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FINANCIAL INFRASTRUCTURE, TECHNOLOGICAL SHIFT, AND INEQUALITY IN ECONOMIC DEVELOPMENT

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This paper presents an overlapping generations model with technology choice and imperfect financial markets, and examines the evolution of the income distribution in economic development. The model shows that improvements in financial infrastructure facilitate economic development both by raising the aggregate capital–labor ratio and by causing a technological shift to more capital-intensive technologies. Although a higher capital–labor ratio under a given technology reduces inequality, a technological shift can lead to a concentration of economic rents among a smaller number of agents. We derive the condition under which an improvement in financial infrastructure actually decreases the average utility of agents.

Keywords: Technological Shift, Income Distribution, Rents, Enforcement, Credit Rationing

1. INTRODUCTION

One important aspect of economic development is that less productive technologies, which are often labor-intensive, are replaced with more productive ones. However, major technological shifts have often been accompanied by conflicts among different individuals or parties in the economy. Mokyr (1990) documents that before and during the Industrial Revolution, there were numerous examples

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of antimachinery agitation in Britain. In 1768, 500 sawyers attacked a mechanical sawmill in London. In 1792, a Manchester-based firm that pioneered Cartwright's power loom was burned down. Between 1811 and 1816, the "Luddite" riots occurred in the Midlands and the industrial counties. Also, in continental Europe, resistance came from guilds of skilled artisans. In 1780, antimachinery vandalism occurred in the city of Rouen and then spread to Paris, destroying spinning machines imported from Britain and locally made devices such as pitchfork-making machines. These episodes clearly show that not everyone benefits from technological shifts.

This paper examines the effect of economic development on income inequality and welfare, particularly focusing on technological shift and improvements in financial infrastructure as sources of economic development. Any resistance to a new technology suggests that there is a group of agents who earn economic rents that are related to the existing technology. It is popularly believed that this fact actually indicates that technological shifts are desirable for the economy as a whole; that is, although some agents may lose their vested interests, the improved productivity of new technologies can be enjoyed by all agents in the economy. This view suggests that the degree of inequality would fall when a new technology was adopted after the resistance to it has been overcome.

However, historical evidence shows that this is not always the case. For example, between 1759 and 1801, the nominal Gini index rose from 52.2 to 59.3 in England, when the textile and many other industries shifted from cottage to manufacturing technologies (Lindert 2000). Furthermore, Morrison (2000) argues that in continental Europe (e.g., France and Germany), the income share of the top decile increased by 5 to 10% in the mid-nineteenth century, making this period synonymous with the peak of the Kuznets curve. If the rise in inequality implies a further concentration of economic rents among a smaller number of agents, the welfare effect of a technological shift is no longer obvious.

In this paper, we theoretically examine the process of economic development and technological shift, as well as their effects on income distribution and welfare, by constructing an overlapping-generations model with multiple technologies and imperfect financial markets. In particular, this study focuses on financial infrastructure, such as legal and accounting systems, because recent studies have suggested that financial infrastructure is closely related to both technological shifts and the income distribution. La Porta et al. (1997, 1998) and Levine et al. (2000) provided convincing evidence that the development of financial markets is strongly influenced by the financial infrastructure that determines the enforceability of financial contracts. Because "financial revolutions" have often preceded major technological shifts (e.g., Sylla 2002),[1] this evidence justifies the consideration of financial markets as an important source of technological shift. Also, Galor and Zeira (1993) and Matsuyama (2000) demonstrated theoretically that limited enforcement of financial contracts gives rise to credit rationing, which limits the number of entrepreneurs who earn economic rents. This paper incorporates multiple technologies with different capital intensities into their settings and examines how technological shifts affect the income distribution.

Our analysis reveals that improvements in financial infrastructure facilitate economic development in two ways, which have contrasting implications for income distribution. First, as long as the same technology is used, improved financial infrastructure makes credit accessible to an increased number of agents, which raises the aggregate capital–labor ratio and hence the per capita income. In this case, the amount of rent received by each entrepreneur declines, and the incomes of wage earners increase. Consequently, inequality falls and welfare improves.

The second way in which improvements in financial infrastructure facilitate economic development is through technological shifts. While the economy's financial infrastructure is underdeveloped, agents must rely on labor-intensive technologies. However, once the financial infrastructure improves to a certain extent, some agents can obtain sufficient funds to adopt capital-intensive technology. At this point, entrepreneurs relying upon labor-intensive technologies are in effect driven from the markets, whereas those who can adopt the capital-intensive technology begin to attract most of the surplus from the higher productivity, without distributing much to others. This implies that technological shifts not only significantly improve aggregate efficiency (or aggregate income) but also drastically affect the extent of income inequality. Because efficiency and inequality change simultaneously, we carefully examine their overall effect on social welfare and derive a condition under which the rise in inequality is so substantial that welfare actually declines following a technological shift.

The rest of this paper is organized as follows. Section 2 briefly reviews the literature related to this topic and compares the distributional implications therein with ours. Section 3 constructs an overlapping generations model with technology choice under imperfect financial markets. In Section 4, we derive the equilibrium distribution of income and explain why significant inequality emerges among agents. Section 5 clarifies how financial infrastructure affects the choice of technology in equilibrium. Section 6 examines the effects of improvements in financial infrastructure on the income distribution and welfare in the steady state. Section 7 presents conclusions. Proofs of propositions are in the Appendix.

2. COMPARISON WITH THE LITERATURE

In the literature, there are various approaches to theoretically analyzing the evolution of the income distribution through the process of economic development. Among these, close to our approach are the studies by Rajan and Zingales (2003) and Erosa and Hidalgo-Cabrillana (2008), who consider the effect of improved financial infrastructure on the income distribution. These studies have found that economic development that results from improvements in financial infrastructure will reduce the amount of economic rent received by each incumbent rent earner, thereby decreasing inequality [see Figure 1(i)]. Such a redistribution of income

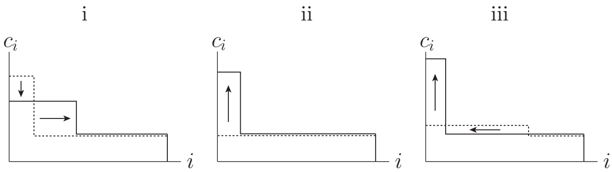

FIGURE 1. Patterns of changes in the consumption distribution.The thick and dashed lines indicate the distribution of consumption before and after the change, respectively. The horizontal axis represents the index of agents. The vertical axis represents each agent's level of consumption. (i) Less inequality, some agents lose, welfare improves. (ii) More inequality, no one loses, welfare improves. (iii) More inequality, some agents lose, welfare may worsen.

generally improves the economy's welfare, although it will not be supported by incumbent rent earners.[2]

Although these studies suggest that income inequality is reduced under a given production technology, other studies focusing on technological shift explain the rise in income inequality in the early stages of economic development. Specifically, Banerjee and Newman (1998) and Greenwood and Jovanovic (1990) show that when agents gradually shift to a new technology, the degree of inequality in the economy rises temporarily because of the disparity in income levels between the new and old sectors. According to these studies, those who move to the new sector are better off, and even those who remain in the old sector are not worse off: they can earn an income as high as the one earned before the technological shift [see Figure 1(ii)].[3] Therefore, although a technological shift increases inequality, it weakly increases every agent's utility and is necessarily welfare-improving. Thus, although these two existing strands of literature provide opposite implications for inequality, both conclude that development always improves welfare.

This paper provides a different welfare implication by simultaneously considering the possibility of technological shift and improvements in financial infrastructure. We obtain a result similar to those of Rajan and Zingales (2003) and Erosa and Hidalgo-Cabrillana (2008) in the context of an improvement in financial infrastructure that does not cause a technological shift. Furthermore, given the satisfaction of a certain condition, we also obtain a result similar to those of Banerjee and Newman (1998) and Greenwood and Jovanovic (1990) in the context of an improvement in financial infrastructure that does cause a technological shift. However, unlike Greenwood and Jovanovic (1990) and Banerjee and Newman (1998), under a different condition, we find that the technological shift makes incumbent entrepreneurs strictly worse off. Specifically, the technological shift deprives the incumbent entrepreneurs of economic rents, and then these rents are redistributed to a smaller number of agents [see Figure 1(iii)]. In this case, social welfare, as measured by the average utility of agents, can deteriorate, especially when people are highly risk-averse.

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A critical difference from existing theories on technological shifts is that in our model, when the technological shift occurs, the previous technology is crowded out of the credit market even when the entrepreneurs with that technology still want to operate with it. Once an improvement in financial infrastructure permits the adoption of a capital-intensive technology, incumbent entrepreneurs with the labor-intensive technology cannot obtain the funds required to operate because banks provide loans only to entrepreneurs with the capital-intensive technology. In addition, credit rationing limits the number of agents who can adopt the new technology. Therefore, there are agents who cannot maintain their previous levels of income because they can no longer use the old technology, nor adopt the new technology. In previous studies, in contrast, agents are allowed to use the previous technology, and some of them voluntarily shift to the new technology because it is in their own interest to do so. This distinguishes the welfare implications of technological shifts in our model from those of previous studies.

Although we focus on capital intensity, there are several other mechanisms through which financial markets affect technological choice. To mention a few, Saint-Paul (1992) shows that without a well-functioning financial market, risk-averse agents may choose less specialized and less productive technologies. Castro et al. (2009) demonstrate that stronger investor protection facilitates economic development, given that the technology for producing investment goods involves a higher idiosyncratic risk than does the technology for producing consumption goods. In contrast, Bencivenga et al. (1995) show that a technological shift resulting from improved financial infrastructure may reduce the growth rate if the new technology requires a longer duration for which investments must be committed. Each of these studies focuses on a particular aspect of technology; however, they are not concerned with income distribution and welfare. In our model, agents choose from technologies with different capital intensities, and in this setting, we demonstrate that improvements in financial infrastructure do not necessarily improve the economy's welfare.

3. THE MODEL

3.1. Economic Environments

Consider an overlapping generations economy with no population growth, in which each generation contains a unit mass of agents who live for two periods (young and old). The life of an agent who was born in period t proceeds as follows. In the first period, he supplies one unit of labor inelastically to the competitive labor market and receives the market wage w_t, measured in terms of consumption goods. For simplicity, we assume that the agent born in period t consumes only in the second period (i.e., in period $t+1$) and receives utility of $u(c_{t+1})$, where the utility function $u(\cdot)$ is increasing and concave. To finance this consumption, the agent makes use of the first-period income w_t in one of two ways. First, he may save it entirely and consume $c_{t+1} = rw_t$ in the second period. We assume that the economy under consideration is a small open economy and that savings yield a constant gross world interest rate of $r \geq 1$.[4] The second option is to become an entrepreneur and to start a project. When starting a project, an agent chooses from among a discrete set of technologies, $\mathcal{J}$. Every technology produces a homogeneous consumption good from capital and labor with constant returns to scale. Specifically, if the agent adopts technology $j \in \mathcal{J}$, the project produces the consumption good according to

$$y_{t+1} = \begin{cases} k_{t+1} f_j(\ell_{t+1}/k_{t+1}), & \text{if } k_{t+1} \geq I_j, \\ 0 & \text{if } k_{t+1} < I_j, \end{cases} \tag{1}$$

where ℓ_{t+1} and k_{t+1} are the amounts of labor and capital inputs, respectively, and $f_j(\cdot)$ is the per unit *capital* (not per *capita*) production function of technology j. Equation (1) shows that exploiting the potential of each technology requires at least a certain amount of investment. The minimal required amount of capital, denoted by $I_j \geq 0$, differs across technologies and depends on technical aspects (such as the scope for scale economies for that technology) and various barriers to the adoption of the technologies, which may be specific to each economy. Capital depreciates completely within one period, and $f_j(\cdot)$ satisfies the standard Inada conditions for all $j \in \mathcal{J}$.

As is standard in overlapping generation models, we assume that the output (of consumption goods) in period t can be used as capital in period $t+1$. However, the agent's first-period income, w_t, often falls short of the minimum required amount of capital, I_j. In that case, the agent must finance the gap by borrowing from competitive financial intermediaries, which we call banks. The banks raise funds from both domestic savings and the international financial market at the world interest rate r, and they offer loans to domestic agents.[5] In order to obtain the loan, $k_{t+1} - w_t$, which is needed to finance an investment of size k_{t+1}, the agent applies to the banks by announcing the plan for a project, comprising three elements—the choice of technology, $j \in \mathcal{J}$, the size of the investment, $k_{t+1} \geq I_j$, and the amount of his or her own funds, w_t—that are verifiable and thus contractable. If the agent is approached by several banks, he chooses a loan contract from the bank that offers the lowest gross interest rate, denoted by R_t. If the agent is denied the loan at any interest rate—i.e., is credit rationed—he gives upon becoming an entrepreneur and lends his entire first-period income to the credit market.

At period $t+1$, an entrepreneur (an agent who has successfully obtained credit or has managed his investment fully using his own funds) decides the number of young workers to hire, $\ell_{t+1} > 0$, at the market wage rate w_{t+1}. The revenue from the project is $y_{t+1} - w_{t+1}\ell_{t+1}$. The entrepreneur is obliged to repay the loan from the project revenue, but also has the option, as Matsuyama (2000) and Foellmi and Oechslin (2007, 2010) similarly assume, of defaulting at the cost of a fraction, $\lambda \in (0, 1)$, of the project revenue. Intuitively, the entrepreneur may choose to abscond (i.e., skip town) with the revenue from the project, $y_{t+1} - w_{t+1}\ell_{t+1}$, but in this event, he loses the fraction λ of the project revenue. This setting is equivalent

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to assuming that lenders can capture only $100\lambda\%$ of the cash flow from any project. Thus, the parameter λ represents the quality of the economy's financial infrastructure, determined by factors such as legal and accounting systems, which determine the enforceability of financial contracts.

At the end of period $t+1$, the agent's consumption is $c_{t+1} = y_{t+1} - w_{t+1}\ell_{t+1} - R_{t+1}(k_{t+1} - w_t)$ if he repays the loan, and is $c_{t+1} = (1-\lambda)(y_{t+1} - w_{t+1}\ell_{t+1})$ if he defaults. In fact, defaults do not occur in equilibrium, because banks only lend funds for projects on which it is beneficial for entrepreneurs to repay the loans. However, as will be shown, the potential of default (i.e., the limited enforcement) creates credit market imperfections and causes credit rationing.

3.2. The Behavior of Households and Banks

This section examines the rational behavior of generation-t households (who become entrepreneurs in period $t+1$ if they obtain credit) and banks, taking as given their first-period income w_t and the market wage rate in the second period w_{t+1}. The decision processes are sequential and therefore can be solved backward. The final decision is to determine the number of workers to hire, ℓ_{t+1}, since the entrepreneur has already chosen the technology, j, and the amount of capital, $k_{t+1} \geq I_j$.

Whether or not the entrepreneur decides to default, her objective at this stage is to maximize revenue $y_{t+1} - w_{t+1}\ell_{t+1}$ with respect to labor input ℓ_{t+1}, where output y_{t+1} is given by (1). Straightforward differentiation shows that it is optimal to choose

$$\ell_{t+1} = f_j'^{-1}(w_{t+1})k_{t+1} \equiv \widetilde{\ell}_j(w_{t+1})k_{t+1}, \tag{2}$$

where $\widetilde{\ell}_j(w_{t+1}) \equiv f_j'^{-1}(w_{t+1})$ represents the optimal labor input per unit of capital as a decreasing function of the market wage w_{t+1}. The rate of return from this project (the amount of maximized revenue divided by the amount of capital) is

$$\rho_j(w_{t+1}) = f_j(\widetilde{\ell}_j(w_{t+1})) - w_{t+1}\widetilde{\ell}_j(w_{t+1}), \tag{3}$$

which is decreasing in the market wage w_{t+1}. From the project revenue $\rho_j(w_{t+1})k_{t+1}$, the entrepreneur repays the loan unless it exceeds the cost of default. That is, the loan will be repaid if and only if

$$R_{t+1}(k_{t+1} - w_t) \leq \lambda\rho_j(w_{t+1})k_{t+1}. \tag{4}$$

Banks offer loans to potential entrepreneurs if and only if the entrepreneurs are willing to repay them *and* the interest rate the entrepreneurs pay on the loan is at least as high as the world interest rate r. As long as repayments from entrepreneurs are expected, competition among banks brings the interest rate down to r because of the free entry-and-exit condition of banks. Banks are assured of repayment if a prospective entrepreneur's planned project, summarized by (j, k_{t+1}, w_t), satisfies condition (4) at interest rate $R_{t+1} = r$. Using the amount of investment for the proposed project, this condition can be written as

$$k_{t+1} \leq \frac{w_t}{1 - \lambda\rho_j(w_{t+1})/r} \quad \text{if} \quad \lambda\rho_j(w_{t+1}) < r. \tag{5}$$

If the proposed plan fails to satisfy (5), the project cannot attract credit at any interest rate.[6] It can be observed from (3) and (5) that the equilibrium wage in period $t+1$, w_{t+1}, must satisfy $\lambda\rho_j(w_{t+1}) < r$ for any technology $j \in \mathcal{J}$. If this condition is not satisfied (which is when the rate of return from the investment satisfies $\rho_j(w_{t+1}) \geq r/\lambda > r$), entrepreneurs can obtain an infinite payoff by investing an infinite amount of capital and hiring an unbounded number of workers, which clearly results in excess demand in the labor market. Thus, the equilibrium wage w_{t+1} must satisfy

$$w_{t+1} > \max_{j \in \mathcal{J}} \rho_j^{-1}(r/\lambda) \equiv \underline{w}(\lambda). \tag{6}$$

Now let us return to the choice of technology and the size of the investment. A prospective entrepreneur chooses j and k_{t+1} in order to maximize second-period consumption,

$$c_{t+1} = rw_t + (\rho_j(w_{t+1}) - r)k_{t+1}. \tag{7}$$

This expression shows that she wants to become an entrepreneur (i.e., to choose some j and set $k_{t+1} > 0$ rather than to save all her first-period income by choosing $k_{t+1} = 0$) only when the rate of return from the investment $\rho_j(w_{t+1})$ is at least as high as the interest rate. Because the rate of return depends on the market wage w_{t+1}, this condition can be written as

$$w_{t+1} \leq \rho_j^{-1}(r) \equiv P_j, \tag{8}$$

which we call the *profitability constraint*. The constant P_j represents the level of market wage at which a project with technology j breaks even. We assume that P_j is smaller than the minimum level of investment I_j.[7]

When the profitability constraint is satisfied, the agent is willing (at least weakly) to start a project. In particular, when the profitability constraint holds with strict inequality, she wants to invest as much as possible. Under (6), however, condition (5) implies that there is an upper bound for the size of the investment, and this upper bound depends on the amount of the entrepreneur's own funds, w_t. In addition, to adopt technology j, at least I_j units of capital must be invested. This implies that the entrepreneur must provide enough of her own funds for the upper bound to be at least as large as I_j. Comparing the right-hand side (RHS) of (5) with I_j, we obtain

$$w_t \geq \left(1 - \frac{\lambda\rho_j(w_{t+1})}{r}\right) I_j \equiv \eta_j(w_{t+1}, \lambda), \tag{9}$$

where the function $\eta_j(\cdot)$ represents the minimum amount of the entrepreneur's own funds required to borrow from banks to start a project with technology j. Note that this minimum requirement is increasing in the market wage w_{t+1}. Therefore,

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as long as the agent cannot self-finance the project (i.e., as long as $w_t < I_j$), condition (9) can be stated in terms of the market wage w_{t+1}, given the amount of the entrepreneur's own funds w_t:[8]

$$w_{t+1} \leq \rho_j^{-1}[(r/\lambda)(1 - w_t/I_j)] \equiv B_j(w_t, \lambda). \tag{10}$$

We call (10), or equivalently (9), the *borrowing constraint* for technology j. Agents can adopt technology j unless the market wage exceeds $B_j(w_t, \lambda)$. This borrowing constraint is relaxed (i.e., $B_j(w_t, \lambda)$ increases) when the agent has more of her own funds w_t or when the economy's financial infrastructure λ improves.

Now we are ready to describe the occupational choice of agents in terms of the market wage w_{t+1} and the amount of own funds w_t. Combining (8) and (10), we see that technology j satisfies both the profitability and borrowing constraints if and only if

$$w_{t+1} \leq \min\{P_j, B_j(w_t, \lambda)\} \equiv \phi_j(w_t, \lambda). \tag{11}$$

If the market wage w_{t+1} is less than or equal to $\phi_j(w_t, \lambda)$, agents with their own funds w_t are able and willing to become entrepreneurs with technology j, rather than merely to save their first-period incomes. Among the potentially usable technologies, $\mathcal{J}$, there exists at least one such technology if[9]

$$w_{t+1} \leq \max_{j \in \mathcal{J}} \phi_j(w_t, \lambda) \equiv \theta(w_t, \lambda). \tag{12}$$

In this case, the agent becomes an entrepreneur and invests as much as she can borrow (see condition (5)), which is

$$k_{t+1} = \frac{w_t}{1 - \lambda\rho_j(w_{t+1})/r} = \frac{w_t}{\eta_j(w_{t+1}, \lambda)} I_j. \tag{13}$$

Note that $w_t/\eta_j(w_{t+1}, \lambda)$ in equation (13) represents the ratio of actual own funds to the amount required to obtain credit, and therefore, it exceeds unity. From (2), (3), and (7), the consumption of the entrepreneur and the individual labor demand from this project are

$$\ell_{t+1} = k_{t+1}\widetilde{\ell}_j(w_{t+1}) = \frac{w_t}{\eta_j(w_{t+1}, \lambda)} I_j \widetilde{\ell}_j(w_{t+1}), \tag{14}$$

$$c_{t+1} = rw_t + (\rho_j(w_{t+1}) - r)\frac{w_t}{\eta_j(w_{t+1}, \lambda)} I_j. \tag{15}$$

The second term in (15) represents the surplus income obtained by virtue of becoming an entrepreneur. If the market wage w_{t+1} is above the threshold $\theta(w_t, \lambda)$, the rate of return from any technology that satisfies the borrowing constraint falls short of r. Thus, it is best for the agent to save her entire first-period income (i.e., $k_{t+1} = \ell_{t+1} = 0$) and to receive $c_{t+1} = rw_t$. If $w_{t+1} = \theta(w_t, \lambda)$, then either the profitability or the borrowing constraint is exactly binding. If the profitability constraint is not binding (and then the borrowing constraint must be binding), the agent strictly prefers to start a project, similarly to the case of $w_{t+1} < \theta(w_t, \lambda)$.

Otherwise, she is indifferent as to whether to start a project: investment k_{t+1} can be zero or anywhere between the minimum amount I_j and the RHS of (13); the labor demand is $\ell_{t+1} = k_{t+1}\widetilde{\ell}_j(w_{t+1})$; and any choice results in $c_{t+1} = rw_t$.

4. INEQUALITY IN EQUILIBRIUM

This section establishes the existence of an equilibrium wage rate at which the aggregate supply of and demand for labor are equalized, and then it examines the extent of inequality that arises at equilibrium. Before proceeding to the formal analysis, we first present an intuitive explanation of how and when significant income inequality arises among old agents in equilibrium. For this purpose, it is convenient to introduce temporarily a small amount of ex ante heterogeneity among agents. In particular, for the time being, we assume that each agent, in his or her youth, experiences an exogenous income shock, ϵ_t, which is chosen randomly from a uniform distribution between 0 and $\overline{\epsilon} > 0$.

Suppose that the agents in each generation are now indexed by $i \in [0, 1]$ and that agent i's realized first-period income is given by $w_t + \epsilon_{it}$. In the preceding section, we showed that agents with their own funds w_t are willing to become entrepreneurs if the market wage w_{t+1} is below the threshold level of $\theta(w_t, \lambda)$, or equivalently, if their threshold $\theta(w_t, \lambda)$ is above the market wage w_{t+1}. Because we now assume that agents have heterogeneous amounts of their own funds, $w_t + \epsilon_{it}$, the threshold $\theta(w_t + \epsilon_{it}, \lambda)$ may also vary across agents. From its definition, the function $\theta(w_t + \epsilon_{it}, \lambda)$ is increasing in $w_t + \epsilon_{it}$, $\theta(0, \lambda) = \underline{w}(\lambda) > 0$, and $\lim_{w_t+\epsilon_{it}\to\infty} \theta(w_t + \epsilon_{it}, \lambda) = \max_{j\in\mathcal{J}} P_j < \infty$. Therefore, the threshold level for any agent is within a (small) finite interval $[\underline{\theta}_t, \overline{\theta}_t]$, where

$$\underline{\theta}_t \equiv \theta(w_t, \lambda), \quad \overline{\theta}_t \equiv \theta(w_t + \overline{\epsilon}, \lambda).$$

From this observation, it follows that the equilibrium level of the market wage, w_{t+1}, must be somewhere between $\underline{\theta}_t$ and $\overline{\theta}_t$. If $w_{t+1} > \overline{\theta}_t$, then no agent will start a project, and therefore the aggregate labor demand will be zero. Conversely, if $w_{t+1} < \underline{\theta}_t$, then all old agents strictly prefer to start projects, which (under Assumption 1 below) necessarily generates excess labor demand. Therefore, if there exists an equilibrium wage level w_{t+1} such that the aggregate labor demand coincides with the aggregate labor supply, it must be within the interval $[\underline{\theta}_t, \overline{\theta}_t]$.

Figure 2 depicts a typical shape of the function $\theta(\cdot)$ against the amount of own funds, $w_t + \epsilon_{it}$, which we call the θ curve. The shape of the θ curve on a short interval $[w_t, w_t + \overline{\epsilon}]$ determines $\underline{\theta}_t$ and $\overline{\theta}_t$. One possibility is that the curve is entirely flat in that interval. In this case, $\underline{\theta}_t$ and $\overline{\theta}_t$ are the same, and the equilibrium wage is uniquely determined at this level. Note that a flat segment of the θ curve corresponds to the profitability constraint for some technology j. In equilibrium, $w_{t+1} = \overline{\theta}_t = \underline{\theta}_t = P_j$ holds, which means from (8) that the rate of return from investment $\rho_j(w_{t+1})$ is the same as the interest rate r. Therefore, all agents are indifferent between becoming entrepreneurs and saving their incomes.

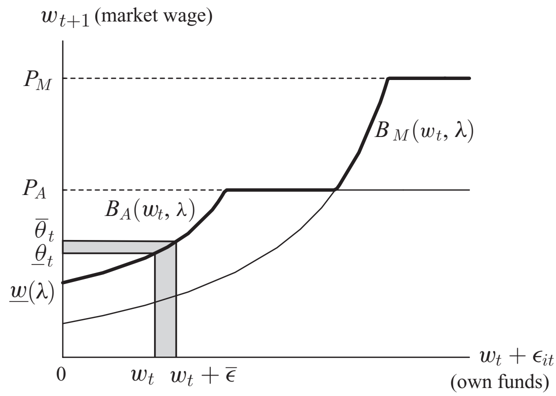

FIGURE 2. An example of the θ curve. It depicts the case of two technologies, $\mathcal{J} = \{A, M\}$. The gray area represents the distribution of the agents' own funds when they face small income shocks.

Irrespective of what they decide, they obtain $c_{t+1} = r(w_t + \epsilon_{it})$. Given that the magnitude of random income ϵ_{it} is small, consumption inequality in the second period is also small.

However, there is a different distributional consequence when the θ curve is upward sloping over the interval $[w_t, w_t + \overline{\epsilon}]$. Because an upward-sloping segment corresponds to a borrowing constraint for a particular technology, the profitability constraint is not binding in this case. This implies that the rate of return from starting a project is strictly higher than r and that every agent strictly prefers to start a project. However, if this were the case, overall labor demand would exceed its aggregate supply. Thus, the equilibrium wage w_{t+1} must be between $\underline{\theta}_t$ and $\overline{\theta}_t$ so that agents with relatively few funds of their own (i.e., those for whom $\theta(w_t + \epsilon_{it}, \lambda)$ is less than w_{t+1}) do not satisfy the borrowing constraint. In other words, some agents must be rationed from the credit market. The consumption of these credit-rationed agents is significantly lower than that of the entrepreneurs, which generates nontrivial inequality among old agents.

In the remainder of this section, we formally establish the existence of the market equilibrium and explicitly derive the equilibrium income distribution. In this economy, the aggregate supply of labor is given by the population of young agents, which is normalized to unity. Given this period's market wage w_{t+1} and labor income from the previous period, w_t, aggregate labor demand is obtained by summing the decisions of all the old agents, as follows:

$$L^D_{t+1}(w_{t+1}; w_t) \equiv \left[\int_{\theta(w_t+\epsilon_{it})>w_{t+1}} \ell_{i\,t+1}\, di,\ \int_{\theta(w_t+\epsilon_{it})\geq w_{t+1}} \ell_{i\,t+1}\, di \right], \qquad \textbf{(16)}$$

where $\ell_{i\,t+1}$ is given by (14), with w_t being replaced by $w_t + \epsilon_{it}$. As shown in (16), function $L^D_{t+1}(w_{t+1}; w_t)$ is a set-valued function (or *correspondence*) because agents may be indifferent with regard to whether or not to start a project (and to hire a worker). We assume that if all old agents start projects, aggregate labor demand exceeds aggregate labor supply. More specifically,

Assumption 1. *$I_j > 1/\widetilde{\ell}_j(P_j)$ for all $j \in \mathcal{J}$.*

From (14), Assumption 1 means that each project requires that more than one worker be hired, which we reasonably assume to be satisfied throughout the paper. Now we can put forward the following proposition.

PROPOSITION 1. *Suppose that Assumption 1 holds and that the number of intersections between the functions $\rho_j(w_{t+1})$ and $\rho_{j'}(w_{t+1})$ for any $j \neq j'$ is not infinite. Then, given the previous period's equilibrium wage $w_t > 0$, there is an equilibrium level of $w_{t+1} \in [\underline{\theta}_t, \overline{\theta}_t]$, for which $1 \in L^D_{t+1}(w_{t+1}; w_t)$ holds.*

Proof. In the Appendix. ■

Although the proof is technical (mainly because aggregate labor demand is given by a set-valued function), the intuition is clear. Aggregate labor demand is greater than unity for $w_{t+1} < \underline{\theta}_t$ and is zero for $w_{t+1} > \overline{\theta}_t$. Moreover, in the Appendix, we show that aggregate labor demand is continuous with respect to w_{t+1}.[10] Thus, it follows that there must be a level of w_{t+1} between $\underline{\theta}_t$ and $\overline{\theta}_t$ at which the aggregate labor demand coincides with its supply. Given the equilibrium wage rate, w_{t+1}, only those agents whose own funds satisfy $w_{t+1} \leq \theta(w_t + \varepsilon_{it}, \lambda)$ can borrow funds from the banks. Thus, in this model, agents are credit rationed if the amounts of their own funds, or equivalently, their income shocks ε_{it}, fall short of a threshold level.

However, it is noteworthy that the result of Proposition 1 does not depend on the size of the heterogeneity term in the income of young agents. In particular, even in the limit, when heterogeneity is negligible (more specifically, when the distribution of $\epsilon_{it} \in [0, \overline{\epsilon}]$ is almost degenerate, i.e., $\overline{\epsilon} \to 0$), Proposition 1 still shows that there exists an equilibrium level of the market wage, w_{t+1}. In that case, we can state the properties of the equilibrium more explicitly.

PROPOSITION 2. *Suppose that the assumptions in Proposition 1 hold and that* ex ante *heterogeneity is negligible ($\overline{\epsilon} \to 0$); then the equilibrium at the limit is characterized as follows.*

(a) *The equilibrium wage w_{t+1} is determined by $\theta(w_t, \lambda)$.*
(b) *The choice of technology in equilibrium, denoted by j^*, is such that $\phi_{j^*}(w_t, \lambda)$ is the highest among all technologies.*
(c) *Credit rationing occurs if and only if the amount of own funds w_t satisfies*

$$w_t < (1 - \lambda) I_{j^*}. \tag{17}$$

(d) *When credit rationing occurs, the number of entrepreneurs, n_{t+1}, and their consumption, c^e_{t+1}, are given by*

$$n_{t+1} = \left(I_{j^*}\widetilde{\ell}_{j^*}(B_{j^*}(w_t, \lambda))\right)^{-1} < 1, \tag{18}$$

$$c^e_{t+1} = r w_t + r((1 - \lambda) I_{j^*} - w_t)/\lambda. \tag{19}$$

The consumption of the other agents is $r w_t < c^e_{t+1}$. *When there is no credit rationing, the consumption of all agents is* $r w_t$.

Proof. In the Appendix. ■

In the remainder of the paper, we continue to consider the limiting case of $\overline{\epsilon} \to 0$, and we omit the ϵ_{it} term from the analysis. Proposition 2 implies that only the technology that can offer the highest wage within its profitability and borrowing constraints can operate in equilibrium. Entrepreneurs with other technologies cannot operate because they cannot hire workers at the market wage level. The proposition also shows that, even at the limit, when there is hardly any ex ante heterogeneity, significant inequality arises between those who can and cannot obtain credit. More specifically, in a credit-constrained equilibrium, each of the n_{t+1} (< 1) entrepreneurs obtains economic rents of $r((1-\lambda)I_{j^*} - w_t)$ that exceed the income of others, $r w_t$. Using this result, the following two sections consider the effects of improvements in financial infrastructure on the equilibrium distribution of income and the welfare of agents.

5. TECHNOLOGICAL SHIFT

Proposition 2 implies that other things being equal, a better financial infrastructure results in a more equal income distribution through the easing of credit rationing. From (18) and (19), stronger enforcement of financial contracts (a larger λ) increases the number of entrepreneurs, n_{t+1}, and reduces the size of the economic rents, $r((1-\lambda)I_{j^*} - w_t)$, received by each of them. In addition, (17) shows that credit rationing disappears when λ is above the threshold level of $1 - w_t/I_{j^*}$. However, these observations do not allow us to conclude that there is a monotonic relationship between the quality of financial infrastructure and the extent of inequality. This is because the threshold level for credit rationing, $1 - w_t/I_{j^*}$, as well as n_{t+1} and c^e_{t+1}, depends on the equilibrium choice of technology j^*, which in turn depends on financial infrastructure, λ. Thus, we need to clarify when an increase in λ causes a technological shift.

This section examines the role of financial infrastructure in determining the equilibrium choice of technology. For concreteness, suppose that the set of usable technologies is composed of Cobb–Douglas technologies, and that their per–unit of capital production functions are given by

$$f_j(\ell/k) = A_j(\ell/k)^{1-\alpha_j}, \quad k \geq I_j, \tag{20}$$

where productivity, A_j, capital intensity, α_j, and the minimum size of investment, I_j differ among technologies. Substituting (20) into (3) and then into (8) and (10) gives the expressions for the profitability and borrowing constraints as

$$w_{t+1} \leq (1-\alpha_j)(\alpha_j/r)^{\widehat{\alpha}_j} A_j^{\widehat{\alpha}_j+1} \equiv P_j, \tag{21}$$

$$w_{t+1} \leq P_j(\lambda I_j/(I_j - w_t))^{\widehat{\alpha}_j} \equiv B_j(w_t, \lambda), \tag{22}$$

where $\widehat{\alpha}_j \equiv \alpha_j/(1-\alpha_j) > 0$. The value of $\phi_j(w_t, \lambda)$ is given by the smaller of P_j and $B_j(w_t, \lambda)$. As stated in Proposition 2, the equilibrium choice of technology is such that $\phi_j(w_t, \lambda)$ is the highest among all technologies.

Although our concern is with the case in which a marginal increase in λ causes a technological shift, it is insightful to see how the pattern of technological specialization is affected by large changes in λ. In particular, when the economy's financial infrastructure is quite primitive ($\lambda \to 0$), then the borrowing constraint becomes very tight ($B_j(w_t, \lambda) \to 0$ for every j), which means that $\phi_j(w_t, \lambda)$ is determined by $B_j(w_t, \lambda)$. In addition, (22) indicates that the higher the capital intensity, the more rapidly $B_j(w_t, \lambda)$ converges to 0 as $\lambda \to 0$. This implies that, with a sufficiently low λ, the economy specializes in labor-intensive technology. Intuitively, if the enforcement of financial contracts is weak, only a small number of agents obtain funds because of tight credit rationing. In such a situation, entrepreneurs who have successfully obtained funds can hire a large number of workers at a low wage, in which case the labor-intensive technology is more suitable. Conversely, when the enforcement of financial contracts is nearly perfect ($\lambda \to 1$), the borrowing constraint becomes weaker than the profitability constraint ($B_j(w_t, \lambda) > P_j$ for every j), which implies that $\phi_j(w_t, \lambda)$ is determined by P_j. Hence, the economy specializes in the technology with the highest profitability P_j, which is largely determined by the technology's productivity A_j. Thus, when development in financial infrastructure triggers a technological shift, the new technology tends to have higher capital intensity *and* higher profitability. If the profitability of a capital-intensive technology is low, it will never be adopted at any stage of economic development. If the capital intensity of a highly profitable technology were low, it would be adopted from the beginning, and therefore there would be no technological shift.

Let us derive the precise condition under which a technological shift occurs. Denoting the technology before the shift as A and denoting that after the shift as M, the previous observation implies that the new technology has a higher capital intensity $\alpha_M > \alpha_A$ and higher profitability, $P_M > P_A$. In this setting, a technological shift occurs when $\phi_A(w_t, \lambda) \equiv \min\{P_A, B_A(w_t, \lambda)\}$ is overtaken by $\phi_M(w_t, \lambda) \equiv \min\{P_M, B_M(w_t, \lambda)\}$. Because $P_M > P_A \geq \phi_A(w_t, \lambda)$, the value of $\phi_M(w_t, \lambda)$ is larger than that of $\phi_A(w_t, \lambda)$ when either $B_M(w_t, \lambda) \geq P_A$ or $B_M(w_t, \lambda) \geq B_A(w_t, \lambda)$ holds.[11] Intuitively, as long as technology A is used, the equilibrium market wage w_{t+1} is bounded above by both the profitability constraint P_A and the borrowing constraint $B_A(w_t, \lambda)$ for technology A. If the borrowing constraint for technology M is weaker than (i.e., $B_M(w_t, \lambda)$ is higher than) either of those two constraints, it means that agents can borrow enough funds to adopt technology M. In fact, agents always shift to technology M if this condition holds, because technology M is more profitable than technology A.

From (22), solving $B_M(w_t, \lambda) \geq P_A$ gives

$$\lambda \geq \left(\frac{P_A}{P_M}\right)^{1/\widehat{\alpha}_M} \left(1 - \frac{w_t}{I_M}\right) \equiv \Lambda_1(w_t). \tag{23}$$

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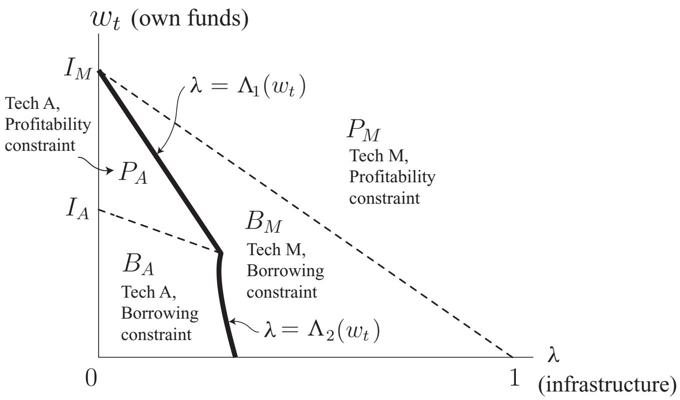

FIGURE 3. Technology choice and the credit regime. Numerically calculated using parameter values of $\alpha_A = 0.20$, $\alpha_M = 0.45$, $r = 2.0$, $P_A = 1.20$, $P_M = 2.25$, $I_A = 1.8$, and $I_M = 3.5$. Region P_A disappears when $I_A > I_M$.

Similarly, $B_M(w_t, \lambda) \geq B_A(w_t, \lambda)$ holds if and only if

$$w < I_A \text{ and } \lambda \geq \left[\frac{P_A}{P_M} \left(1 - \frac{w_t}{I_M}\right)^{\widehat{\alpha}_M} \left(1 - \frac{w_t}{I_A}\right)^{-\widehat{\alpha}_A} \right]^{1/(\widehat{\alpha}_M - \widehat{\alpha}_A)} \equiv \Lambda_2(w_t). \quad \textbf{(24)}$$

By defining $\Lambda_2(w_t) = \infty$ for $w_t \geq I_A$, condition (24) can be simply stated as $\lambda \geq \Lambda_2(w_t)$. Combining conditions (23) and (24) shows that the economy shifts to technology M whenever

$$\phi_M(w_t, \lambda) \geq \phi_A(w_t, \lambda) \Leftrightarrow$$
$$\lambda \geq \min\{\Lambda_1(w_t), \Lambda_2(w_t)\} \equiv \Lambda(w_t). \quad \textbf{(25)}$$

This clearly shows that the economy shifts from a labor-intensive technology to a more capital-intensive technology when financial infrastructure improves to a certain point. A typical shape of the function $\Lambda(w_t)$ is illustrated in Figure 3.

Figure 3 also depicts the regions in which credit rationing occurs. [From condition (17), credit rationing occurs whenever $\lambda < 1 - w_t/I_{j^*}$.] There is not a simple relationship between the degree of contract enforcement and the existence of credit rationing. Specifically, economies in region B_M experience credit rationing even though they have better financial infrastructure than economies in region P_A, where no such rationing occurs. Similarly, credit rationing in region B_M can be more strict than in B_A, particularly when technology M requires a larger scale of production. In other words, better financial infrastructure enables the economy to adopt more productive technologies but at the same time may cause greater inequality. Although it may appear strange, this is not particularly at odds with reality. Credit rationing is not necessarily most prevalent in the initial stages of economic development, when financial infrastructure is underdeveloped. Our model shows that such nonmonotonic behavior arises because the degree of enforcement, λ, not only affects the difficulty of obtaining credit for a given technology but also is a determinant of the economy's technology specialization.

6. DYNAMIC EFFECTS OF IMPROVED FINANCIAL INFRASTRUCTURE

In this section, we investigate the dynamic effects of financial infrastructure. It is known in the literature that the degree of credit market imperfection can differ markedly across countries although changing only slowly within countries [Li et al. (1998)]. This implies that improvements in financial infrastructure, λ, if any, must be gradual. This section examines how such gradual improvements affect the economy's income distribution and welfare.

6.1. Wage Dynamics over Generations

Until now, we have assumed as given the value of w_t, the amount of each entrepreneur's own funds. However, given that an entrepreneur's own funds are her first-period income, w_t is in fact endogenously determined by the equilibrium level of the wage in the previous period. In other words, the equilibrium wage in this period determines the amount of an entrepreneur's own funds for the next generation, which in turn affects the equilibrium wage in the next period. In this way, the equilibrium wage evolves dynamically over generations.

Recall from Proposition 2 that w_t evolves over generations according to $w_{t+1} = \theta(w_t, \lambda)$. When credit rationing is absent, i.e., when the (λ, w_t) pair is in region P_A or P_M of Figure 3, the equilibrium wage is determined by the profitability constraint: $w_{t+1} = P_A$ or P_M. When credit rationing occurs, i.e., when the (λ, w_t) pair is in region B_A or B_M, the equilibrium wage is determined by the borrowing constraint: $w_{t+1} = B_A(w_t, \lambda)$ or $B_M(w_t, \lambda)$. In the latter case, equation (22) implies that the equilibrium wage w_{t+1} is higher (or lower) than the previous period's wage w_t if financial infrastructure λ is better (or worse) than

$$B_j^*(w_t) \equiv (1 - w_t/I_j)(w_t/P_j)^{1/\widehat{\alpha}_j}. \tag{26}$$

The function $B_j^*(w_t)$ indicates the quality of financial infrastructure required for the market wage to become stationary at w_t under technology j.

Figure 4 plots the steady-state level of w_t against λ and shows how w_t moves when it is not at the steady-state level. A number of properties can be observed from this. First, for a given level of λ, there is at least one steady state. There can be multiple steady states, but the *lowest* steady state (i.e., the steady state with the lowest w_t) is always stable. This means that as long as the amount of own funds held by old agents in the initial period, w_0, is sufficiently small, the economy converges to the lowest steady state, which we denote by $w^*(\lambda)$. We assume that this is the case and that the economy always stays near the lowest steady state $w^*(\lambda)$ in the long run.

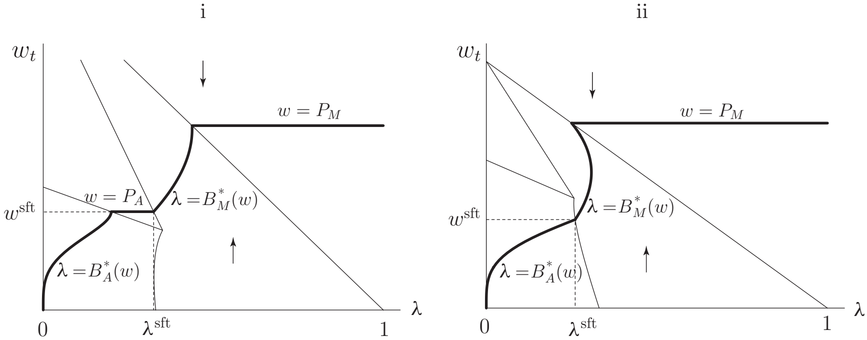

FIGURE 4. The steady-state level of the wage for different sizes of minimum investment: (i) $I_A \leq \zeta(I_M)$; (ii) $I_A > \zeta(I_M)$. Numerically calculated using the same parameter values as in Figure 3. Minimum sizes of investment are $I_A = 1.8$, $I_M = 3$ (i); $I_A = 1.5$, $I_M = 4$ (ii).

Second, the steady-state income of young agents, $w^*(\lambda)$, increases as financial infrastructure develops. This implies that the income of credit-rationed agents, $rw^*(\lambda)$, increases when financial infrastructure improves. Third, there is a threshold level of financial infrastructure, denoted by λ^{sft}, such that a technology shift occurs. More specifically, in the steady state, the economy specializes in technology $j^*(\lambda) = A$ if $\lambda < \lambda^{\text{sft}}$ and $j^*(\lambda) = M$ if $\lambda \geq \lambda^{\text{sft}}$. In particular, observe that the steady-state wage $w^*(\lambda)$ is continuous with respect to λ even at λ^{sft}. This means that a technological shift only marginally affects the income of credit-rationed agents, $rw^*(\lambda)$. In other words, if a technological shift changes the income distribution drastically, it occurs only through changes in the way in which economic rents are distributed.

Third, the precise pattern of evolution of income distribution depends on the minimum size of investment. The locus of the steady state transits region P_A, as shown by panel (i), only if I_A is below the threshold given by

$$\zeta(I_M) \equiv P_A \left(1 - (P_A/P_M)^{1/\widehat{\alpha}_M}(1 - P_A/I_M)\right)^{-1}. \qquad \textbf{(27)}$$

In this case, there are no rent earners immediately before the technological shift. However, if the minimum size of investment in the old technology exceeds $\zeta(I_M)$, credit rationing exists immediately before the technological shift, as shown by panel (ii). This means that there exist a group of agents who lose economic rents when the technological shift occurs.

6.2. Income Distribution in the Steady State

With the economy's technological specialization $j^*(\lambda)$ and the amount of own funds $w^*(\lambda)$ determined, we can characterize the income distribution of agents in the steady state as a function of the economy's financial infrastructure, λ. More specifically, we consider the distribution of consumption among old agents (which coincides with their gross income) because in this economy, only old agents are assumed to obtain utility from consumption.

When $w^*(\lambda) \geq (1-\lambda)I_{j^*(\lambda)}$, there is no credit rationing. In this case, the consumption of all old agents is $rw^*(\lambda)$. When $w^*(\lambda) < (1-\lambda)I_{j^*(\lambda)}$, credit rationing occurs, and only a limited number of agents can start projects. By substituting (2), (20), and (22) into (18), we obtain the following expression for the number of entrepreneurs in a credit-rationing steady state:

$$n^*(\lambda) \equiv \frac{\widehat{\alpha}_j P_j}{rI_j}\left(\frac{\lambda I_j}{I_j - w^*(\lambda)}\right)^{1+\widehat{\alpha}_j}, \quad \text{where } j = j^*(\lambda). \tag{28}$$

With $n^*(\lambda)$ of old agents starting projects, from (19), their earnings are

$$c^{e*}(\lambda) \equiv \frac{1-\lambda}{\lambda} r\left(I_j - w^*(\lambda)\right). \tag{29}$$

The remaining $1 - n^*(\lambda)$ agents are rationed from the credit market and end up consuming $rw^*(\lambda)$ in the steady state.

Before the effect of financial infrastructure on the income distribution and welfare is formally characterized it is illustrative to consider its effects on the economy's aggregates, such as aggregate consumption (which coincides with gross national production in our model) and the Gini coefficient. Recall that only old agents consume, and their population is normalized to unity. When there is no credit rationing, aggregate consumption, $C^*(\lambda)$, is the same as every agent's consumption, $rw^*(\lambda)$. Because there is no inequality, the consumption Gini coefficient $G^*(\lambda)$ is clearly zero. With credit rationing, aggregate consumption and the consumption Gini coefficient are, from (28) and (29),

$$C^*(\lambda) = rw^*(\lambda) + n^*(\lambda)(r/\lambda)((1-\lambda)I_{j^*(\lambda)} - w^*(\lambda)), \tag{30}$$

$$G^*(\lambda) = (1 - n^*(\lambda))(1 - rw^*(\lambda)/C^*(\lambda)). \tag{31}$$

When financial infrastructure improves (λ increases), aggregate consumption and the Gini coefficient respond as stated in the following proposition.

PROPOSITION 3.

(a) *$C^*(\lambda)$ is weakly increasing in λ for all λ;*
(b) *$G^*(\lambda)$ is weakly decreasing in λ for all λ except at $\lambda = \lambda^{sft}$.*

Proof. In the Appendix. ■

Property (a) shows that improvements in financial infrastructure facilitate economic development, in the sense that they increase aggregate consumption. In particular, the function $C^*(\lambda)$ is strictly upward-sloping when the economy faces credit rationing, and it rises discretely when a technological shift occurs. Flat segments of the function $C^*(\lambda)$ correspond to the region of λ under which no credit rationing occurs. In that case, a marginal change in λ has no effect.

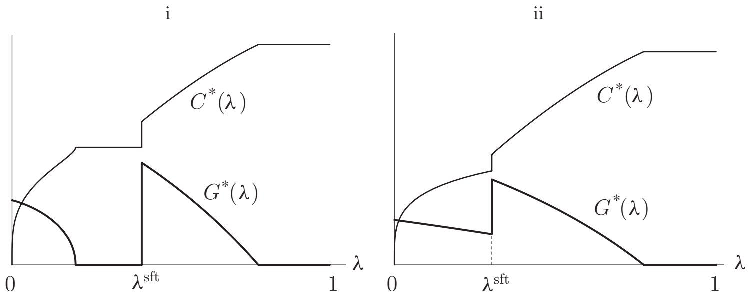

FIGURE 5. Aggregate consumption and the consumption Gini coefficient at the lowest steady state: (i) $I_A \leq \zeta(I_M)$; (ii) $I_A > \zeta(I_M)$. Parameters: $I_A = 1.5$, $I_M = 10$ (i); $I_A = 8.5$, $I_M = 10$ (ii).

Property (b) shows that better financial infrastructure reduces inequality as long as the same technology is used. However, we cannot determine its effect on inequality when a technological shift occurs. In fact, as Figure 5 shows, the Gini coefficient tends to increase when a technological shift occurs. [Observe that $G^*(\lambda)$ rises discontinuously at $\lambda = \lambda^{sft}$.] This confirms our earlier observation that the degree of inequality changes nonmonotonically when an economy develops though improvements in financial infrastructure.

6.3. Distribution of Economic Rents and Welfare Effects

This analysis has shown that without a technological shift, improvements in financial infrastructure increase aggregate income and reduce inequality, both of which are socially beneficial. However, when a technological shift occurs, the increase in aggregate income and the possible rise in inequality may have opposing effects on welfare. In the remainder of this section, we explicitly consider the overall welfare effects of improvements in financial infrastructure.

Let us define the social welfare of generation t by the average utility of all agents in generation t, as follows:[12] $U_t = \int_{i=0}^{1} u(c_{i\,t+1})di$, where $c_{i\,t+1}$ is the second-period consumption of individual i in generation t. Then, from Proposition 2 and equations (28) and (29), in the steady state, social welfare becomes

$$U^* = U^*(\lambda) \equiv \begin{cases} n^*(\lambda)u(c^{e*}(\lambda)) + (1 - n^*(\lambda))u(rw^*(\lambda)) & \text{with credit rationing} \\ u(rw^*(\lambda)) & \text{without credit rationing.} \end{cases}$$

As already observed, a marginal improvement in financial infrastructure affects income distribution and welfare differently, depending on whether it causes a technological shift and also on whether there is credit rationing. Let us consider these possible cases in turn.

Case 1: When an increase in λ does not cause a technological shift. As long as the same technology is used, an increase in λ affects the income distribution only when credit rationing occurs, i.e., only when the economy is in either region B_A or region B_M. In that case, the equilibrium wage $w^*(\lambda)$ rises (see Figure 4), which means that consumption by credit-rationed agents, $rw^*(\lambda)$, also rises. In addition, from equations (28) and (29), we observe that the number of rent earners, $n^*(\lambda)$, increases, and the income of each of them, $c^{e*}(\lambda)$, falls. Intuitively, an improved financial infrastructure enables more agents to obtain funds and thereby increases the aggregate supply of capital. In turn, an increased supply of capital raises the equilibrium wage, and therefore the incomes of credit-rationed agents. However, a rise in the equilibrium wage erodes the rate of return from the project, and as a result, the income of each entrepreneur falls.

As illustrated in Figure 1(i), these changes imply that some of the economic rents received by the initial entrepreneurs are redistributed to credit-rationed agents, some of whom have now become entrepreneurs and received rents, whereas the others also benefit from the increased labor income. Moreover, aggregate consumption, or the total pie, also increases. As a result, $U^*(\lambda)$ unambiguously improves.

Case 2: When a technological shift occurs in an economy without credit rationing. Consider the case in which there is an absence of credit rationing in the economy when λ is slightly less than λ^{sft}. Such a case occurs when the minimum size of investment for technology A satisfies $I_A \leq \zeta(I_M)$ [see Figure 4(i)]. Before the technological shift, the economy is in region P_A, and all old agents earn $rw^*(\lambda) = rP_A$. When λ is raised slightly above λ, the economy moves to region B_M, where credit rationing occurs. This creates substantial income inequality, but nonetheless, all agents are better off. To understand this, recall that the equilibrium wage $w^*(\lambda)$ does not fall following a technological shift. In fact, we have seen that $w^*(\lambda)$ is weakly increasing and continuous at $\lambda = \lambda^{\text{sft}}$. Thus, the income of credit-rationed agents, $rw^*(\lambda)$, is at least as high as the income before the shift [see Figure 1(ii)]. In addition, in region B_M, entrepreneurs earn surplus rents over $rw^*(\lambda)$, and therefore, their incomes are now discretely higher than before. Intuitively, the high productivity of the new technology allows them to earn economic rents without exploiting workers. As a result, welfare $U^*(\lambda)$ unambiguously improves.

Case 3: When a technological shift occurs in a credit-rationed economy. Now, let us consider the case in which $I_A > \zeta(I_M)$, for which a comparison must be made between income distribution in region B_A and that in region B_M [see Figure 4(ii)]. In this case, a technological shift may not make every agent better off, because some entrepreneurs adopting technology A may lose their economic rents when the economy shifts to technology M. Nonetheless, the technological shift will be welfare-improving if it distributes economic rents more widely among agents. However, the following proposition shows that this is not necessarily the case.

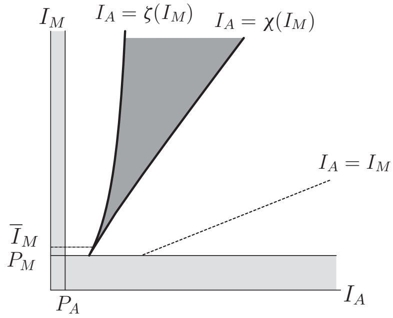

FIGURE 6. Welfare consequences of a technological shift. The number of rent earners declines discretely at the point of the technological shift if the (I_A, I_M) pair is within the dark gray area between the two thick curves. The curves are calculated numerically under parameter values of $\alpha_A = 0.20$, $\alpha_M = 0.45$, $r = 2.0$, $P_A = 1.20$, and $P_M = 2.25$.

PROPOSITION 4. *When a slight increase in λ causes a technological shift, the number of old agents who earn more than rw_t in the lowest steady state decreases if and only if $I_A \in (\zeta(I_M), \chi(I_M))$ and $I_M > \overline{I}_M$, where $\chi(I_M)$ is a continuous function satisfying $\zeta(I_M) < \chi(I_M) < I_M$ and $\overline{I}_M \equiv P_A(\widehat{\alpha}_M/\widehat{\alpha}_A - 1)/((P_M/P_A)^{1/\widehat{\alpha}_M} - 1)$.*

Proof. In the Appendix. ■

Figure 6 shows the representative shapes of the functions $\zeta(I_M)$ and $\chi(I_M)$. When the size of investment is within an intermediate range, $I_A \in (\zeta(I_M), \chi(I_M))$, the technological shift reduces the number of rent earners discretely even though it increases the amount of rent received by each entrepreneur.[13] Such a concentration of economic rents lowers welfare $U^*(\lambda)$ if agents are sufficiently risk-averse (i.e., if the utility function $u(\cdot)$ is sufficiently concave).

To see the deterioration in welfare more precisely, note that the technological shift caused by a marginal increase in λ affects the steady-state income distribution and hence welfare in three respects [see Figure 1(iii)]. First, the income of credit-rationed agents, $rw^*(\lambda)$, increases with λ. However, because $w^*(\lambda)$ changes continuously at $\lambda = \lambda^{\text{sft}}$, this change affects welfare only marginally, and therefore, it is necessarily dominated by the two discrete welfare effects arising from the concentration of economic rents, as is to be described. Second, the amount of income received by each entrepreneur, $c^{e*}(\lambda)$, rises discretely. However, the increase in the individual entrepreneur's utility can be limited because the utility function $u(\cdot)$ is concave. Thus, if the degree of risk aversion is high (i.e., when marginal utility falls sharply with consumption), the positive effect on welfare is limited. Third, the number of entrepreneurs, $n^*(\lambda)$, falls discretely, which also means a fall in the possibility that an agent can become an entrepreneur. The decrease in $n^*(\lambda)$ linearly reduces welfare $U^*(\lambda)$. Because this negative effect is linear in $n^*(\lambda)$, it dominates the second positive effect [relying on the concavity of $c^{e*}(\lambda)$] if the degree of risk aversion is sufficiently high. In this case, welfare $U^*(\lambda)$ falls discretely at $\lambda = \lambda^{\mathrm{sft}}$.

This result can also be interpreted as a certain type of *crowding-out* effect. Note that the rate of return from the capital-intensive technology responds less sensitively to changes in the market wage, because it relies less on labor than does labor-intensive technology. As a result, the borrowing constraint for technology M is less sensitive to an increase in w_{t+1} than is the borrowing constraint for technology A [see equation (22)]. Therefore, once financial infrastructure has improved to be slightly greater than λ^{sft}, entrepreneurs adopting the capital-intensive technology can attract workers by paying wage rates that are marginally higher than the wages that could be offered by entrepreneurs using the labor-intensive technology, within their respective borrowing constraints. Once the wage level is determined by the entrepreneurs using the capital-intensive technology, entrepreneurs with labor-intensive technologies can no longer satisfy their borrowing constraints and thus cannot obtain funds from the banks. In effect, they are crowded out from the credit markets.

7. CONCLUDING REMARKS

We have presented a model in which economic development is facilitated by improvements in financial infrastructure and have examined how the resultant economic development affects the distribution of income among agents in a setting in which multiple technologies are potentially available. It was shown that as long as the economy relies on the same technology, improvements in financial infrastructure gradually increase aggregate income, mitigate inequality, and hence necessarily improve social welfare. However, when the improvement in financial infrastructure reaches a certain point, the economy shifts from a labor-intensive technology to a technology that has higher capital intensity. This certainly increases aggregate income but at the same time substantially aggravates income inequality, potentially to the extent of lowering social welfare. More precisely, the technological shift causes incumbent entrepreneurs to be crowded out from the credit market and deprives them of economic rents. However, this does not generate equality because almost all the surplus from the technological shift goes not to the general public but to new entrepreneurs using the capital-intensive technology, and credit rationing limits the number of new entrepreneurs. In this framework, we have explicitly characterized the condition under which the finance-led technological shift causes the economic rents to be concentrated on fewer agents, which means that the average (or expected) utility of agents can fall following the technological shift if agents are sufficiently risk averse. Thus, this paper identifies a particular channel through which financial development increases inequality, and finds that such an increase can lower social welfare.

There are several possible directions for future research. First, a possible extension of our model is to incorporate human capital. If education is costly and requires borrowing, as in the model of Galor and Zeira (1993), an improvement in financial infrastructure will allow more agents to receive education, which directly mitigates inequality. However, as suggested in the literature on skill-biased technological change [e.g., Acemoglu (2002)], finance-led technological shifts will increase the demand for human capital and thereby widen the inequality between educated and uneducated agents. Which of these effects dominates is an open question.

Second, although we considered a small open economy, analyzing a closed economy version of the model would enable us to examine the effect of capital accumulation. In particular, if we were to incorporate both entrepreneurial investments and educational investment as explained earlier, the qualitative effect of capital accumulation on inequality might depend on the stage of development, given that accumulation may have varying effects on entrepreneurship and education. In particular, Galor and Moav (2004) showed that the relative importance of physical and human capital investment changes as the economy develops, although they did not focus on technological shift.

Third, our model could be incorporated into a politico-economic framework. Financial infrastructure, or more specifically the enforcement of financial contracts, as represented by λ, is largely determined by the efficacy of legal and accounting systems and by general law enforcement. It is a highly political issue whether or not such systems can be successfully improved. We have shown that an improvement in λ can lower social welfare when λ reaches a certain threshold ($\lambda = \lambda^{sft}$). This may be a temporary phenomenon, in the sense that if financial infrastructure continues to improve toward perfection ($\lambda \to 1$), later generations will eventually enjoy higher welfare. However, when the political process fails to take full account of the welfare gains of future generations, people in the current generation may want to keep λ below the threshold to avoid the potential welfare loss arising from greater inequality among themselves. If the necessary reforms are postponed indefinitely over generations on this basis, the economy will be trapped in a state of having poor financial infrastructure, unproductive technology, and hence low income. Although this conjecture implicitly assumes that political power is equally distributed, at least within a generation, a more detailed analysis of the political process may enable a focus on the conflict of interest among parties that have different degrees of political power (e.g., incumbent entrepreneurs and young agents).[14] These interesting extensions remain to be considered.

NOTES

1. Sylla (2002) reports, "The Dutch financial revolution had occurred by the first decades of the seventeenth century, *before* the Dutch Golden Age. . . . The British financial revolution in the late seventeenth and early eighteenth centuries, *before* the English industrial revolution. The U.S. financial revolution occurred . . . *before* the U.S. economy accelerated its growth in the 'statistical dark age' of the early nineteenth century." He also notes, "In the early Meiji era of the 1870s and 1880s, Japan had a financial revolution. . . . Once their financial revolution was in place, the Japanese were off and running." See also Dickson (1967) for similar arguments. Levine (1997, 2005) provides an extensive

survey of the role of a country's financial system in economic development. Christopoulos and Tsionas (2004) show that the causality runs from finance to economic development and not vice versa.

2. See Drazen (2000) for general discussion of the conflicting interests in economic reforms.

3. In fact, these researchers show that the reduced labor supply in the old sector increases the wages of those who remain in the old sector. Aghion and Bolton (1997) also show that the rise in inequality in the early phases of development is beneficial to the poor because it promotes capital accumulation. See Barro (2000, p. 9) for a survey of related studies.

4. Recent empirical studies suggest that financial markets promote economic development not by enhancing overall capital accumulation but by efficiently allocating capital across sectors [e.g., Wurgler (2000)]. The open economy assumption enables us to focus on the role of financial markets in determining the demand for capital and its composition rather than the supply of capital.

5. We introduce banks for expositional purposes, and for simplicity, we assume away any costs of financial intermediation for the banks. We also assume that individual agents cannot raise funds directly from the international financial market because of the possibility of default.

6. Note that a higher interest rate makes condition (4) stricter and gives borrowers more incentive to default. Thus, banks cannot make a profit (even a zero profit) by offering a loan for projects that do not satisfy (5) with an interest rate higher than r.

7. If entrepreneurs have ample funds of their own, they can adopt the most profitable technology without relying on the financial market. However, historical instances where financial markets affected economic performance imply that entrepreneurs usually have insufficient funds to self-finance their projects. Accordingly, we assume that $I_j > P_j$, where P_j is the upper bound of the first-period income when the economy specializes in technology j.

8. If the amount of own funds w_t exceeds I_j, the entrepreneur need not borrow, and therefore, there is no borrowing constraint. Formally, we define $B_j(w_t, \lambda) = \infty$ for $w_t \geq I_j$, which means that condition (10) never binds in such a case.

9. When condition (12) is satisfied, (11) implies that there must be some technology j such that $w_{t+1} < \theta(w_t, \lambda) = \min\{P_j, B_j(w_t, \lambda)\}$. It follows that $w_{t+1} < B_j(w_t, \lambda)$ and $w_{t+1} < P_j$; that is, technology j satisfies the borrowing constraint, and its rate of return is strictly larger than r. If there are more than two such technologies, the entrepreneur chooses the most profitable technology.

10. Since $L^D_{t+1}(\cdot)$ is set-valued, the notion of continuity is slightly different from that for a function. To be precise, in the Appendix, we show that $L^D_{t+1}(\cdot)$ is convex-valued, nonempty, and *upper hemicontinuous*, which implies that the graph of labor demand in (L^D_{t+1}, w_{t+1}) space is jointed.

11. To verify this, observe that $\phi_M \geq \phi_A \Leftrightarrow \min\{P_M, B_M\} \geq \phi_A \Leftrightarrow P_M \geq \phi_A$ *and* $B_M \geq \phi_A$. In the last condition, $P_M \geq \phi_A$ always holds because $P_M > P_A \geq \min\{P_A, B_A\} \equiv \phi_A$. Therefore, $\phi_M \geq \phi_A \Leftrightarrow B_M \geq \phi_A \Leftrightarrow B_M \geq \min\{P_A, B_A\} \Leftrightarrow B_M \geq P_A$ *or* $B_M \geq B_A$.

12. U_t can also be interpreted as the expected utility of young agents as of period t, when they are uncertain about whether they will obtain funds or be credit rationed in the future. Technically, $U_t = \int_{i=0}^{1} u(c_{i\,t+1})di = E_t[u(c_{i\,t+1})]$ holds, where $E_t[\cdot]$ is the expectation at the beginning of period t, when the income shock ϵ_{it} has not been revealed. Given that reform of the financial infrastructure takes a long time, it is reasonable to evaluate the desirability of a change in λ based on its effect on the young generation's expected utility.

13. Strictly speaking, Proposition 4 requires the additional condition $I_M > \overline{I}_M$. However, under reasonable parameter values for α_A, α_M, and P_M/P_A, we found that $\overline{I}$ is often close to P_M or even less than P_M. Given that $I_M > P_M$, condition $I_M > \overline{I}_M$ is usually satisfied.

14. For example, Galor and Moav (2006) and Galor et al. (2009) developed political economic models to analyze the relationship between conflicts of interest, income inequality, and economic development. Galor and Moav (2006) showed that when physical capital has accumulated to a certain level, capitalists agree to pay the costs of public schooling in order to exploit the complementarity between human and physical capital. Galor et al. (2009) introduced another production sector that

uses land rather than physical capital and showed that if political power is more concentrated among landowners than among capitalists, economic development suffers because landowners have less incentive to pay the costs of public schooling.

15. By making this technical assumption, we ignore the unlikely possibility that there is a temporary shift back to the old, labor-intensive technology as a result of improvements in financial infrastructure. Specifically, for this assumption to be violated, the functions $B_A^*(w)$ and $B_M^*(w)$ must have multiple points of intersection in the range $w \in (0, P_A)$, which we numerically found to occur only for a very narrow range of parameters.

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APPENDIX

A.1. PROOF OF PROPOSITION 1

Here, we establish the existence of the equilibrium market wage at period $t+1$, denoted simply by w, assuming as given the predetermined market wage of the previous period w_t. As explained in the text, $\min L^D_{t+1}(w; w_t) > 1$ for all $w \in (\underline{w}(\lambda), \underline{\theta}_t)$ and $L^D_{t+1}(w; w_t) = \{0\}$ for all $w > \overline{\theta}_t$. Thus, the intermediate value theorem implies the existence of $w \in [\underline{\theta}_t, \overline{\theta}_t]$ such that $L^D_{t+1}(w; w_t) \ni 1$ if the graph of $L^D_{t+1}(w; w_t)$ is jointed for all $[\underline{\Theta}, \overline{\Theta}]$, where $\underline{\Theta}$ and $\overline{\Theta}$ are arbitrary constants with $\underline{\Theta} \in (\underline{w}(\lambda), \underline{\theta}_t)$ and $\overline{\Theta} > \overline{\theta}$ [note that we can always choose such constants since $\overline{\theta}$ is finite and $\underline{\theta}_t > \underline{w}(\lambda)$ from $w_t > 0$]. For this, it is sufficient to show that $L^D_{t+1}(w; w_t)$ is convex-valued, nonempty, and upper hemicontinuous (hereafter, u.h.c.) for all $w \in [\underline{\Theta}, \overline{\Theta}]$.

From the definition that $L^D_{t+1}(w; w_t)$ is an aggregation of individual labor demand, its nonemptiness and convexity are obvious; there is a continuum of agents, and the set of most preferred actions of each agent is convex-valued and nonempty and does not depend on the action of others given w. It is also easy to see that the set $L^D_{t+1}(w; w_t)$ is compact. For any $w \in [\underline{\Theta}, \overline{\Theta}]$, condition (6) implies that the size of investment of each project is finite. Then, there exists a finite upper bound in labor demand $\overline{L}$ in $L^D_{t+1}(w; w_t)$, from (2). Since $L^D_{t+1}(w; w_t)$ is closed by construction, it is compact.

According to the definition of Stokey and Lucas (1989, p. 56), a compact-valued correspondence $L_{t+1}^D : [\underline{\Theta}, \overline{\Theta}] \to [0, \overline{L}]$ is u.h.c. at w^*, if, for every sequence $\{w_n\}$ such that $w_n \in [\underline{\Theta}, \overline{\Theta}]$ and $w_n \to w^*$, and for every sequence $\{L_n\}$ such that $L_n \in L_{t+1}^D(w_n; w_t)$, there exists a convergent subsequence of $\{L_n\}$ whose limit point L^* is in $L_{t+1}^D(w^*; w_t)$. To show that $L_{t+1}^D(\cdot; w_t)$ is u.h.c., fix $w^* \in [\underline{\Theta}, \overline{\Theta}]$, and pick any arbitrary sequences $\{w_n\}$ and $\{L_n\}$ such that $w_n \in [\underline{\Theta}, \overline{\Theta}]$, $w_n \to w^*$, and $L_n \in L_{t+1}^D(w_n; w_t)$.

If $\bigcup_{n=1}^{\infty} w_n$ is finite, there must be some $N > 0$ such that $w_n = w^*$ for all $n \geq N$. Then, since $L_n \in L_{t+1}^D(w^*; w_t)$ for all $n \geq N$ and $L_{t+1}^D(w^*; w_t)$ is compact, there is a convergent subsequence of $\{L_n\}$ whose limit point L^* is in $L_{t+1}^D(w^*; w_t)$. Therefore, the conditions for u.h.c. are satisfied.

The remaining case is the one in which $\bigcup_{n=1}^{\infty} w_n$ is infinite. Note that, since the $\rho_j(w)$s intersect with each other only a finite number of times, we can choose a subsequence of $\{w_n\}$ such that each element in the set $\{\rho_j(w_n)\}_{j\in\mathcal{J}} \cup r$ has a distinct value, i.e., such that there is always a strict ordering of profitability among technologies as well as between investment and saving. Because the set of technologies $\mathcal{J}$ is finite, the number of patterns in the ordering of profitability that possibly appear in that subsequence is also finite. Therefore, there is at least one pattern of the strict ordering of profitability that appears an infinite number of times in sequence $\{w_n\}$, from which we can construct a subsequence $w_{n_k} \to w^*$ such that

$$\rho_1(w_{n_k}) < \cdots < \rho_{\widehat{j}-1}(w_{n_k}) < r < \rho_{\widehat{j}}(w_{n_k}) < \cdots < \rho_J(w_{n_k}) \text{ for all } k, \tag{A.1}$$

where each of the available technologies is numbered in ascending order of profitability. In this particular ordering, J is the number of technologies and $\widehat{j}$ is the index of the least profitable technology whose rate of return is higher than that on saving.

Let us derive $L_{t+1}^D(w_{n_k}; w_t)$ using (A.1). Note that, from (5) and (7), it is optimal for each agent to invest in the most profitable technology (that with the largest index) under his specific borrowing constraint $w_t + \epsilon_{it} \geq \eta_j(w_{n_k}, \lambda)$ as long as there is such a technology greater than $\widehat{j}$. More specifically, he adopts technology j if and only if the amount of his own funds is within the range $w_t + \epsilon_{it} \in W_j(w_{n_k})$, where $W_j(w_{n_k})$ is defined recursively for $j = J, J-1, \ldots, \widehat{j}$ by

$$W_j(w_{n_k}) \equiv \begin{cases} [\eta_J(w_{n_k}, \lambda), \infty) & \text{for } j = J; \\ [\eta_j(w_{n_k}, \lambda), \infty) \setminus W_{j+1}(w_{n_k}) & \text{for } j = J-1, \cdots, \widehat{j}. \end{cases}$$

From (10) and (14), an entrepreneur with his own funds $w_t+\epsilon_{it}$ demands $\widetilde{\ell}_j(w_{n_k})(w_t + \epsilon_{it})/(1 - \lambda\rho_j(w_{n_k})/r)$ units of labor whenever $w_t + \epsilon_{it} \in W_j(w_{n_k})$ for some $j \geq \widehat{j}$. Recall that $\widetilde{\ell}_j(w_{n_k}) \equiv f_j'^{-1}(w_{n_k})$, which means that the individual labor demand is a value rather than a set. Since ϵ_{it} is distributed uniformly between 0 and $\overline{\epsilon}$, aggregate labor demand is given by

$$\sum_{j=\widehat{j}}^{J} \int_{w_t+\epsilon\in E_j(w_{n_k}; w_t)} \frac{(w_t + \epsilon)\widetilde{\ell}_j(w_{n_k})}{1 - \lambda\rho_j(w_{n_k})/r} \frac{d\epsilon}{\overline{\epsilon}} \equiv \widetilde{L}_{t+1}(w_{n_k}; w_t), \tag{A.2}$$

where the set $E_j(w_{n_k}; w_t) \equiv \{\epsilon \in [0, \overline{\epsilon}] | \, w_t + \epsilon \in W_j(w_{n_k})\}$ represents the range of random incomes for which technology j is chosen. Note that $\widetilde{L}_{t+1}(w_{n_k}; w_t)$ is a function and, therefore, set $L_{t+1}^D(w_{n_k}; w_t)$ has only one element. The only choice of sequence $\{L_{n_k}\}$ is such that $L_{n_k} = \widetilde{L}_{t+1}(w_{n_k}; w_t)$ for all k. When viewed as a correspondence, $E_j(w; w_t)$ is well defined and continuous for all $w \in [\underline{\Theta}, \overline{\Theta}]$. From (A.2), the function $\widetilde{L}_{t+1}(w; w_t)$ is also well defined and continuous for all $w \in [\underline{\Theta}, \overline{\Theta}]$. Thus, since $w_{n_k} \in [\underline{\Theta}, \overline{\Theta}]$ converges to w^*, $L_{n_k} = \widetilde{L}_{t+1}(w_{n_k}; w_t)$ converges to $L^* \equiv \widetilde{L}_{t+1}(w^*; w_t)$.

The final task is to show that $L^* \in L^D_{t+1}(w^*; w_t)$. Consider the relative profitability of each technology when the market wage is given by w^*. Since $\rho_j(w)$ is continuous, taking the limit $w_{n_k} \to w^*$ in (A.1) implies that

$$\rho_1(w^*) \leq \cdots \leq \rho_{\widehat{j}-1}(w^*) \leq r \leq \rho_{\widehat{j}}(w^*) \leq \cdots \leq \rho_J(w^*). \qquad \textbf{(A.3)}$$

In the limit, we have only a weak ordering of profitability. Nonetheless, agents with their own funds $\epsilon_{it} \in W_j(w^*)$ for some $j \geq \widehat{j}$ find it at least weakly optimal to choose technology j, whereas other agents find it at least weakly optimal to save. Thus, $L^D_{t+1}(w^*; w_t) \ni \widetilde{L}_{t+1}(w^*; w_t) = L^*$. This establishes that $L^D_{t+1}(w; w_t)$ is u.h.c. at $w = w^*$. Since $w^* \in [\underline{\Theta}, \overline{\Theta}]$ is arbitrary, the correspondence $L^D_{t+1}(w; w_t)$ is u.h.c. for all $w \in [\underline{\Theta}, \overline{\Theta}]$. This completes the proof. ■

A.2. PROOF OF PROPOSITION 2

Proof of Parts (a) and (b). These properties are directly obtained by taking the limit $\overline{\epsilon} \to 0$ in Proposition 1.

Proof of Part (c). Given technology j^*, credit rationing occurs if the borrowing constraint (10) is stronger than the profitability constraint (8). A comparison of these conditions gives $B_{j^*}(w_t, \lambda) < P_{j^*} \Leftrightarrow w_t < (1 - \lambda)I_{j^*}$.

Proof of Part (d). In what follows, the income distribution when credit rationing occurs is derived. Note that, from the continuity of the function $B_j(\cdot)$ for all j, we can choose a sufficiently small $\overline{\epsilon} > 0$ so that $\theta(w_t + \epsilon_{it}, \lambda) = B_{j^*}(w_t + \epsilon_{it}, \lambda)$ for all $\epsilon_{it} \in [0, \overline{\epsilon}]$. Then Proposition 1 states that $w_{t+1} \in [B_{j^*}(w_t, \lambda), B_{j^*}(w_t + \overline{\epsilon}, \lambda)]$ or, equivalently, $\eta_{j^*}(w_{t+1}, \lambda) \in [w_t, w_t + \overline{\epsilon}]$ from (9). Now, consider the limiting case in which the degree of heterogeneity $\overline{\epsilon}$ is infinitesimally small ($\overline{\epsilon} \to 0$). In the limit, the previous relationships indicate that $w_{t+1} \to B_{j^*}(w_t, \lambda)$ and $\eta_{j^*}(w_{t+1}, \lambda) \to w_t$. Applying these results to (14) shows that the limiting value of labor demand by any entrepreneur is $I_{j^*}\widetilde{\ell}_{j^*}(B_{j^*}(w_t, \lambda))$. Since total labor demand should be unity in equilibrium, it follows that the number of entrepreneurs in the limit is $(I_{j^*}\widetilde{\ell}_{j^*}(B_{j^*}(w_t, \lambda)))^{-1}$, which is smaller than the number of agents (i.e., unity) under Assumption 1. Similarly, taking the limit in (15) and eliminating $B_{j^*}(\cdot)$ by (10) shows that the limiting value of the consumption of entrepreneurs is $rw_t + r((1 - \lambda)I_{j^*} - w_t)/\lambda$, where the second term is positive since $w_t < (1 - \lambda)I_{j^*}$ from part c. ■

A.3. LEMMA 1 AND PROOF

In this Appendix, we formally establish the four properties discussed in Section 6.1. Recall that we have assumed $P_M > P_A$. In what follows, we assume a slightly stronger version, $P_M/P_A \geq 1/(1 - \alpha_M)$, in order to reduce the number of cases to be analyzed without affecting the main findings.

LEMMA 1. *Let $j^*(\lambda)$ and $w^*(\lambda)$ denote the choice of technology and the wage rate, respectively, at the lowest steady state. Suppose that $P_M/P_A \geq 1/(1 - \alpha_M)$ and that*

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increases in λ do not cause $j^*(\lambda) \in \{A, M\}$ to change more than twice.[15] *Then the following properties hold.*

(a) *For any $\lambda \in (0, 1)$, there exists a $w^*(\lambda) \in (0, P_M]$. In addition, if $w_t \leq w^*(\lambda)$ for some t, then w_t converges to $w^*(\lambda)$.*
(b) *There exists a $\lambda^{\text{sft}} \in (0, 1)$ such that $j^*(\lambda) = A$ if $\lambda < \lambda^{\text{sft}}$ and $j^*(\lambda) = M$ if $\lambda > \lambda^{\text{sft}}$.*
(c) *$w^*(\lambda)$ is weakly increasing in λ; in addition, it is continuous at $\lambda = \lambda^{\text{sft}}$.*
(d) *$w^*(\lambda^{\text{sft}}) < P_A$ whenever $I_A > \zeta(I_M)$.*

Proof of Part (a). As shown by Figure 2, $0 < \underline{w}(\lambda) \leq \theta(w, \lambda) \leq P_M$ for all $w \geq 0$ from the definition of the function $\theta(\cdot, \lambda)$ in (12) and from the assumption that $P_M > P_A$. Therefore, $\theta(0, \lambda) - 0 > 0 \geq \theta(P_M, \lambda) - P_M$. Since $\theta(w, \lambda) - w$ is continuous with respect to w, the intermediate value theorem shows that there is at least one $w \in (0, P_M]$ such that $\theta(w, \lambda) - w = 0$, the smallest of which is denoted by $w^*(\lambda)$.

Note that $\theta(w, \lambda) - w > 0$ for all $w \in [0, w^*(\lambda))$. Suppose that $w_t \in [0, w^*(\lambda))$. Then w_t follows $w_{t+1} = \theta(w, \lambda) > w_t$ and thus gets higher overtime. In addition, since $\theta(w, \lambda)$ is weakly increasing in w, $w_{t+1} = \theta(w_t, \lambda) \leq \theta(w^*(\lambda), \lambda) = w^*(\lambda)$, which means that w_t never exceeds $w^*(\lambda)$. Therefore, w_t converges to $w^*(\lambda)$ whenever $w_t \leq w^*(\lambda)$ for some t. ■

Proof of Part (b). Define $h(\lambda) \equiv \Lambda(w^*(\lambda)) - \lambda$. Then (25) implies that $j^*(\lambda) = A$ if $h(\lambda) > 0$ and $j^*(\lambda) = M$ if $h(\lambda) < 0$. From (23), (24), (25), and $P_M > P_A$, it follows that $\Lambda(0) = \Lambda_2(0) = (P_A/P_M)^{1/(\widehat{\alpha}_M - \widehat{\alpha}_A)} > 0$ and that $\Lambda(w) \leq \Lambda_1(w) \leq (P_A/P_M)^{1/\widehat{\alpha}_M} < 1$ for any $w > 0$. Using these results, we first show that $j^*(\lambda) = A$ if λ is sufficiently small. From (22), as $\lambda \to 0$, $B_j(w, \lambda) \to 0$ for every j and w and, therefore, $\theta(w, \lambda) \to 0$ for all w from (12). Then, $w^*(\lambda) = \theta(w^*(\lambda), \lambda) \to 0$. Therefore, $\lim_{\lambda \to 0} h(\lambda) = \Lambda(0) - 0 = (P_A/P_M)^{1/(\widehat{\alpha}_M - \widehat{\alpha}_A)} > 0$, which means that $\lim_{\lambda \to 0} j^*(\lambda) = A$. Conversely, $\lim_{\lambda \to 1} h(\lambda) = \Lambda(1) - 1 \leq (P_A/P_M)^{1/\widehat{\alpha}_M} - 1 < 0$, which implies that $\lim_{\lambda \to 1} j^*(\lambda) = M$. Since we are considering the situation in which $j^*(\lambda)$ does not change more than twice, it immediately follows that there is a unique threshold $\lambda^{\text{sft}} \in (0, 1)$ such that $j^*(\lambda) = A$ if $\lambda < \lambda^{\text{sft}}$ and $j^*(\lambda) = M$ if $\lambda > \lambda^{\text{sft}}$. ■

Proof of Part (c). Choose arbitrary $\lambda_1, \lambda_2 \in (0, 1)$ such that $\lambda_1 < \lambda_2$, and suppose that $w^*(\lambda_1) > w^*(\lambda_2)$. Since $w < \theta(w, \lambda_1)$ for all $w < w^*(\lambda_1)$, as shown in part a, we have $w^*(\lambda_2) < \theta(w^*(\lambda_2), \lambda_1)$. Recall that $\theta(w, \lambda)$ is weakly increasing with respect to λ, as can be confirmed from its definition (12). Then, $\theta(w^*(\lambda_2), \lambda_1) \leq \theta(w^*(\lambda_2), \lambda_2) = w^*(\lambda_2)$. Combining the previous two results yields $w^*(\lambda_2) < w^*(\lambda_2)$, which is a contradiction. Therefore, $w^*(\lambda)$ is weakly increasing.

For later use, we prove that if the B_M curve crosses the 45° line at $w \leq P_A$, then its slope at the intersecting point is less than unity. Differentiating $B_M(w, \lambda)$ with respect to w and equating $B_M(w_t, \lambda)$ with w shows that the gradient is less than unity whenever $w < (1 - \alpha_M)I_M$. Since the parameters satisfy $I_M > P_M$ and $P_M \geq P_A/(1 - \alpha_M)$ (as assumed at the beginning of this Appendix), this property holds for all $w \leq P_A \leq (1 - \alpha_M)P_M < (1 - \alpha_M)I_M$.

Using this property, now we prove the continuity of $w^*(\lambda)$ at λ^{sft}. Suppose that contrary to our claim, $w^*(\lambda)$ increases discretely at λ^{sft}. Since the θ curve shifts upward continuously with λ, such a jump means that the θ curve is tangential to the 45° line at $w^*(\lambda^{\text{sft}})$ when

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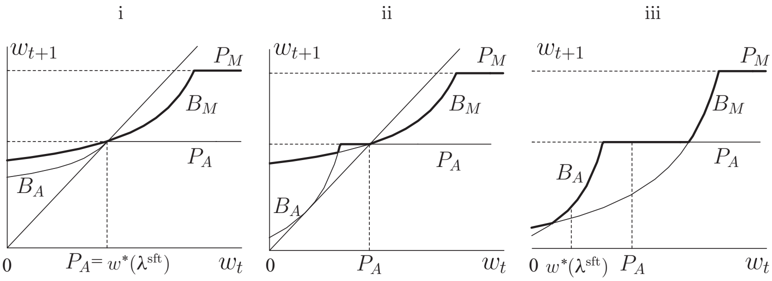

FIGURE A.1. Contradictions.

$\lambda = \lambda^{sft}$. Note that $w^*(\lambda^{sft}) \in (0, P_A]$ is implied by the continuity of the θ curve with respect to w and λ. In addition, $w^*(\lambda^{sft})$ cannot be P_A since it is impossible for the θ curve to be tangent to the 45° line at P_A given that the slope of the B_M curve is less than unity on the 45° line [see Figure A.1(i)]. Therefore, $w^*(\lambda^{sft}) \in (0, P_A)$. This implies that $B_M(P_A, \lambda^{sft})$ must be smaller than P_A since, otherwise, the B_M curve would be above the 45° line for all $(0, P_A)$, and then so would be the θ curve, which contradicts the premise that $w^*(\lambda) \in (0, P_A)$ is a steady state [see Figure A.1(ii)]. However, if $B_M(P_A, \lambda^{sft}) < P_A$, then $\theta(P_A, \lambda^{sft}) = P_A$, which means that P_A is the second smallest steady state [see Figure A.1(iii)]. In that case, a slight increase in λ lets $w^*(\lambda)$ jump to P_A, where $j^*(\lambda) = A$ still holds. This contradicts the definition of λ^{sft}, proving that the jump assumed at the beginning cannot occur. ■

Proof of Part (d). Let us first consider the timing of the resolution of the borrowing constraint under technology A, by tentatively assuming that only technology A is available. Similarly to the first half of the proof of part (c), it can be shown that $w^*(\lambda)$ is weakly increasing in λ. Thus, there is a unique $\widehat{\lambda} \in (0, 1)$ such that

$$w^*(\lambda) < P_A \text{ for all } \lambda < \widehat{\lambda}; \quad w^*(\widehat{\lambda}) = P_A. \tag{A.4}$$

This means that the borrowing constraint for technology A is resolved in the steady state when $\lambda = \widehat{\lambda}$. Note, from property (c) of Proposition 2, that $\widehat{\lambda} \geq 1 - P_A/I_A$ holds.

Now we show that the economy shifts to technology M before λ reaches $\widehat{\lambda}$, under the assumption that $I_A > \zeta(I_M)$. Rearrangement using (24) shows that the condition $I_A > \zeta(I_M)$ is equivalent to $\Lambda_2(P_A) < 1 - P_A/I_A$. From the definition of $\Lambda(\cdot)$ in (25) and the property of $\widehat{\lambda}$ obtained earlier, it follows that $\Lambda(P_A) \leq \Lambda_2(P_A) < 1 - P_A/I_A \leq \widehat{\lambda}$. We have seen earlier that if the economy has not shifted to technology M, the steady state when $\lambda = \widehat{\lambda}$ is $w_t = w_{t+1} = P_A$. However, the derived inequality $\Lambda(P_A) \leq \widehat{\lambda}$ means that the economy has already shifted to technology M when $\lambda = \widehat{\lambda}$ and $w_t = P_A$. This means that the economy shifts to technology M before credit rationing is resolved. Therefore, $w^*(\lambda^{sft}) < P_A$. ■

A.4. PROOF OF PROPOSITION 3

Proof of Part (a). Recall that $\lambda = B_j^*(w)$ holds in any steady state with credit rationing, where $B_j^*(w)$ is defined in (26). Using this to eliminate λ from (28) and (30) shows that the

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number of entrepreneurs and the aggregate consumption can be expressed as the following functions of the steady state wage w and technology j:

$$N^{ss}(j, w) \equiv \widehat{\alpha}_j w^{1/\widehat{\alpha}_j} \left(r I_j P_j^{1/\widehat{\alpha}_j}\right)^{-1}, \quad \text{(A.5)}$$

$$C^{ss}(j, w) \equiv r w + \widehat{\alpha}_j w\left(1 - (w/P_j)^{1/\widehat{\alpha}_j}\right). \quad \text{(A.6)}$$

Although this is derived by assuming that there is credit rationing in the economy, (A.6) still gives the appropriate level of aggregate consumption even when there is no credit rationing; i.e., $C^{ss}(j, P_j) = r P_j$. Therefore, $C^*(\lambda) = C^{ss}(j^*(\lambda), w^*(\lambda))$ holds for all λ.

Suppose that λ is less than λ^{sft}, in which case, $j^*(\lambda) = A$, from Lemma 1. The slope of $C^*(\lambda)$ is then given by

$$\frac{dC^*(\lambda)}{d\lambda} = \frac{dC^{ss}(A, w^*(\lambda))}{dw} \cdot \frac{dw^*(\lambda)}{d\lambda}. \quad \text{(A.7)}$$

Differentiating (A.6) shows that the first term on the RHS is $r + \widehat{\alpha}_A - (1+\widehat{\alpha}_A)(w^*(\lambda)/P_A)^{1/\widehat{\alpha}_A} \geq 0$, where the inequality follows from $r \geq 1$ and $w^*(\lambda) \leq P_A$. Lemma 1 shows that the second term in (A.7) is also nonnegative. Therefore, $C^*(\lambda)$ is weakly upward-sloping for all $\lambda \in (0, \lambda^{\text{sft}})$. The same argument applies when $\lambda \in [\lambda^{\text{sft}}, 1)$, except that A should be replaced by M, which proves that $C^*(\lambda)$ is weakly upward-sloping in that range as well. The remaining task is to prove that $C^*(\lambda)$ does not decrease at the threshold λ^{sft}. Applying $\widehat{\alpha}_A < \widehat{\alpha}_M$ and $P_A < P_M$ to the definition of $C^{ss}(j, w)$ in (A.6) shows that $C^{ss}(A, w) \leq C^{ss}(M, w)$ for any w. In addition, $w^*(\lambda)$ is continuous at the threshold. Therefore, the left-hand limit $\lim_{\lambda \to \lambda^{\text{sft}} - 0} C^*(\lambda) = C^{ss}(A, w^*(\lambda^{\text{sft}}))$ is lower than $C^*(\lambda^{\text{sft}}) = C^{ss}(M, w^*(\lambda^{\text{sft}}))$. ■

Proof of Part (b). Substituting (A.5) and (A.6) for (31) gives the consumption Gini coefficient in the steady state as $G(j, w, B_j^*(w)) = (1 - N^{ss}(j, w))(1 - rw/C^{ss}(j, w)) \equiv G^{ss}(j, w)$. Note that $N^{ss}(j, w)$ is increasing in w and that $rw/C^{ss}(j, w) = [1 + (\widehat{\alpha}_j/r)(1 - (w/P_j)^{1/\widehat{\alpha}_j})]^{-1}$ is decreasing in w, which together imply that $G^{ss}(j, w)$ is decreasing in w. When $\lambda \in (0, \lambda^{\text{sft}})$, $G^*(\lambda) = G^{ss}(A, w^*(\lambda))$, from Lemma 1. It is weakly decreasing in λ since a rise in λ weakly increases $w^*(\lambda)$, from Lemma 1, which weakly decreases $G^{ss}(j, w^*(\lambda))$, as shown previously. The same argument applies to the case of $\lambda \in [\lambda^{\text{sft}}, 1)$, except that A should be replaced by M. ■

A.5. PROOF OF PROPOSITION 4

If $I_A \leq \zeta(I_M)$, there is no credit rationing and there are no rent earners before the technological shift occurs. Therefore, it is sufficient to consider only the case $I_A > \zeta(I_M)$, when the economy is credit-constrained both before and after the technological shift. Then we can use (A.5) and express the steady state number of entrepreneurs at levels of enforcement slightly below and above the threshold as $N^{ss}(A, w^{\text{sft}})$ and $N^{ss}(M, w^{\text{sft}})$, respectively, where $w^{\text{sft}} \equiv w^*(\lambda^{\text{sft}})$ denotes the steady state wage at the threshold.

Let Q denote the ratio of these two numbers. Using (A.5),

$$Q \equiv \frac{N^{ss}(M, w^{\text{sft}})}{N^{ss}(A, w^{\text{sft}})} = \frac{\widehat{\alpha}_M}{\widehat{\alpha}_A} \frac{I_A (w^{\text{sft}}/P_A)^{1/\widehat{\alpha}_A}}{I_M (w^{\text{sft}}/P_M)^{1/\widehat{\alpha}_M}}. \quad \text{(A.8)}$$

Our concern is whether $Q \geq 1$ or $Q < 1$. Note that the continuity of the steady state wage at the threshold means that $B_A^*(w^{\rm sft}) = \lambda^{\rm sft} = B_M^*(w^{\rm sft})$, where $B_j^*(\cdot)$ is given by (26). Using this relationship, (A.8) can be simplified to

$$Q = \frac{\widehat{\alpha}_M}{\widehat{\alpha}_A} \frac{I_A - w^{\rm sft}}{I_M - w^{\rm sft}}. \tag{A.9}$$

From the assumptions $I_j > P_j$ and $P_M > P_A$, it follows that both $I_A - w^{\rm sft}$ and $I_M - w^{\rm sft}$ are positive, guaranteeing that $Q > 0$. Moreover, it is implied that $Q > 1$ whenever $I_A \geq I_M$ (recall that $\widehat{\alpha}_A < \widehat{\alpha}_M$).

Let us examine how Q responds to changes in I_A when $I_A < I_M$. Differentiating (A.9) with respect to I_A gives

$$\frac{dQ}{dI_A} = \frac{\widehat{\alpha}_M}{\widehat{\alpha}_A (I_M - w^{\rm sft})^2} \left[(I_M - w^{\rm sft}) - (I_M - I_A) \frac{dw^{\rm sft}}{dI_A} \right], \tag{A.10}$$

the sign of which depends on that of $dw^{\rm sft}/dI_A$. Note that the functions $B_A^*(w)$ and $\Lambda(w)$ intersect at the point $(\lambda^{\rm sft}, w^{\rm sft})$, as shown by Figure 4(ii). Equations (26) and (25) show that with an increase in I_A, $B_A^*(w)$ shifts to the right, whereas $\Lambda(w)$ shifts to the left, pushing the intersecting point downward. This means that $dw^{\rm sft}/dI_A < 0$ and, therefore, $dQ/dI_A > 0$ from (A.10).

We confirmed that $Q > 1$ when $I_A = I_M$ and that it gradually decreases as I_A falls for all $I_A > \zeta(I_M)$. If $\lim_{I_A \to \zeta(I_M)} Q < 1$, the intermediate value theorem shows that there exists a value of I_A below which $Q < 1$ holds. We now calculate this limiting value. From the definition $\zeta(I_M)$ and the continuity of $w^{\rm sft}$ with respect to I_A, observe that $w^{\rm sft} \to P_A$ when $I_A \to \zeta(I_M)$ (i.e., the point of technological shift approaches region P_A, where $w = P_A$). By substituting this into $Q = N^{ss}(M, w^{\rm sft})/N^{ss}(A, w^{\rm sft})$ and using the definition of $\zeta(I_M)$ in (27), we can show that

$$\lim_{I_A \to \zeta(I_M)} Q = \frac{\widehat{\alpha}_M}{\widehat{\alpha}_A} \left(\frac{P_A}{P_M} \right)^{1/\widehat{\alpha}_M} \frac{\zeta(I_M)}{I_M} \lesseqgtr 1 \;\Leftrightarrow\; I_M \gtreqless \overline{I}_M.$$

Therefore, since $I_M > \overline{I}_M$, there exists a $\chi(I_M) \in (\zeta(I_M), I_M)$ such that $Q < 1$ for $I_A \in (\zeta(I_M), \chi(I_M))$. Finally, since Q is continuous with respect to I_M from (A.9), the implicit function theorem guarantees that the value of I_A at which $Q = 1$ changes continuously with respect to $I_M > \overline{I}_M$ and approaches $\zeta(I_M)$ as $I_M \to \overline{I}_M$. This indicates the continuity of the function $\chi(I_M)$. ■