_k^\ell = 1$, then $o_k^{\ell,i}$ should be equal to $c_k^i \varepsilon^\ell$, otherwise 0.

$$o_k^{\ell,i} \geq \varepsilon^\ell \left(c_k^i - (1 - O_k^\ell) \right) \quad \forall i, k, \ell = 1, \ldots, I-1 \tag{MILP1.15}$$

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$$o_k^{\ell,i} \le \varepsilon^\ell c_k^i \quad \forall i, k, \ell = 1, \ldots, I-1 \tag{MILP1.16}$$

and

$$o_k^{\ell,i} \le \varepsilon^\ell O_k^\ell \quad \forall i, k, \ell = 1, \ldots, I-1. \tag{MILP1.17}$$

We modify the constraints (MILP1.9) and (MILP1.10) as well:

$$\pi_k c_k^i + \sum_{\ell=1}^{I-1} o_k^{\ell,i} + g_k^i \ge \lambda^i c_k^i \quad \forall i, k \tag{MILP1.18}$$

$$\pi_k c_k^i + \sum_{\ell=1}^{I-1} o_k^{\ell,i} - g_k^i \le \lambda^i c_k^i \quad \forall ik. \tag{MILP1.19}$$

In the MILP for the joint optimization of transition rules and premiums, we would minimize (MILP1.obj), subject to (MILP1.1)–(MILP1.8) and (MILP1.13)–(MILP1.19).

If we consider the profit constraint (3.13), then with finitely many premium changes we may not get the global optimum. Additionally, by Theorem 2, there can be one additional premium in the optimal solution. We can include another layer for this extra premium with this unique premium's level. However, we do not know the exact value of the ε of this layer, beforehand. With adding multiple additional layers of premium changes, we can approximate the optimal solution with arbitrary precision.

We increase the number of layers (L) and separate them into two sets $\mathcal{L}_1$ and $\mathcal{L}_2$, thus $L = |\mathcal{L}_1| + |\mathcal{L}_2|$. The first type of layers denotes the modifications used earlier to achieve the expected claims of the risk-groups $\varepsilon^\ell = \lambda^\ell - \lambda^1$, if $\ell \in \mathcal{L}_1$ hence $|\mathcal{L}_1| = I - 1$. The other type of layer is for the unique premium only. For this, we arbitrarily determine every ε^ℓ, if $\ell \in \mathcal{L}_2$. By Theorem 2, there can only be one type of unique premium, that is, there can be at least one active layer in $\mathcal{L}_2$. For this, we introduce binary variables S_ℓ, for all $\ell \in \mathcal{L}_2$. This variable equals to 1, if the classes' layer ℓ is active:

$$\sum_{k=0}^{K} O_k^\ell \le (K+1) S_\ell \qquad \forall \ell \in \mathcal{L}_2. \tag{MILP1.20}$$

There can be at least one active layer in $\mathcal{L}_2$:

$$\sum_{\ell \in \mathcal{L}_2} S_\ell \le 1. \tag{MILP1.21}$$

In this case, we also have to include the profit constraint in the model:

$$\sum_{i=1}^{I} \phi^i \sum_{k=0}^{K} \left(\pi_k c_k^i + \sum_{\ell=1}^{L} o_k^{\ell,i} \right) \ge \sum_{i=1}^{I} \phi^i \lambda^i. \tag{MILP1.22}$$

The values ε^{ℓ} if $\ell \in \mathcal{L}_2$ are arbitrary, as well as $|\mathcal{L}_2|$. In theory, if we include a large number of $\mathcal{L}_2$ type layers, then the model may give a good solution close to the global optimum. If $|\mathcal{L}_2|$ is large, then the computational time increases dramatically because of the 'big-M' constraints, such as (MILP1.15)–(MILP1.17). In the numerical experiments, we used only one second-type layer ($|\mathcal{L}_2| = 1$). In this case, we iteratively reran the model to find the best ε of this layer. This reduced computational time, especially for larger instances without significantly affecting the optimal solution.

4. Multi-period model

In Section 3.3, we presented a MILP model based on the stationary distribution. In some cases, however, for the probabilities to reach the stationary level more time periods are needed than the policyholders may remain in the system. In these cases, instead of the stationary distribution, using the probabilities in each period of the insurance contract in the optimization would be more appropriate. In this section, we introduce a modification of the model in Section 3.4, where we do not use the stationary probabilities.

Take the first Θ periods of the insurance contract. The index of time is denoted by t, $(t = 0, \ldots, \Theta)$ where $t = 0$ means the beginning of the contract and Θ is the end of it. The variables c^i_k, g^i_k, $d^i_{k,j,m}$ depend now on time, so we use the notation $c^i_{k,t}$, $g^i_{k,t} d^i_{k,j,m,t}$ accordingly. In the starting period (indexed with 0) each policyholder is assigned to the same initial class. We introduce B_k binary variables for all classes to determine the initial class. When the B_k variable takes the value 1, then class k is the initial class. Assume that there is only one initial class:

$$\sum_{k=0}^{K} B_k = 1 \tag{4.1}$$

$$c^i_{k,0} = B_k \qquad \forall i, k. \tag{4.2}$$

Transition rules are determined in the same way as previously. This means that the constraints (MILP1.1)–(MILP1.4) remain unchanged in the multi-period model. Constraints (MILP1.5)–(MILP1.7) now become

$$\sum_{k=0}^{K} c^i_{k,t} = 1 \qquad \forall i, t = 1, \ldots, \Theta \tag{4.3}$$

$$d^i_{k,j,m,t} \geq \lambda^i_m c^i_{k,t} - (1 - T_{j,m,k}) \qquad \forall i, j, k, m, t = 0, \ldots, \Theta - 1 \tag{4.4}$$

$$c^i_{k,t} = \sum_{j=-(K-k)}^{k} \sum_{m=0}^{M} d^i_{k-j,j,m,t-1} \qquad \forall i, k, t. \tag{4.5}$$

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Constraint (MILP1.8) can only be used for irreducibility if Θ is large enough. In the multi-period model, another approach for irreducibility would be more appropriate. Theorem 2 is still valid in the multi-period case, so for the joint optimization of the premiums and transition rules we can use the same modifications as in Section 3.4. Variables $o_k^{\ell,i}$, however, should be time-dependent and replaced by variables $o_{k,t}^{\ell,i}$.

There are two ways for including the profit constraint in the multi-period model. Either we prescribe the profitability over all of the $\Theta+1$ periods or we do so for each period. If we consider the overall profit, then the model should include the following constraint:

$$\sum_{t=0}^{\Theta}\sum_{i=1}^{I}\phi^i\sum_{k=0}^{K}\left(\pi_k c_{k,t}^i+\sum_{\ell=1}^{L}o_{k,t}^{\ell,i}\right)\geq\sum_{i=1}^{I}(\Theta+1)\phi^i\lambda^i. \tag{4.6}$$

On the other hand, if we consider profitability in each period, then we have

$$\sum_{i=1}^{I}\phi^i\sum_{k=0}^{K}\left(\pi_k c_{k,t}^i+\sum_{\ell=1}^{L}o_{k,t}^{\ell,i}\right)\geq\sum_{i=1}^{I}\phi^i\lambda^i\quad\forall t. \tag{4.7}$$

Furthermore, in the objective function, we should consider the absolute deviation of every period:

$$\min\sum_{t=0}^{\Theta}\sum_{i=1}^{I}\sum_{k=0}^{K}\phi^i g_{k,t}^i. \tag{MILP2.obj}$$

5. Numerical experiments

For calculations, we used an AMD Ryzen 5 2600 Six-Core CPU 3,40 GHz computer with 16 GB DDR4 RAM. We ran the program in Python 3.7.3. and used the Gurobi 8.1.0 solver for the optimization. To reduce the numerical problems caused by the large number of ‘big-M’ constraints, we did concurrent optimization within Gurobi for solving the LP which means that the solver uses multiple algorithms simultaneously and returns the solution obtained first.

5.1. Model without profitability constraint

5.1.1. *Results of stationary model.*
For the calculations, we considered two types of policyholders, a ‘good’ with lower risk and a ‘bad’ with high risk. For the sake of simplicity, we assumed $M=1$ in every case and the same proportion of types. Four alternative scenarios were investigated, two non-realistic with high claim probabilities (10%; 20%) and (10%; 50%), and two scenarios with more realistic parameters (1%; 2%) and (1%; 5%), respectively.

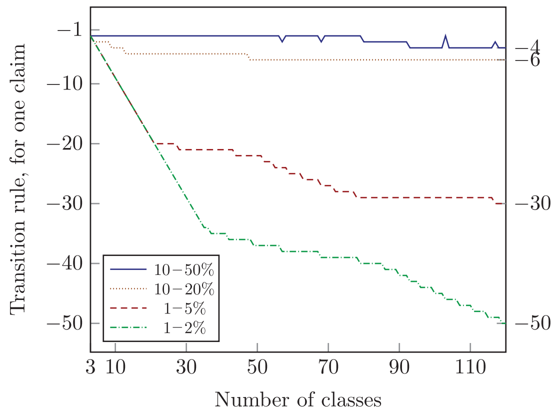

FIGURE 1: Changes of the transition rule for one claim as a function of the number of classes.

We considered a 10-class BMS for each scenario and used the model introduced in Section 3.4. We jointly optimized the transition rules with the premiums without the profitability constraint. We investigated the cases where transition rules are allowed to be different in each class (introduced in Section 3.3) and when the transition rules are unified (introduced in Section 3.3.2).

Table 2 presents the transition rules of a 10-class BMS in each scenario. If there is no claim in a period, then in every situation, the policyholder moves one class upward. If there is a claim during the period, then only unrealistic, ‘high’ probability cases resulted in non-unified transition rules. In the ‘small’ probability situations, the results of the non-unified models gave the same solution as the unified ones. In these cases, the policyholders will be classified into class 0 if they have a claim. In the ‘high’ risk situations, however, the higher classes have less strict transition rules if transition rules are non-unified. Overall, if the claim-risks are higher, the transition rules are less strict. We investigated every scenario’s BMS with more classes as well. The computational time of the models was limited to 3 h. When the limit had been reached, we stopped and the best solution found was recorded.

Table 3 presents the differences in the objective function as well as running time between the unified (U) and non-unified (NU) models. As we increase the number of classes, for the ‘high’ probability scenarios, the NU models gave an even better solution. In the cases of ‘small’ probabilities, however, the change of U to NU was of no significance. Besides, the NU models are computationally much harder because of the large number of binary variables. While the U models produced optimal solutions in every case within a second, in many cases we could not compute the optimal solution of the NU models in 3 h. With unified transition rules, we investigated the effect of the number of classes. Every optimal solution was determined from a 3-class to a 120-class BMS.

In Figure 1, the unified transition rule for the case of a claim in each BMS can be seen.

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TABLE 2

THE TRANSITION RULES OF THE 10-CLASS BM SYSTEMS FOR EACH SITUATION.

		One claim							
Current		1–2%		1–5%		10–20%		10–50%	
class	Zero claim	Non-unified	Unified	Non-unified	Unified	Non-unified	Unified	Non-unified	Unified
0	+1	0	0	0	0	0	0	0	0
1	+1	−1	−1	−1	−1	−1	−1	−1	−1
2	+1	−2	−2	−2	−2	−2	−2	−2	−2
3	+1	−3	−3	−3	−3	−3	−3	−3	−2
4	+1	−4	−4	−4	−4	−4	−4	−4	−2
5	+1	−5	−5	−5	−5	−5	−4	−5	−2
6	+1	−6	−6	−6	−6	−6	−4	−6	−2
7	+1	−7	−7	−7	−7	−7	−4	−4	−2
8	+1	−8	−8	−8	−8	−7	−4	−1	−2
9	0	−9	−9	−9	−9	−2	−4	−1	−2

TABLE 3

DIFFERENCES IN RUNNING TIME AND OBJECTIVES OF 10–15-CLASSES BM SYSTEMS. THE TIME U PRESENTS THE RUNNING TIME OF THE UNIFIED MODEL, THE TIME NU IS THE NON-UNIFIED MODEL'S COMPUTATIONAL TIME (BOTH IN SECONDS). THE OBJ. CH. COLUMNS SHOW THE IMPROVEMENT OF THE OBJECTIVE VALUE OF THE MODEL WITH THE NON-UNIFIED TRANSITION RULES.

	1–2%			1–5%			10–20%			10–50%		
Class	Obj. ch. (%)	Time NU	Time U	Obj. ch. (%)	Time NU	Time U	Obj. ch. (%)	Time NU	Time U	Obj. ch. (%)	Time NU	Time U
10	0	22	0.2	0	6	0.3	−0.08	539	0.2	−0.52	5	0.2
11	0	589	0.2	0	41	0.2	−0.10	9311	0.3	−0.60	16	0.2
12	0	10,606	0.3	0	289	0.3	−0.13	10,800	0.3	−0.67	423	0.3
13	0	10,800	0.4	0	2992	0.3	−0.15	10,800	0.4	−0.68	475	0.3
14	0	10,800	0.3	0	10,800	0.3	−0.16	10,800	0.4	−0.72	1229	0.4
15	0	10,800	0.4	0	10,800	0.4	−0.17	10,800	0.5	−0.78	10,800	0.5

As the number of classes increases, the transition rule gets stricter in general, in every scenario. Though, after a certain number of classes the penalty for a claim tends to decrease. In addition, if the probabilities are higher, then the point where the rate of decline changes occurs at a much lower number of classes. The $m = 0$ case resulted in a one positive step in each situation.

For visualization of the sorting capability of the BMS, we use an indicator that represents the overpayment for a type i policyholder that we denote by OP^i. OP^i shows the ratio of the paid and the ideal payment of type i policyholders.

$$OP^i = \frac{\sum_{k=0}^{K} \pi_k c_k^i}{\lambda^i} - 1. \tag{5.1}$$

Therefore, if OP^i is positive, then the expected payment of risk-group i is more, if negative, it is less than their expected claim. We also introduce the $\Omega = \sum_{i=1}^{I} |OP^i|$ that can be interpreted as the righteousness of the BMS. If it is close to zero, then every policyholder pays close to his/her ideal level.

Ω differs from the model's objective due to the risk aversion of the policyholders. Hence, it is possible to design a BMS where the Ω is smaller than our model's optimal result, though deviations of the premiums can be considerably higher in that case.

Ω decreases in every situation with the increase in the number of classes (Figure 2) which means that a BMS with a larger number of classes would be a better sorting system. If the risk-groups' parameters are higher, then Ω tends to the zero level, which means the BMS almost perfectly sorts the types. For the 10–50% case, the BMSs over 20 classes sort almost perfectly the types, the 10–20% situation needs about 100 classes. In lower-risk situations even 120 classes are not enough for a viable sorting system, but we can see some decrease as the number of classes increases. Also, we can see that in the lower-risk situations, the smaller number of classes brings the payment of 'good' policyholders closer to their ideal level. As the number of classes increases, payment for 'bad' types gets closer to their ideal level, too.

The dotted line represents the profit-ratio of the insurer, which is the expected overall payment of the policyholders divided by the expected total claims minus one. If the profit-ratio is positive, then the BMS is profitable. Certainly, if every policyholder's expected payment equals to his/her ideal level, then the profit-ratio is zero. With a lower number of classes, the policyholders pay less but as the number of classes increases the payment is increasing as well.

5.1.2. *Consideration of more than one claim per period.*

In the previous models, the maximal number of claims that can happen in a period is one. We also considered a model when $M = 2$, where the probabilities of the claims are calculated according to the Poisson distribution. Figure 3 presents the transition rules of the models. On the left-side, the first claim's

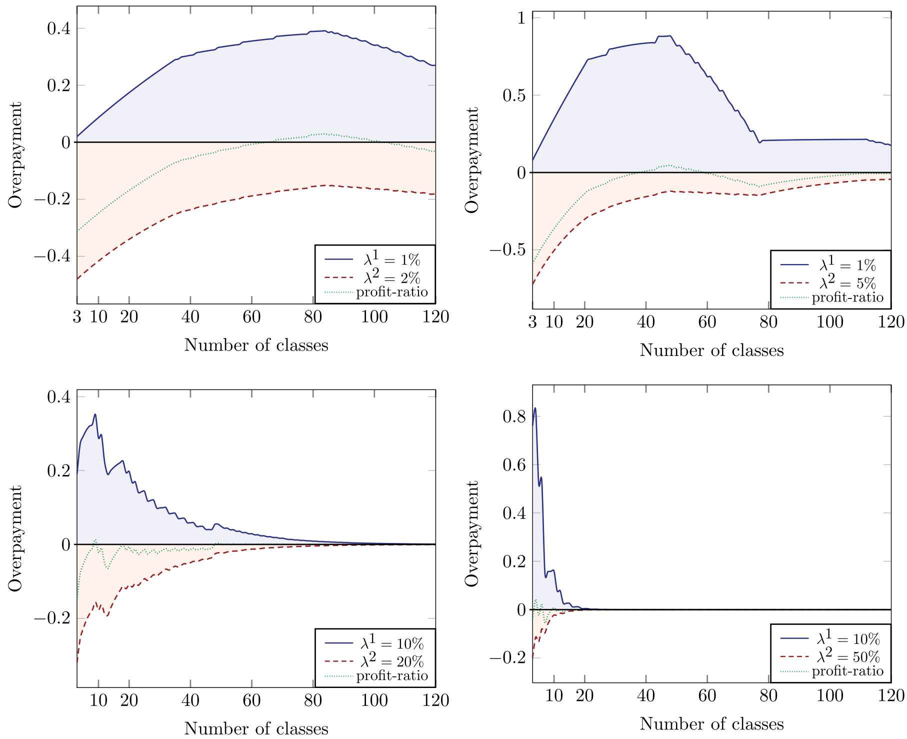

FIGURE 2: Changes of the OP^i and the profit-ratio as a function of the number of classes.

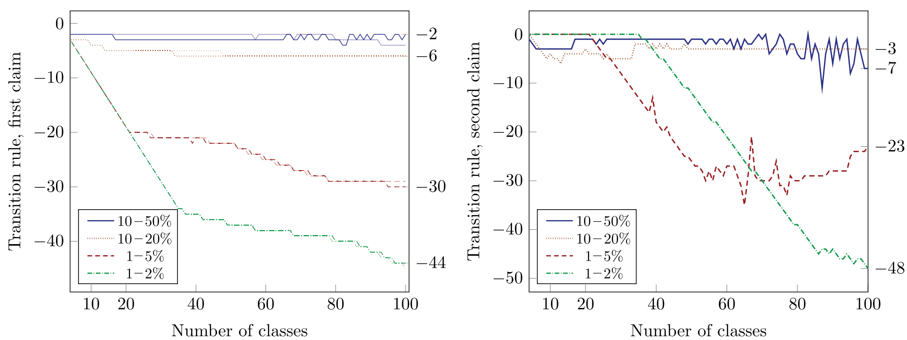

FIGURE 3: The first and second claim's transition rules.

transition rule is shown. The other graph presents the additional reduction that the second claim would cause.

In the figure of the first claim transition rule, we also display with paler colors the results of the models with $M = 1$. These lines are almost the same; hence, the first claim is treated similarly in all of the scenarios.

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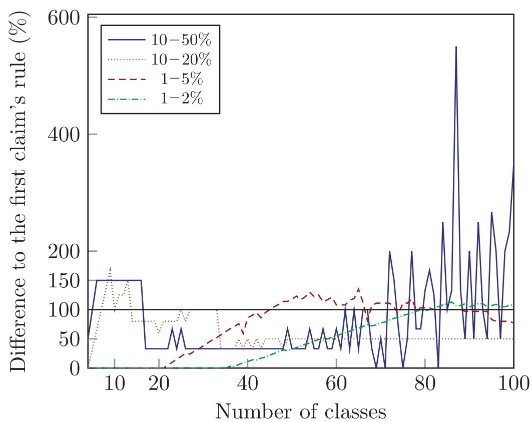

FIGURE 4: The ratio between the reduction of the first and second claim.

For the second claim, when there are only a few classes in every scenario, the second claim is not treated too strictly. Interestingly in the scenario of 1–2%, the second claim is not considered at all, if there are less than 40 classes. However, as the number of the classes increases, the smaller probability scenarios' transition rules decrease. The higher probability scenarios' transition rules do not decrease that much.

Figure 4 presents the ratio between the first and the second claims' reduction in the classification. If the value is 100%, then the second claim is treated the same way as the first one is. When it is less than 100%, then the second claim results in less class-reduction. Mostly, the transition rule for the second claim is not that strict as the first one. In the 10–50% scenario as the number of classes increases, the rule of the second claim gets a bit unstable. The reason behind it is because the Ω in these BM systems is nearly zero. Hence, the second claim does not influence the solution that much.

In practice, usually, the second claim is treated similarly to the first one. The occurrence of two claims in the same period in the real world has a very small probability. We also found that the consideration of multiple claims does not change the Ω values of the models. Besides, the first claims' transition rule was also very similar to the $M = 1$ case. Therefore, in the following numerical examples, we considered only one possible claim, for the sake of computational simplicity.

5.1.3. *Results of multi-period models.*
For the multi-period model, we studied the $\Theta = 20$ and $\Theta = 40$ cases. For treating irreducibility, constraints (MILP1.8) may not be suitable. Therefore, we fixed the $m = 0$ transition rule to 1. Because of the excessive amount of computations required by the multi-period model, we only calculated BMSs where the number of classes is divisible by 5, up to 30 classes. Figure 5 shows the differences of the $m = 1$ transition rule in each case.

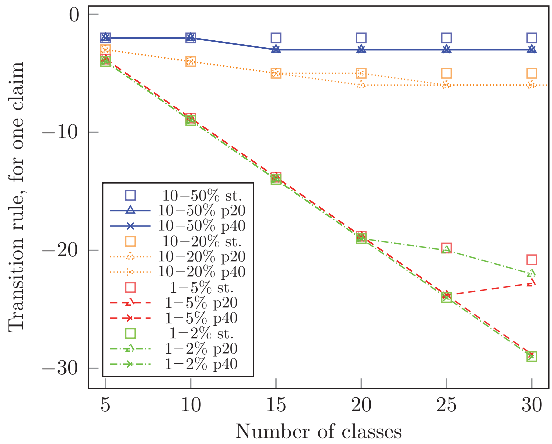

FIGURE 5: Changes of the transition rule for one claim as a function of the number of classes.

Figure 5 shows the transition rule of the multi-period models, compared to the results of the stationary models. In the 'high' probability situations and in the 1–5% case, the transition rules are much stricter in the multi-period model. In the 1–2% risk situation, however, the transition rules become less strict when there are 25 or 30 classes. In these cases, the value represents the average overpayment (or underpayment) of the types during the insurance contract. Figure 6 shows the differences between the Ω values from the stationary model.

As the number of the periods increases, the Ω of the multi-period model gets closer to the result of the stationary model. As Figure 6 shows if Θ is smaller, then the overpayment is less for the 'good' policyholders, and the underpayment is more for the 'bad' ones. In addition, because of the limitation of the periods, if we increase the number of classes, then the sorting capability of the system does not necessarily improve. In the 10–50% case, in the stationary model, each type of policyholders pays their ideal expected premium in class 25, in contrast, in the multi-period models we obtained considerably worse results.

5.2. Model with profitability constraint

With the inclusion of the profitability constraint (MILP1.22), there is one unique premium. In Section 3.4, we introduced a modification of the MILP where we can determine the unique premium with additional premium changing layers. Multiple layers require more binary variables producing a notable increase in computational time. Moreover, even with a fixed number of layers, we cannot be certain that we have found the global optimum. Therefore, for finding an (almost) globally optimal solution within a reasonable time, we included an iterated local search (ILS) algorithm (Figure 7; see, for instance, Lourenço *et al.*, 2010).

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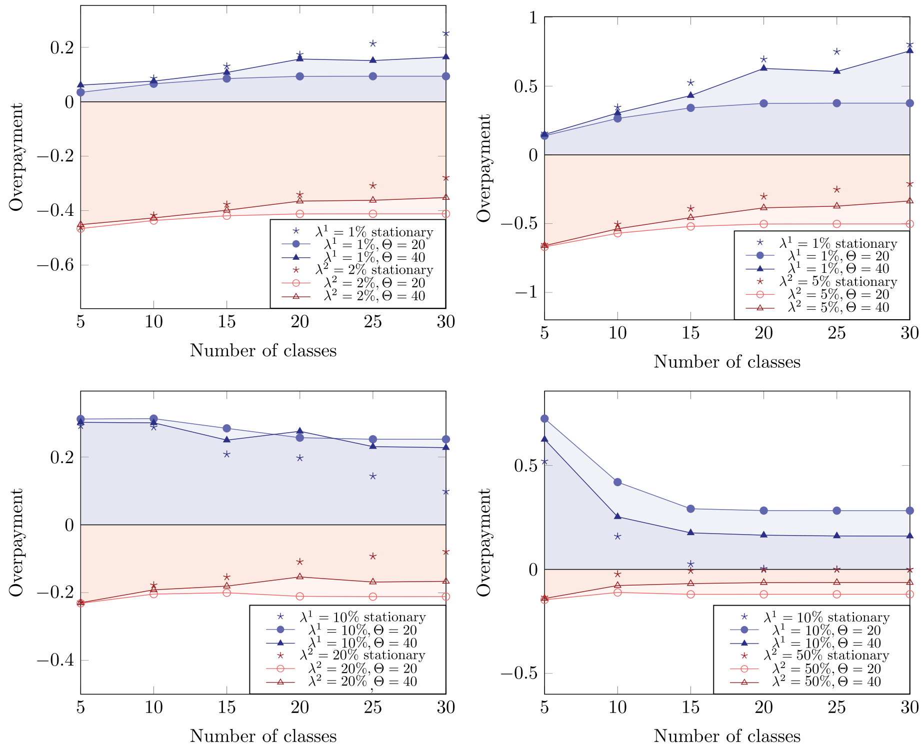

FIGURE 6: Changes of the OP^i as a function of the number of classes.

S_0 = Initial solution
S^* =LocalSearch(S_0)
repeat
  S^p= Perturbation(S^*, history)
  S^{ls} = LocalSearch(S^p)
  if $\min(S^p; S^{ls}) < S^*$ **then**
    $S^* = \min(S^p; S^{ls})$
  end
until *termination condition met*;

FIGURE 7: Iterated local search (source: Lourenço *et al.*, 2010).

We started the initial solution (S_0) as zero, which means that the first model is when there is not any unique premium. We used a randomized *Perturbation* function, which increases over the iteration if we do not find a better solution. However, if we find a better solution, then the following perturbation will be a smaller increase again. To make the running time faster, we only make *LocalSearch* on the perturbed solution, if the perturbed solution is close to the best solution. We made three random restarts of this algorithm for finding a better solution.

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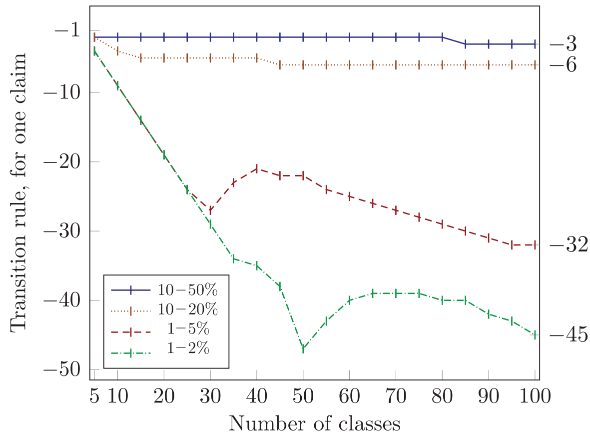

FIGURE 8: Changes of the transition rule for one claim as a function of the number of classes.

5.2.1. *Results of stationary model.*
Figure 8 shows the results of the transition rule of the $m = 1$ case as the number of classes increases with the inclusion of the profitability (MILP1.22) constraint. Similarly to the case without considering profitability, the parameter j decreases as the number of classes increases, but after a certain number of classes the reduction changes its trend. Interestingly, initial decrease turns to increase and then it again starts decreasing.

Figure 9 shows the OP^i values of the different situations. With the dashed lines, we indicate the case when profitability is not considered. Profitability typically affects the minority of BMSs. Overall, profitability increases the payment of the policyholder, that is, the 'good' policyholders will pay even more from their ideal expected premium, but the 'bad' policyholders pay closer to their ideal level.

5.2.2. *Results of multi-period models.*
Figure 10 depicts the Ω values of the multi-period models if we consider the profitability in each period (constraints (4.7)).

We can observe similar results to those without profitability constraints. The improvement of the Ω is considerably smaller in the multi-period case if we increase the number of classes compared to those of the results of the stationary models.

5.3. Case study on real data

Using data from a Hungarian insurance company, we could work with realistic claim probabilities. We distinguished five different risk-groups whose ratios and expected claims can be seen in Table 4.

TABLE 4

PARAMETERS OF THE RISK-GROUPS.

Type	1	2	3	4	5
Expected claim	1.8%	2.7%	3.2%	4.1%	5.0%
Ratio	11%	44%	26%	11%	7%

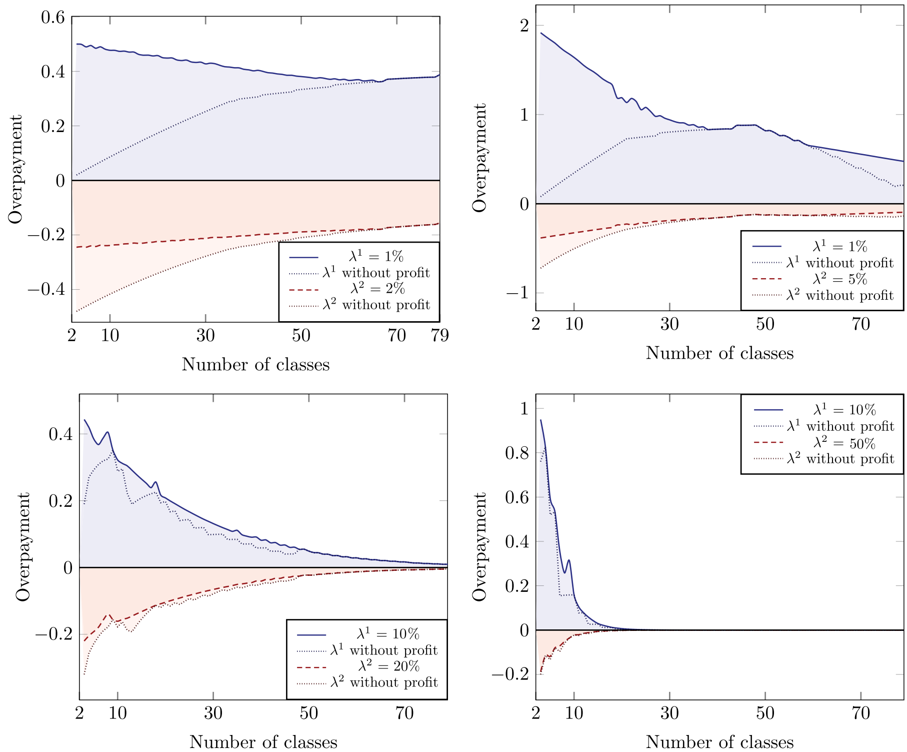

FIGURE 9: Changes of the OP^i as a function of the number of classes.

We solved the stationary models and the multi-period models with $\Theta = 20$ and $\Theta = 40$. In both cases, we worked both with and without the consideration of the profitability constraint. Because of the excessive time requirement, we only considered the unified cases of a 20-class BMS. Table 5 presents the transition rules and the OP^i and Ω values of the models.

If we did not consider the profitability, the result of the stationary model and both multi-period models would not be significantly different. The transition rule is less strict in the $\Theta = 20$ case, but the values of overpayments have not differed that much.

Considering profitability, however, results in a much different outcome. The value of the objective function gets much worse, and of course, the absolute overpayment increased as well compared to the financially not balanced

TABLE 5

THE RESULTS OF THE MODELS WITH REALISTIC PARAMETERS. T_0, T_1 DENOTES THE TRANSITION RULES. OBJECTIVE IS THE VALUE OF THE OBJECTIVE FUNCTION, COMPARED TO THE STATIONARY MODEL WITHOUT PROFITABILITY. IN THE CASE OF MULTI-PERIOD MODELS, IT HAS BEEN DIVIDED BY THE NUMBER OF PERIODS FOR THE SAKE OF COMPARISON. THE Ω SHOWS THE ABSOLUTE OVERPAYMENTS, AND THE OP^i IS THE OVERPAYMENT OF THE TYPE i POLICYHOLDERS.

	Without profitability			With profitability		
	Stationary	$\Theta = 20$	$\Theta = 40$	Stationary	$\Theta = 20$	$\Theta = 40$
T_0	1	1	1	1	1	1
T_1	−6	−5	−6	−8	−2	−16
Objective	100%	100.01%	100.01%	155.53%	183.08%	163.91%
Ω	1.46	1.46	1.46	1.49	1.54	1.48
OP^1	0.50	0.50	0.50	0.68	0.78	0.69
OP^2	0.00	0.00	0.00	0.13	0.19	0.13
OP^3	−0.16	−0.16	−0.16	−0.05	0.00	−0.04
OP^4	−0.34	−0.34	−0.34	−0.25	−0.22	−0.24
OP^5	−0.46	−0.46	−0.46	−0.38	−0.36	−0.38

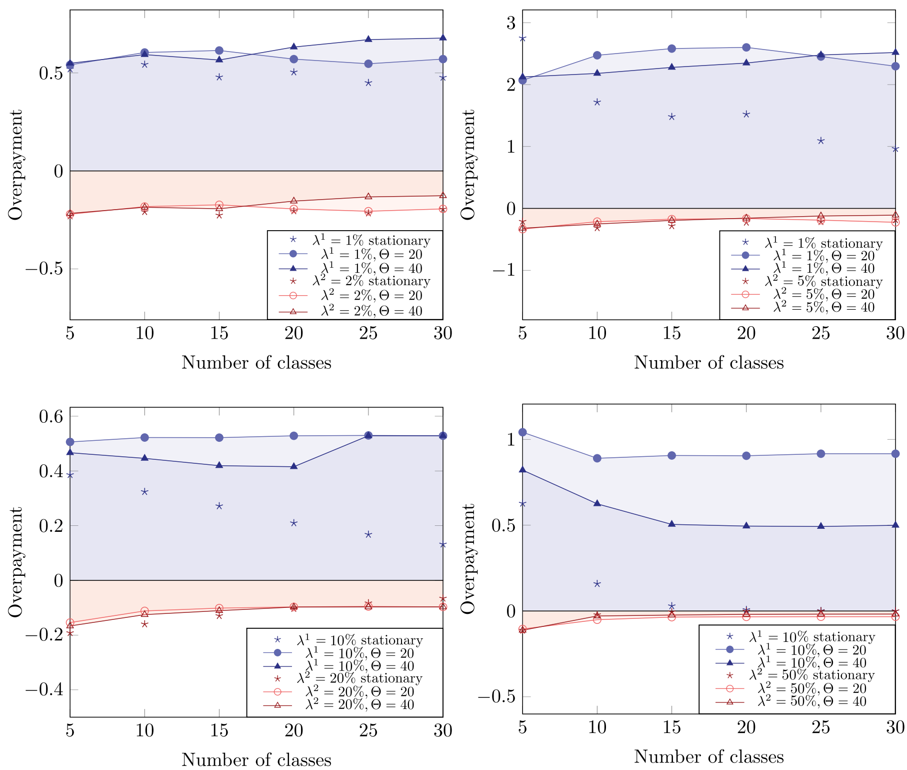

FIGURE 10: Changes of the OP^i as a function of the number of classes.

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TABLE 6

PREMIUM SCALES OF THE REALISTIC MODELS.

	Without profitability					With profitability					
Premium:	0.018	0.027	0.032	0.041	0.05	0.018	0.027	**0.03**	0.032	0.041	0.05
Stationary	–	5–19	0–4	–	–	–	–	19	0–18	–	–
$\Theta = 20$	–	5–19	0–4	–	–	–	–	–	8–19	1–7	0
$\Theta = 40$	–	4–19	0–3	–	–	–	–	7–19	2–6	0–1	–

models. The difference between the stationary and the multi-period models is more notable. The time of the multi-period model is more crucial here since the 20-period model's result differs more from the outcome of the stationary model than the result of the 40-period model. Even the $\Theta = 40$ case results in a much worse optimal-solution than the stationary case.

Table 6 presents the optimal premiums. Each cell shows the classes, where the column's premium is present. Therefore, the 5–19 in the column of 0.027 means that class $5, 6, \ldots, 19$ has the premium 0.027.

When we did not consider the profitability constraint, the optimal premium scale only contained two premiums: the 0.027 and 0.032. It means that three risk-groups cannot pay their fair price. Therefore, 11% of the policyholders surely overpay and 18% pay undoubtedly less. When the profit constraint was considered, for the types with 0.018 and 0.027 expected claim there was no class with a fair payment for them. The premiums were adjusted upward, but there were still up to three different premiums. However, because in this case we considered a metaheuristic, there may exist a better solution. In these cases, we got the 0.03 as unique premium (with boldface in Table 6), which only appeared in the stationary and the $\Theta = 40$ models. In the $\Theta = 20$ model, we did not get any unique premium.

Interestingly, we got fewer premiums than the number of risk-groups in every case. The other objective of the BMS is to reduce the moral hazard. In other words, a system is needed which motivates the policyholders to reduce their risks. For this, the designer of the BMS may want more variability on the premiums. One possibility would be to specify a minimal difference between the premiums of each class. However, this would be rather difficult in the joint optimization approach. In this model, we can specify that each type's fair premium should appear in the premium scale. For this, we have to add the following constraints:

$$\sum_{k=0}^{K+1} \sum_{\ell=1}^{L} O_k^\ell \leq K \tag{5.2}$$

$$\sum_{k=0}^{K+1} O_k^\ell \geq 1 \quad \forall \ell \in L \tag{5.3}$$

TABLE 7

THE RESULTS WHEN EACH TYPE HAS A PREMIUM.

Premium:	0.018	0.027	0.032	0.041	0.05	T_1	ℓ^1 change	ℓ^2 change
Stationary						−17	1.709	1.151
$\Theta = 20$	19	3–18	2	1	0	−3	1.662	1.131
$\Theta = 40$						−16	1.719	1.172

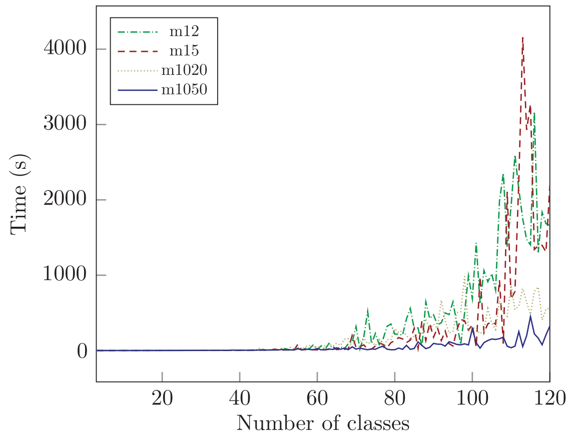

FIGURE 11: Running times of the stationary models without profit.

Table 7 presents the results of the extended models. We only considered the models without profitability constraint, because in these cases, we obtained the exact solution. In these models, we got the same premium scales, only the transition rules differed. Again the $\Theta = 20$ had a much different transition rule. When the considered time was longer, the model had a similar solution to the stationary model. The ℓ^1-change column presents the increase in the objective value if we add these constraints. The ℓ^2 column shows the increase if we consider the ℓ^2 norm, that is, the squared deviations instead of the absolute value of the deviations. In this case, the solution also becomes worse which means more variation on the premiums was not that good in this case as well.

6. COMPUTATIONAL CONSIDERATIONS

In the introduced MILP model as we increase the number of classes, the number of types or the maximal number of potential claims, finding the optimal solution may take more time. For example, the running times of the stationary model without the profitability constraint changed as Figure 11 shows.

When $K = 3$, the running time, in general, was less than half-second, and when the number of classes was 120, it increased to above 30 min. When we

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investigated the multi-period model, even the smaller instances needed a bit more time. The $\Theta = 40$ case required in all of the cases more than 1 h. When the profitability was considered, the running time also increased considerably because of the ILS' multiple runs. The realistic case also resulted in a long running time. The profitless $\Theta = 40$ case needed more than 1 day for finding the optimal solution.

It is important to note that despite the running time was lengthy in some cases, the aim to optimize such a BMS is not a time urgent problem. An optimized system should be valid for several years in the third-party liability insurance application. Also, the running time can be reduced by better hardware and with more significant tolerance level.

7. Conclusion

In this article, we studied the optimization of BMSs. In the literature, mostly stationary probabilities are used for the optimization of a BMS. We developed multiple models to optimize a BMS, both using stationary and multi-period optimization. Our results show that when models considering several periods instead of the (single) stationary period have different outcomes.

Numerical results show that BMSs with more classes can have a better sorting capability. With more classes, however, the policyholders may not reach the period of the stationary probabilities. This means that other transition rules and premiums may be better than the ones obtained in models with stationary probabilities. In addition, if there are fewer classes, then the BMSs are usually not financially balanced. Thus, consideration of the profitability constraint can be necessary for the insurance company.

We analyzed a case study with realistic claim-probabilities. The results in the stationary and the multi-period models are not too far apart if the profitability constraint is not included. By adding financial balancedness to the model, the outcomes of the models differ more.

Acknowledgements

We thank Ferenc Forgó for helpful comments and suggestions.

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