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FERTILITY, HUMAN CAPITAL, AND INCOME: THE EFFECTS OF CHINA'S ONE-CHILD POLICY

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This paper studies the effects of China's one-child policy on human capital and income. I build and calibrate a quantitative OLG model with intergenerational transfers. The model generates a quantity–quality trade-off, so a restriction on fertility leads to an increase in human capital, and higher human capital then contributes to higher individual income and welfare. Calibrating the model to match survey data on urban households, I find that the one-child policy increases the human capital of affected agents by about 47% relative to a counterfactual with no fertility restrictions. However, the effect on aggregate income is negative as the size of the labor force falls.

Keywords: One-Child Policy, Fertility, Human Capital

1. INTRODUCTION

After 35 years of mandatory fertility restrictions, China ended its controversial one-child policy at the end of 2015. As the first generation born under the one-child policy are still young adults, its repercussions are yet to fully unfold. The policy was instituted with the dual purposes of curbing population growth and promoting modernization. Fertility did certainly fall, but the policy's effect on growth remains unclear. Recent deceleration in China's GDP growth has led to concerns over the policy's effect on the labor force. For example, the New York Times article Myers et al. (2019) 'China's Looming Crisis: A Shrinking Population'. This paper formally investigates the effect of China's one-child policy on human capital, income, and welfare.

Fertility interventions in China began with propaganda campaigns in the 1970s. In 1979, these were followed by a strict one child limit in urban areas. As fertility declined, education expenditure in single child families increased significantly. Data from the Urban Household Survey (UHS) show that, in 1992, less than 5% of total household wage income was spent on education for a 20-year-old adult child.[1] In 2002, this share had reached about 15%. Such an increase in education

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spending, if translated into higher human capital, has the potential to increase individual and aggregate income. This paper examines these issues in a general equilibrium model with endogenous fertility.

I first describe a three-period OLG model with intergenerational transfers. In this model, agents optimally choose fertility and investment in their children's human capital. They not only derive utility from having children but also receive transfers in old age from their adult children. These transfers are determined by the transfer function of Choukhmane et al. (2014).[2] The transfer received is increasing in both fertility and the human capital level of children. This model embodies a quantity–quality trade-off. In the presence of a binding constraint on fertility, parents choose the maximum permitted level of fertility and increase their investment in children's human capital. The quantity–quality trade-off has been theorized since Becker and Lewis (1973) and Becker and Tomes (1976). In Becker and Lewis (1973) and Becker and Tomes (1976), the trade-off is driven by intergenerational altruism, whereas here it is driven by the intergenerational transfer.

A binding fertility restriction results in higher individual human capital, but reduces the size of the future labor force. It is thus unclear whether aggregate income will rise or fall. A quantitative assessment is required. I conduct this assessment using an extended quantitative OLG model. I calibrate the model to match moments from the UHS and compare the outcome under fertility restrictions to a counterfactual with no such restrictions. To capture the decline in fertility prior to the implementation of the strict one-child policy, I impose a fertility policy consisting of a 2.3 child limit from 1971 to 1975, a 1.6 child limit from 1976 to 1980, and a strict one-child policy from 1981 onward.[3] In addition, I feed an exogenous time-varying education cost into the model.

When calibrated to match moments from the UHS, the model generates a counterfactual decline in fertility from about 1.5 (three children per family) in the 1960s to 0.98 (just below two children per family) in the 2010s. Given that I abstract from other factors that may decrease fertility, such as housing price increases, it is safe to conclude that, on average, the current two-children policy is not binding for urban households.

The calibrated model also predicts that human capital is 47% higher for generations born under the one-child policy than their equivalents in the counterfactual. This finding is in line with other macroeconomic studies of China's one-child policy, despite different measures being used. For example, Liao (2013) used the share of skilled workers, and Zhu et al. (2014) used years of schooling. The estimate from this paper is, however, large relative to the empirical literature. For example, using data on twins, Rosenzweig and Zhang (2009) found that the one-child policy had, at most, a moderately positive effect on human capital. Li and Zhang (2017) also found a modest effect using prefecture level data. A possible explanation for the difference is that empirical studies omit general equilibrium effects. To show this, I decompose my estimate into a fertility effect and a general equilibrium effect. The fertility effect is the change in human capital when fertility declines and the interest rate is held constant. The general equilibrium effect captures the change in interest rates. In my estimates, the general equilibrium effect accounts for half of the increase. Banerjee et al. (2014) also emphasized the importance of the general equilibrium effect, but they focus on the relationship between fertility policy and savings.

Given the positive effect of fertility restriction on human capital, it is not surprising that GDP per worker is higher. The model is able to generate a time series for GDP per capita that closely matches urban GDP per capita. As human capital is a crucial determinant of GDP per capita, accuracy in predicting GDP per capita indicates accuracy in predicting human capital.[4]

The quantitative model also allows me to study welfare effects. As the fertility restriction is a binding policy, it is immediate that it lowers the welfare of generations giving birth during the policy implementation. However, for generations born under the policy, higher human capital and a higher physical capital to labor ratio increase their welfare. However, their fertility choices are also restricted, they transfer a larger share of their income to their parents, and they receive a smaller transfer from their children. Quantitatively, the model predicts that these agents have higher welfare. Similar results are found in Liao (2013), despite the mechanism being slightly different. In particular, Liao (2013) omitted intergenerational transfers, but emphasises the role of the higher capital to labor ratio.

This paper also explores the effect of the one-child policy on aggregate variables. When the model is calibrated to match moments for urban households, aggregate GDP is lower than the counterfactual from 1995 to 2000 due to the labor force being smaller. Similar results are obtained when the model is calibrated to match the whole of China. This result is in contrast to Zhu et al. (2014), who found that the overall effect is positive as the increase in human capital overturns the decline in the size of the labor force. However, their analysis omits the fertility decision and compares a calibrated model to three assumed counterfactual scenarios. In this paper, fertility is endogenous, and the counter factual is the equilibrium outcome with no fertility restrictions.[5] The prediction of this paper echoes the recent deceleration in China's growth and also the discussion of fertility and growth in general.[6]

In a summary, I make two contributions to the literature. First, in relation to the literature on fertility and human capital, I emphasize the effect of fertility restrictions on human capital through the general equilibrium channel. This helps reconcile the sizeable effects of fertility restrictions found in macro models with the modest effects found in the empirical literature. Second, this paper is the first to study the effect of one-child policy on aggregate GDP in a framework with endogenous fertility.[7] The predictions of this paper call for more studies of the potential negative impacts of low fertility on long-term aggregate growth. The recent deceleration of GDP growth in China has spurred media discussion of this topic, but more academic research is needed.

The remainder of the paper is organized as follows. Section 2 presents a simple OLG model of fertility and human capital choice to illustrate the main mechanisms. Section 3 studies the implications of a binding fertility restriction in this context. Section 4 extends the model to a 16-period quantitative OLG model. Section 5 discusses calibration of the model when moments from the UHS are targeted and presents results. Section 6 extends the calibration to the whole of China. Section 7 concludes.

2. THE MODEL

This section presents a general equilibrium OLG model with intergenerational transfers, a channel emphasized by Ehrlich and Lui (1991).

2.1. Production

Consider a Cobb–Douglas production function with labor augmenting technology:

$$Y_t = K_t^{1-\alpha}(A_t L_t)^{\alpha}, \tag{1}$$

where L_t is the efficiency units of labor supplied by $N_{y,t}$ young agents with human capital h_{t-1}.

$$L_t = N_{y,t} h_{t-1}. \tag{2}$$

Fertility restrictions reduce $N_{y,t}$. However, this fall may not reduce the total supply of efficiency units of labor if h_{t-1} rises sufficiently in response. The overall effect of fertility restrictions on the supply of efficiency units of labor, and aggregate output, thus depends on the response of human capital.

2.2. The Agent's Problem

Agents live for three periods. An agent born at time $t-1$ is a child in $t-1$, a young agent in t, and an old agent in $t+1$.

Children do not make any decisions. They receive human capital investment from their parents in the form of educational goods. Young agents inelastically supply a unit of labor and earn wage income. They choose fertility n_t and the level of human capital investment in each child E_t. Human capital formation has a similar form to Choukhmane et al. (2014):

$$h_t = A_h E_t^{\gamma}, \tag{3}$$

with $0 < \gamma < 1$.[8] A_h reflects the efficiency of the human capital formation. For simplicity, I assume that p_E units of consumption goods can be converted into a single unit of human capital good. Total spending on human capital goods is then $n_t p_{E,t} E_t$.

For inter-generational transfers, I use the transfer function of Choukhmane et al. (2014). Young agents at time t, together with their siblings, make a transfer to support their parents, who reach old age at time t. Each young agent gives a fraction $\psi \frac{n_{t-1}^{\omega-1}}{\omega}$ of their wage income, so the total transfer received by their parents is $\psi \frac{n_{t-1}^{\omega}}{\omega} w_{y,t}$. ψ captures children's generosity toward the parents, and $\omega < 1$ captures free riding between siblings. When n_{t-1} increases, $\frac{n_{t-1}^{\omega-1}}{\omega}$ falls, so each

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agent transfers a smaller share of their wage income $w_{y,t}$ to their parents, but, in total, their parents receive more relative to wage income $w_{y,t}$. Similarly, when agents who are young at time t become old at time $t+1$, the total transfer they receive will be $\psi \frac{n_t^{\omega}}{\omega} w_{y,t+1}$.

Assuming log utility, the maximization problem for an agent born at time t is

$$\max_{\{n_t, E_t, c_{y,t}, a_{y,t}, c_{o,t+1},\}} \quad U_t = ln(c_{y,t}) + \upsilon ln(n_t) + \beta ln(c_{o,t+1}),$$

subject to:

$$c_{y,t} + a_{y,t} = \left(1 - n_t\phi_f - \psi \frac{n_{t-1}^{\omega-1}}{\omega}\right) w_{y,t} - n_t p_{E,t} E_t, \tag{4}$$

$$c_{o,t+1} = R_{t+1} a_{y,t} + \psi \frac{n_t^{\omega}}{\omega} w_{y,t+1}, \tag{5}$$

here υ captures utility derived from having children. These agents choose the fertility rate n_t, human capital investment E_t, young age consumption $c_{y,t}$, young age saving $a_{y,t}$, and old age consumption $c_{o,t+1}$. ϕ_f in (4) is a per-child fixed cost of raising children, and $\psi \frac{n_{t-1}^{\omega-1}}{\omega}$ is the fraction of income transferred to parents.

2.3. Optimality Conditions and Equilibrium

Define $k_t = \frac{K_t}{A_t L_t} = \frac{K_t}{A_t N_{y,t} h_{t-1}}$ to be the ratio of capital to efficiency units of labor. The wage and interest rate in this economy are given by

$$w_t = \alpha A_t k_t^{1-\alpha} h_{t-1}, \tag{6}$$

$$R_t = (1-\alpha) k_t^{-\alpha} + 1 - \delta, \tag{7}$$

where R_t is the gross interest rate and δ is the depreciation rate.

The equilibrium is given by series of factor prices $\{w_t, R_t\}_{t=0}^{\infty}$ and choice variables $\{n_t, E_t, c_{y,t}, a_{y,t}, c_{o,t+1}\}_{t=0}^{\infty}$ that solve the individual maximization problem and satisfy capital market clearing:

$$K_t = N_{y.t-1} a_{y,t-1}. \tag{8}$$

The consumption and saving decisions are standard and are described in Appendix A.1. Here, I focus on the fertility and human capital decisions. The first-order condition for human capital investment E_t is

$$n_t p_{E,t} = \frac{1}{R_{t+1}} \frac{\psi n_t^{\omega}}{\omega} \frac{\partial w_{y,t+1}}{\partial h_t} \frac{\partial h_t}{\partial E_t}. \tag{9}$$

The left-hand side is the cost of buying a single unit of human capital goods for each child. The right-hand side is the increase in the transfer received in the next period. Using (6), the expression for wage rate (9) can be rewritten as

$$n_t p_{E,t} E_t = \frac{\psi n_t^{\omega} \gamma}{\omega} \frac{\alpha A_{t+1} k_{t+1}^{1-\alpha} A_h E_t^{\gamma}}{R_{t+1}}. \tag{10}$$

Holding the interest rate and the price of human capital goods constant, (10) implies a negative relationship between fertility and human capital investment per child. This is because the marginal cost of human capital investment increases linearly with the number of children, but the marginal benefit is diminishing. Thus, with higher fertility, parents invest less in human capital for each child. This equation embodies the quantity–quality trade-off in this model. In addition, when the interest rate R is higher, parents invest less in human capital. This is because investment in physical capital is now more profitable.

The first-order condition for fertility n_t is

$$\frac{\upsilon}{n_t} = \frac{1}{c_{y,t}} \left(\phi_f w_{y,t} + p_{E,t} E_t - \frac{1}{R_{t+1}} \psi n_t^{\omega-1} w_{y,t+1} \right). \tag{11}$$

The left-hand side is the direct gain in utility, while the right-hand side is the discounted net marginal cost of a child. $\phi_f w_{y,t}$ is the fixed cost of raising an additional child, and $p_{E,t}E_t$ is the additional spending on human capital goods. $\psi n_t^{\omega-1} w_{y,t+1}$ is the marginal transfer received during next period.

The fertility first-order condition (11), combined with the consumption decision (A.1) and the human capital first-order condition (10), gives:

$$\frac{\upsilon}{n_t} = \frac{(1+\beta)\left(\phi_f w_{y,t} + \left(1 - \frac{\omega}{\gamma}\right) p_{E,t} E_t\right)}{\left(1 - \psi \frac{n_{t-1}^{\omega-1}}{\omega} - n_t \phi_f\right) w_{y,t} - \left(1 - \frac{1}{\gamma}\right) n_t p_{E,t} E_t}. \tag{12}$$

This can be rearranged to

$$(1+\beta+\upsilon)(1-\lambda) \frac{p_{E,t} E_t}{w_{y,t}} = -(1+\beta+\upsilon)\phi_f + \frac{\upsilon}{n_t}\left(1 - \psi \frac{n_{t-1}^{\omega-1}}{\omega}\right), \tag{13}$$

where $\lambda = \frac{(1+\beta)\omega+\upsilon}{\gamma(1+\beta)+\gamma\upsilon}$.

ASSUMPTION 1. *Assume $\omega > \gamma$. This is a sufficient condition for $\lambda > 1$.*

When Assumption 1 is satisfied, (13) generates a positive relationship between the fertility rate n_t and the level of human capital investment E_t in each child. An increase in human capital increases both the marginal cost and marginal benefit of having children, but when $\omega > \gamma$ the benefit increases more than the cost. Intuitively, a larger ω means less free-riding between siblings, increasing the benefit from having more children.

The equilibrium must also satisfy capital market clearing condition (8). Substituting in the saving decision (A.3) and the human capital condition (10):

$$\begin{aligned} K_{t+1} &= N_{y,t} a_{y,t} \\ &= N_{y,t} \left\{ \frac{\beta}{1+\beta} \left[\left(1 - \psi \frac{n_{t-1}^{\omega-1}}{\omega} - n_t \phi_f\right) w_{y,t} - n_t p_{E,t} E_t \right] - \frac{1}{1+\beta} \frac{n_t p_{E,t} E_t}{\gamma} \right\}. \end{aligned}$$

Divide both sides by $N_{y,t}$ and rewrite K_{t+1} as $k_{t+1}A_{t+1}N_{y,t+1}h_t$:

$$(1+\beta)\, k_{t+1}A_{t+1}A_h E_t^{\gamma} n_t = \beta \left[\left(1 - \psi \frac{n_{t-1}^{\omega-1}}{\omega} - n_t\phi_f \right) w_{y,t} - n_t E_t p_{E,t} \right] - \frac{n_t E_t p_{E,t}}{\gamma}. \tag{14}$$

The human capital condition (10), fertility condition (13), physical capital condition (14), wage rate (6), and interest rate (7) together characterize an equilibrium of the model in $\left\{n_t, E_t, w_{y,t}, R_t, k_t\right\}_{t=0}^{\infty}$.

PROPOSITION 1. *With full depreciation $\delta = 1$, an increase in future productivity A_{t+1} does not affect fertility n_t or human capital investment E_t.*

Formal proof is presented in Section A.2. A higher A_{t+1} increases future wages. This increases the transfer parents receive for all n_t and all E_t. As a result, both fertility and investment in children's human capital will increase. However, there is a further general equilibrium effect. An increase in the supply of efficient units of labor lowers the ratio of capital to efficient units of labor and increases the gross rate of interest R_{t+1}. This effect encourages investment in physical capital and reduces both fertility and investment in children's human capital. With full depreciation ($\delta = 1$), the two effects are of equal magnitude.

When $\delta < 1$, the direct effect can dominate, leading n_t and E_t to increase with A_t. The model is thus able to generate an increasing h_t series. However, if p_E grows at the same rate as wage, the model admits a steady state with constant n_t and h_t. I consider this steady state now.

2.4. Steady State

Assume that productivity grows exogenously at rate $\frac{A_{t+1}}{A_t} = g_A$ and that the price of human capital goods grows at the same rate as wages so that $\frac{p_{E,t}}{w_{y,t}} = \frac{p_E}{w_y}$. Substitute the wage rate (6), interest rate (7), and human capital formation (3) into, respectively, the human capital condition (10), the fertility condition equation (13), and the physical capital condition (14) and impose assumptions above.

Equation (13) becomes:

$$(1+\beta+\upsilon)\,(1-\lambda)\,\frac{p_E}{w_y}E = -\,(1+\beta+\upsilon)\,\phi_f + \frac{\upsilon}{n}\left(1 - \psi\frac{n^{\omega-1}}{\omega}\right). \tag{15}$$

Equation (10) becomes:

$$\frac{p_E}{w_y}E = \frac{g_A}{R}\frac{\gamma\psi n^{\omega-1}}{\omega}. \tag{16}$$

Equation (14) becomes:

$$(1+\beta)\frac{1-\alpha}{\alpha}\frac{g_A n}{R+\delta-1} = \beta\left(1-\psi\frac{n^{\omega-1}}{\omega} - n\phi_f\right) - \left(\beta + \frac{1}{\gamma}\right)\frac{p_E}{w_y}nE. \tag{17}$$

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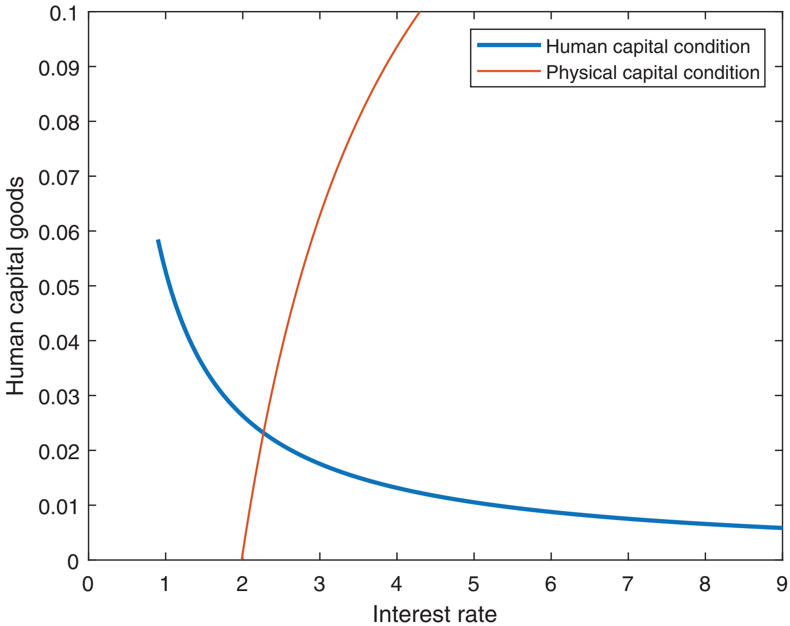

Notes: $\beta = 0.85$; $\upsilon = 0.32$; $\alpha = 0.59$; $g_A = 1.3$; $\phi_f = 0.1$; $A_h = 0.5$; $\gamma = 0.38$; $\psi = 0.09$; $\omega = 0.65$; $\frac{p_E}{w_y} = 0.01$; $n = 1$.

FIGURE 1. Human capital and Physical capital conditions, given fertility.

Then, (15), (16), and (17) characterize steady-state human capital investment E_{ss}, fertility n_{ss}, and the interest rate R_{ss}.

For a given fertility rate n ($n = 1$), Figure 1 plots the conditions for human and physical capital (16) and (17) over different levels of human capital investment E and interest rates R. The human capital condition slopes downward. An increase in the gross interest rate raises the rate of return on physical capital relative to human capital. The physical capital condition is upward sloping. Higher spending on human capital implies less spending on physical capital, thus the ratio of physical capital to efficient units of labor is lower, and the interest rate is higher.

Figure 2 illustrates how the human and physical capital conditions shift in response to a decline in fertility. In this example, n falls from 1 to 0.7. The human capital condition shifts upward because, for any given interest rate, fewer children means that the marginal return from investment in human capital is higher. The physical capital condition shifts to the left because, for a given level of human capital, a fall in fertility decreases the total spent on consumption and human capital. Saving thus increases, and the interest rate falls. These shifts together mean that human capital spending per child increases when fertility declines. Figure 3 represents this negative relationship as a downward-sloping curve, labeled as the capital condition. A mathematical description of this curve can be obtained

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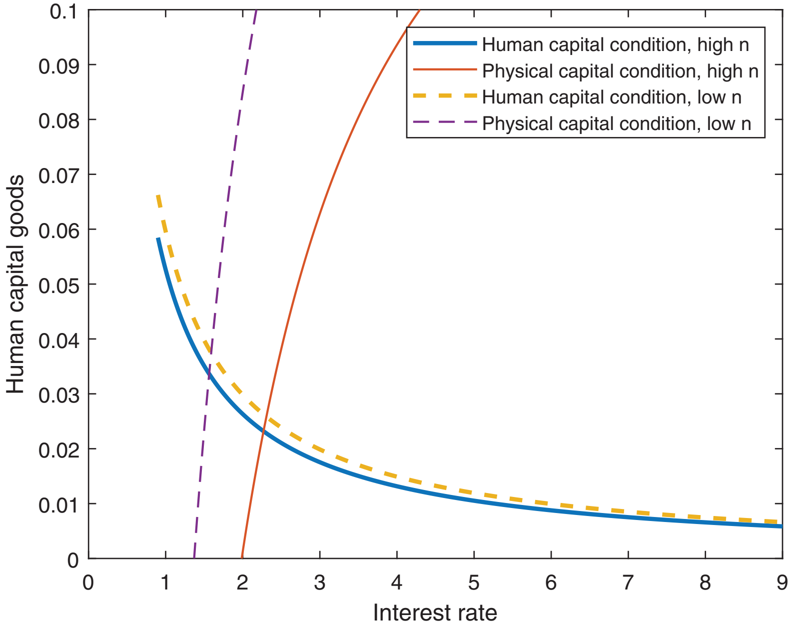

Notes: $\beta = 0.85$; $\upsilon = 0.32$; $\alpha = 0.59$; $g_A = 1.3$; $\phi_f = 0.1$; $A_h = 0.5$; $\gamma = 0.38$; $\psi = 0.09$; $\omega = 0.65$; $\frac{p_E}{w_y} = 0.01$; $n_{\text{high}} = 1$; $n_{\text{low}} = 0.7$.

FIGURE 2. Human capital and physical capital conditions, changing fertility.

by combining (16) and (17) to substitute out R. It inherits the quantity–quality trade-off from (16).

The upward-sloping curve in Figure 3 is the fertility condition (15). The intersection of the two curves gives steady-state fertility n_{ss} and human capital investment E_{ss}. In steady state, the interest rate R_{ss} and ratio of capital to efficient units of labor k_{ss} will also be constant. Wages grow at the same rate as TFP. All other aggregate variables grow at a rate equal to TFP growth multiplied by fertility.

$$\frac{w_{y,t+1}}{w_{y,t}} = g_A$$

$$\frac{L_{t+1}}{L_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = g_A n_{ss}.$$

2.5. Steady-State Comparative Statics

In this section, I discuss how the steady state responds to changes in parameter values.

PROPOSITION 2. *If the fixed cost of raising a child ϕ_f increases, the fertility rate will fall.* $\frac{\partial n}{\partial \phi_f} < 0$.

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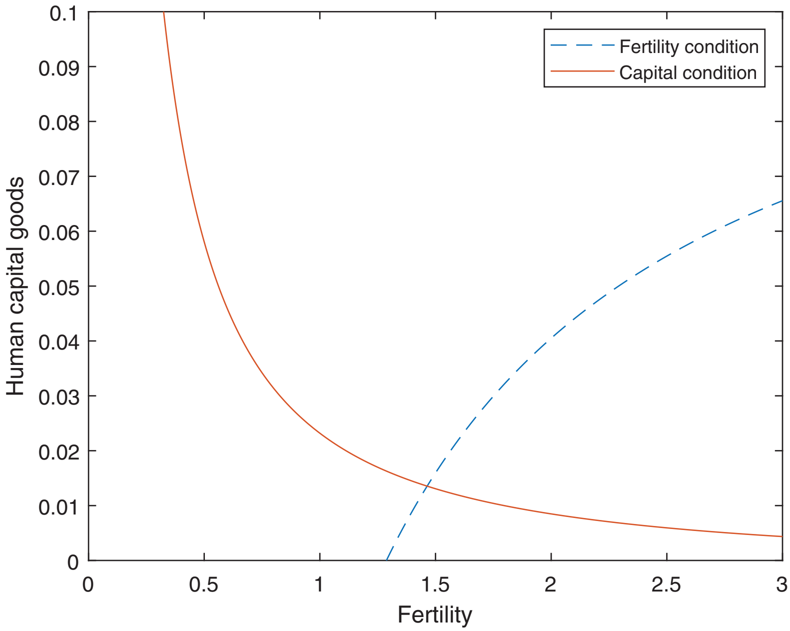

Notes: $\beta = 0.85$; $\upsilon = 0.32$; $\alpha = 0.59$; $g_A = 1.3$; $\phi_f = 0.1$; $A_h = 0.5$; $\gamma = 0.38$; $\psi = 0.09$; $\omega = 0.65$; $\frac{p_E}{w_y} = 0.01$.

FIGURE 3. Steady state fertility and human capital goods.

Appendix A.3 provides a graphical explanation. As raising children becomes more expensive, parents have fewer children. However, the effect of an increase in ϕ_f on human capital investment, $\frac{\partial E}{\partial \phi_f}$, is ambiguous. There are two forces that operate in opposite directions. As parents reduce fertility, the quantity–quality trade-off means that they invest more in each child. However, the higher fixed cost also implies that parents have a smaller budget and buy less of everything, including human capital goods. Hence, the effect on human capital investment E is ambiguous.

PROPOSITION 3. *When the price of human capital goods increases relative to income, parents decrease their investment in human capital per child.* $\frac{\partial E}{\partial \frac{p_E}{w_y}} < 0$.

The proof is similar to that of the Proposition 2. When the price of human capital goods increases, parents purchase less of them. However, the effect on fertility $\frac{\partial n}{\partial \frac{p_E}{w_y}}$ is ambiguous. If the cost of human capital goods is higher, investing in children's human capital as a means of increasing income in old age is less attractive. However, less spending on human capital lowers the total cost of raising a child and, because parents derive utility from having children, this increases fertility. The net effect is ambiguous and will depend on parameters.

PROPOSITION 4. *Changes in labor productivity A_t and human capital formation A_h do not affect the fertility n, interest rate R, or spending on educational goods consumption E.*

This is apparent from equations (15), (16), and (17). Neither A_t nor A_h appears in any of these three equations.

3. THE MODEL WITH FERTILITY RESTRICTION

This section presents the equilibrium when a fertility restriction is imposed. Here I use an upper bar to denote constrained steady-state values. I assume that the fertility restriction is binding $\bar{n} < n_{ss}$.

3.1. Equilibrium Characterization

In this case, agents are not allowed to have a fertility higher than $\bar{n}$, so they choose $\bar{n}$. The fertility optimality condition (15) is irrelevant. The equilibrium is characterized by the human capital condition, the physical capital market clearing condition, and $n = \bar{n}$. The human capital condition (16) now reads:

$$\frac{p_E}{w_y}E = \frac{g_A}{R}\frac{\gamma\psi\bar{n}^{\omega-1}}{\omega}, \tag{18}$$

and the physical capital market condition (17) becomes:

$$(1+\beta)\frac{1-\alpha}{\alpha}\frac{g_A\bar{n}}{R} = \beta\left(1-\psi\frac{\bar{n}^{\omega-1}}{\omega}-\bar{n}\phi_f\right)-\left(\beta+\frac{1}{\gamma}\right)\frac{p_E}{w_y}\bar{n}E. \tag{19}$$

The above two equations determine the steady-state investment in human capital $\bar{E}$ and gross interest rate $\bar{R}$ under a binding fertility restriction.

3.2. Comparative Statics

PROPOSITION 5. *As long as $\bar{n} < n_{ss}$, $\frac{\partial\bar{E}}{\partial\bar{n}} < 0$.*

When there is a binding constraint on fertility, the upward-sloping fertility condition in Figure 3 is irrelevant, and the downward-sloping capital condition in Figure 3 alone determines equilibrium human capital. When parents are allowed fewer children, they invest more in each child's human capital.

The effect on total human capital spending, $\bar{n}\bar{E}$, is ambiguous. To see this, it is helpful to rewrite equation (18) as

$$\frac{p_E}{w_y}nE = \frac{g_A}{R}\frac{\gamma\psi\bar{n}^{\omega}}{\omega}, \tag{20}$$

$\frac{p_E}{w_y}nE$ is the share of income invested in human capital. Holding the interest rate constant, a decrease in fertility makes the right-hand side of (20) smaller, which means that the share of income invested in human capital must fall. However,

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TABLE 1. The Timing of child-birth and transfers for an agent born at time $t-4$

		Age		Education Investment	Transfers
$t-4$	1	0–4	born	from parents	
$t-3$	2	5–9		from parents	
$t-2$	3	10–14		from parents	
$t-1$	4	15–19		from parents	
t	5	20–24	h_t	from parents	
$t+1$	6	25–29	n_{t+1}	in children: compulsory	
$t+2$	7	30–34		in children: compulsory	
$t+3$	8	35–39		in children: compulsory	to parents
$t+4$	9	40–44		in children: discretionary	to parents
$t+5$	10	45–49	h_{t+5}	in children: discretionary	to parents
$t+6$	11	50–54			to parents
$t+7$	12	55–59			
$t+8$	13	60–64			from children
$t+9$	14	65–69			from children
$t+10$	15	70–74			from children
$t+11$	16	75–79	die		from children

Notes: The human capital level h_t of the agent born in time $t-4$ depends on the human capital investment they receive from time $t-4$ to t. Each agent's human capital level is determined during their fifth period of life.

a fall in human capital investment leads to a rise in savings, which reduces the interest rate and increases the right-hand side of (20). Hence, the overall change is ambiguous.

Since the supply of efficient units of labor is equal to total human capital, this is also ambiguous, and the effect on aggregate output is unclear. To see how a fertility restriction affects aggregate income, a quantitative exercise is necessary.

4. A QUANTITATIVE OLG MODEL

This section extends the three-period model into sixteen periods and quantitatively assesses the effect of fertility restrictions on income and welfare. This extended model allows me to match the observed timing of education expenditure. One period in the model corresponds to five years. Table 1 summarizes the timing of lifetime events.

Human Capital Investment: During the first five periods of an agent's life, they make no active decisions and receive human capital investment from their parents. During the first three periods, the level of investment is compulsory. During the fourth and fifth, investment is discretionary.

Working and Saving: During the fifth period of an agent's life, their human capital is finalized and they begin working. For simplicity, I assume that agents consume all their wage income in this period. Starting from period six, agents optimally choose consumption and savings. Agents work till period 12, which corresponds to ages 56 to 60. The official retirement age in China is 60 for men, 55 for women

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working in public sector, and 50 for other female workers. When calibrating the model, I base the age efficiency parameters on the average income of all individuals in the relevant age cohort, including both working and retired individuals where relevant.

Childbirth: Agents decide how many children to have at the beginning of the 6th period. During the 6th, 7th, and 8th period, agents pay the fixed cost of education for each child, and in the 9th and 10th period, they choose how much to invest in discretionary education.

Transfer: Agents make transfers to their parents in periods 8 to 11. These periods are the last four periods of their parents' lifetime. Similarly, when they are in the last four periods of their own lifetime, they receive transfers from their own children.

The utility function of an agent born at time $t-4$ and entering the labor market at time t is

$$U_t = \frac{1}{\beta} ln(c_t(5)) + \upsilon ln(n_{t+1}) + \sum_{s=6}^{10} \beta^{s-6} ln\left(\frac{c_{t+s-5}(s)}{1+n_{t+1}}\right) + \sum_{s=11}^{16} \beta^{s-6} ln(c_{t+s-5}(s)),$$

where the subscripts t and s denote time and the agent's age.

Note that during periods 6 through to 10, the periods during which agents raise children, the agent's consumption is $\frac{c}{1+n}$. c_s here represents total consumption for the whole family, with every member being given an equal share. Family consumption is modeled in this way as I do not have reliable data on child rearing costs other than education. The UHS reports household expenditure by category, but not by recipient, so it is unclear how much is spent on children. The only exception is clothing. The UHS data from 1992 to 2006 include spending on children's clothing. Figure A.2 in the Appendix plots expenditure on children's clothing expenditure as a share of household wage income. However, the calculated share is between 1% and 2%, which is too small to be plausible as the total cost of raising a child.[9]

Taking into account intergenerational transfers and human capital investment, the agent faces the following constraints:

$$c_t(5) = w_t(5)$$

$$c_{t+1}(6) + a_{t+1}(6) = (1-n_{t+1}\phi_1)w_{t+1}(6)$$

$$c_{t+2}(7) + a_{t+2}(7) = (1-n_{t+1}\phi_2)w_{t+2}(7) + R_{t+2}a_{t+1}(6)$$

$$c_{t+3}(8) + a_{t+3}(8) = \left(1-n_{t+1}\phi_3 - \frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+3}(8) + R_{t+3}a_{t+2}(7)$$

$$c_{t+4}(9) + a_{t+4}(9) = \left(1-\frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+4}(9) + R_{t+4}a_{t+3}(8) - n_{t+1}p_{E_4,t+4}E_{t+4}(4)$$

$$c_{t+5}(10) + a_{t+5}(10) = \left(1-\frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+5}(10) + R_{t+5}a_{t+4}(9) - n_{t+1}p_{E_5,t+5}E_{t+5}(5)$$

$$c_{t+6}(11)+a_{t+6}(11)=\left(1-\frac{\psi n_{t-4}^{\omega-1}}{\omega}\right)w_{t+6}(11)+R_{t+6}a_{t+5}(10)$$
$$c_{t+7}(12)+a_{t+7}(12)=w_{t+7}(12)+R_{t+7}a_{t+6}(11)$$
$$c_{t+8}(13)+a_{t+8}(13)=\frac{\psi n_{t+1}^{\omega}}{\omega}w_{t+8}(8)+R_{t+8}a_{t+7}(12)$$
$$c_{t+9}(14)+a_{t+9}(14)=\frac{\psi n_{t+1}^{\omega}}{\omega}w_{t+9}(9)+R_{t+9}a_{t+8}(13)$$
$$c_{t+10}(15)+a_{t+10}(15)=\frac{\psi n_{t+1}^{\omega}}{\omega}w_{t+10}(10)+R_{t+10}a_{t+9}(14)$$
$$c_{t+11}(16)=\frac{\psi n_{t+1}^{\omega}}{\omega}w_{t+11}(11)+R_{t+11}a_{t+10}(15).$$

Here, ϕ_1, ϕ_2, and ϕ_3 are the costs for compulsory education. $p_{E_4,t+4}$ ($p_{E_5,t+5}$) is the time $t+4$ ($t+5$) price of human capital goods that children receive in their fourth (fifth) period of life. $E_{t+4}(4)$ ($E_{t+5}(5)$) is the corresponding quantity of human capital goods. Parameters ψ and ω are the same as the simple model. The agent born at time $t-4$ chooses fertility n_{t+1}, human capital spending $E_{t+4}(4)$ and $E_{t+5}(5)$, and consumption $\{c_t(s)\}$ and saving $\{a_t(s)\}$ for periods t through to $t+11$.

Human capital formation is modified to

$$h_{t+5}=A_h\left[E_{t+4}(4)^{\tau}E_{t+5}(5)^{1-\tau}\right]^{\gamma}, \quad \mathbf{(21)}$$

where $0<\tau<1$ and $\gamma<1$. A_h remains the efficiency of human capital formation, but human capital now depends on investment received in the fourth and fifth periods of life.[10]

The production function $Y_t=K_t^{1-\alpha}(A_tL_t)^{\alpha}$ and the definition of the capital to efficiency units of labor ratio $k\equiv\frac{K_t}{A_tL_t}$ are unchanged. However, L_t is now the sum of the efficient units of labor supplied by agents in periods 5 to 12 of their lifetimes:

$$L_t=\big[e_5N_t(5)h_t+e_6N_t(6)h_{t-1}+e_7N_t(7)h_{t-2}+e_8N_t(8)h_{t-3}+e_9N_t(9)h_{t-4}$$
$$+e_{10}N_t(10)h_{t-5}+e_{11}N_t(11)h_{t-6}+e_{12}N_t(12)h_{t-7}\big]. \quad \mathbf{(22)}$$

Here $N_t(s)$ is the number of age-s agents at time-t, and e_s is their exogenous relative efficiency level.

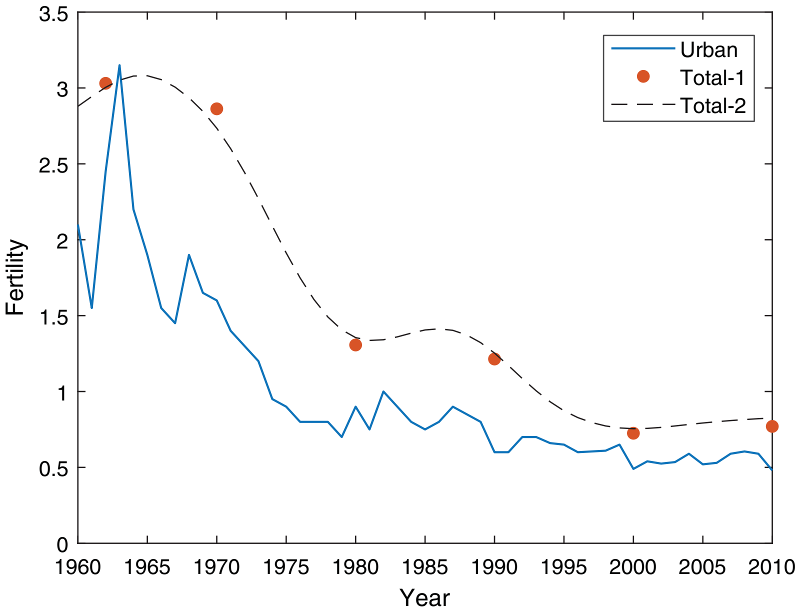
K_t is the summation of the asset holdings of all agents at the end of period $t-1$:

$$K_t=N_{t-1}(6)a_{t-1}(6)+N_{t-1}(7)a_{t-1}(7)+N_{t-1}(8)a_{t-1}(8)+N_{t-1}(9)a_{t-1}(9)$$
$$+N_{t-1}(10)a_{t-1}(10)+N_{t-1}(11)a_{t-1}(11)+N_{t-1}(12)a_{t-1}(12)$$
$$+N_{t-1}(13)a_{t-1}(13)+N_{t-1}(14)a_{t-1}(14)+N_{t-1}(15)a_{t-1}(15).$$

The wage income of an age s agent at time t is

$$w_t(s)=\alpha A_tk_t^{1-\alpha}e_sh_{t+5-s}. \quad \mathbf{(23)}$$

The interest rate R_t is again determined by (7).



Notes: Urban and Total-1 data are from Zhang (2017). Total-2 is from the World Bank.

FIGURE 4. Fertility: urban and total.

5. CALIBRATION AND QUANTITATIVE RESULTS: URBAN CHINA

This section describes the calibration. The primary data sources are the National Bureau of Statistics China (NBSC), the UHS, the Penn World Table (PWT) 8.0, and the China Household Income Project (CHIP). See Section A.5 for a more detailed description.

Before discussing calibration, I provide more details on fertility interventions in China and the Chinese educational system. The strict one-child policy, under which the urban Han families were allowed only a single birth, was implemented in 1979. However, China's family planning policy started long before then, mostly through propaganda campaigns, and had already led to a voluntary decline in fertility. In 1971, the propaganda slogan 'One-child isn't too few, two are just fine, and three are too many' was introduced. In 1973, couples were encouraged to marry late, increase the time gap between their first and the second child, and have fewer children (Zhang, 2017).[11] Figure 4 shows that the fertility rate fell sharply between 1971 and 1978. Urban fertility, measured as the number of children per family divided by two, fell to about 0.75 (1.5 children per family) prior to 1979. After the implementation of the one-child policy, it continued to fall to close to 0.5 (one child per family).

Compulsory education in China lasts for nine years: six years of primary education and three years of junior high school. Children are expected to start primary

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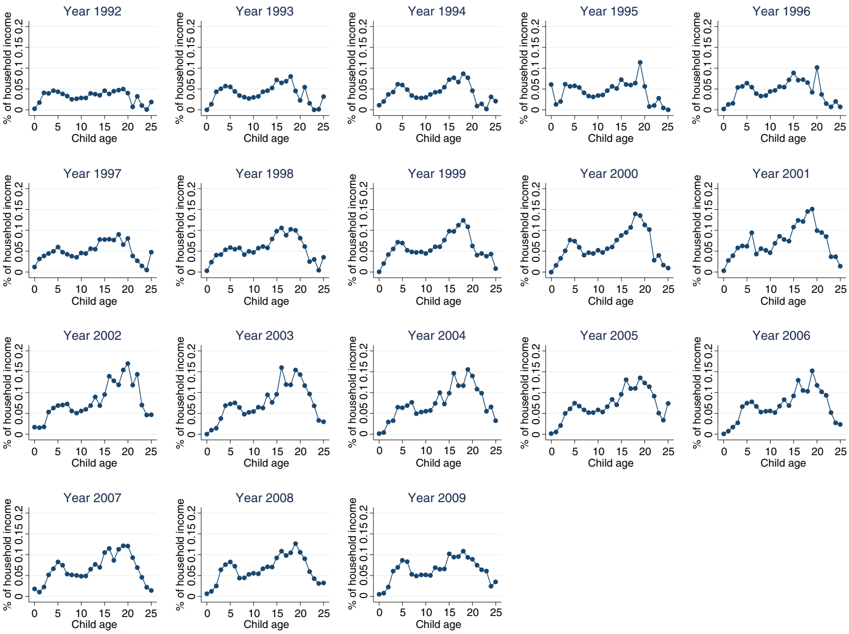

Data source: UHS. The sample consists of two parent one child households. The share is calculated as average education spending over corresponding average family wage income. Children who do not receive education are included in the calculation.

FIGURE 5. Education spending on single child: 1992 to 2009.

school at age seven. Compulsory education finishes around the age of 16. This roughly corresponds to the second and third periods in the model. During compulsory education, tuition fees are funded by the government, but schools still charge miscellaneous fees, which are covered by parents.

High school and college education are not compulsory, and all fees are paid by parents. High school education usually lasts three years. A college degree typically takes three to four years, depending on the type of college, so children normally finish undergraduate education at age 22 or 23. The fourth period in the model, ages 16 to 20, corresponds to a combination of high school and college education. The fifth period, ages 20 to 25, roughly corresponds to college.

I now present evidence on education spending in urban China. I use UHS data and focus on two parent one child households. I calculate the average spending by child age and divide it by average wage income for the corresponding families. Figure 5 shows the results from 1992 to 2009. The spending pattern has been relatively stable since 2000. Note that, as high school and college are not mandatory, enrolment in school declines with age, and this leads the spending share to begin declining at age 20. Although this framework does not include years of schooling explicitly, average education spending does capture that not every child attends high school or college.

TABLE 2. Baseline calibration: exogenous parameters

Parameters	Value	Target
A_h	1	normalisation
α	0.59	Guerriero (2019)
g_A pre-1980	1	no TFP growth
g_A post-1980	1.05	5% labor productivity growth
ψ	0.09	Choukhmane et al. (2014)
ω	0.65	Choukhmane et al. (2014)
δ	0.06	per annum

TABLE 3. Baseline calibration

Parameters	Value	Target
β	0.986	Per annum
e_5	0.3	UHS 1992
e_6	0.77	UHS 1992
e_7	0.87	UHS 1992
e_8	0.91	UHS 1992
e_9	0.97	UHS 1992
e_{10}	1	UHS 1992
e_{11}	0.78	UHS 1992
e_{12}	0.51	average of UHS 1992 to 1995
ϕ_1, ϕ_2, ϕ_3	time varying	UHS 1992 to 2009
γ	0.39	absolute spending share
τ	0.65	relative spending share
υ	3.33	2.92 children per family pre-1970
p_E	timing varying	changes in fees

5.1. Calibration

The model is calibrated as follows. First, I assume that the fertility constraint is binding. Then, ignoring the fertility optimality condition, I use the observed cost of raising children for 1996 to 2009 to find values for most parameters such that the model matches data moments. Second, I hold the value of these parameters fixed, set the cost of education to its 1992 level, and find a value for the 'love of children' parameter υ such that the optimal unconstrained fertility is about 1.46 (2.92 children per family). This was the level of fertility before policy intervention. I use data on educational costs in 1992 as this is the first year for which I have data. Tables 2 and 3 present the calibrated values.

5.1.1. Constant Parameters. This section describes the calibration of model parameters that are constant over time. For the depreciation rate δ, I assume

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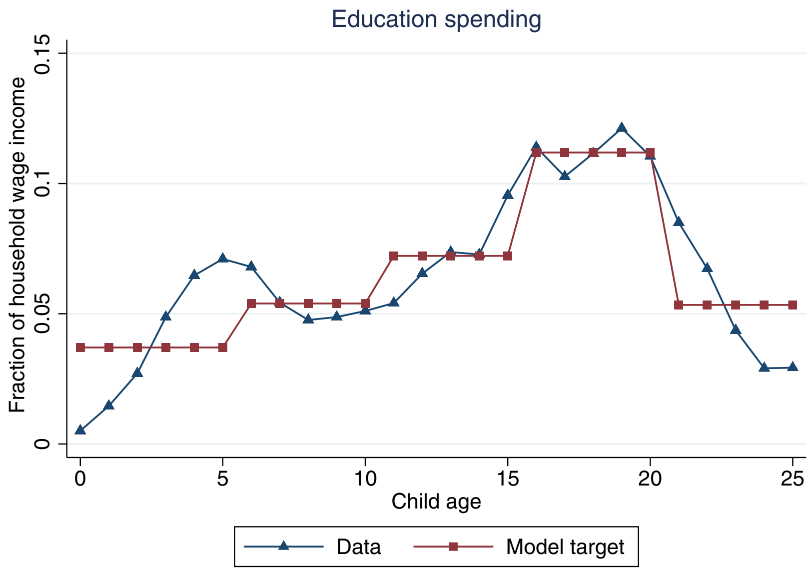

Note: This is the average from UHS 1996 to 2009.

FIGURE 6. Education spending on single child.

an annual rate of 6%. The parameters of the transfer function, ψ and ω, are set to the values estimated by Choukhmane et al. (2014). For the labor share, α, I use the estimate of 0.59 from Guerriero (2019). This estimate adjusts for self-employment income.[12]

The values of e_s affect the relative income of agents of different ages. Using UHS 1992 data, I calculate the average wage rate for each age category and normalise e_{10} to 1. The calibrated values are listed in Table 3.

γ and τ are parameters of the human capital formation function. They are related to discretionary spending on children's education in the 9th and 10th periods of an agent's life. Relative spending in the two periods can be written as

$$\frac{p_{E_4}E_4}{p_{E_5}E_5} = \frac{\tau}{1-\tau}\frac{1}{R_{ss}}, \tag{24}$$

τ captures the relative importance of spending in the two periods. γ is the return to human capital. β, the discount factor, affects educational spending through the interest rate. I choose these three parameters such that, with fertility restricted to $n = 0.5$ and the fixed cost of child raising set to the average observed in 1996 to 2009, the model's steady-state annual interest rate is 4% and the spending shares of education in the 9th and 10th period are, respectively, 10.93% and 5.4%. This is shown in Figure 6.

In this calibration γ is fixed throughout. This calibration would be invalid if the return to human capital in China was increasing over time. As a test of the validity of this calibration, I run the following regression for each year from 1992 to 2009

$$ln(wage)_{i,j} = \eta_1 yos_{i,j} + \eta_2 exp_{i,j} + \eta_3 exp^2_{i,j} + \epsilon_{i,j},$$

$ln(wage)_{i,y}$ denotes the natural logarithm of the wage of individual i in year j, yos denotes years of schooling, exp is years of work experience, and exp^2 is its square. Here, the η_1 is the Mincer return, capturing the effect of one extra year of schooling on $ln(wage)$. It can be thought of as the empirical counterpart of the γ parameter. Table A.1 in the Appendix presents regression results. The mincer return increased in the 1990s but has been stable since around 2000. This stable period is the relevant for individuals born under the one-child policy, so a constant γ is a reasonable assumption.

5.1.2. Time-varying parameters. g_A is growth in labor productivity. I assume that there was no growth before 1980.[13] After 1980, I assume that labor productivity grew at 5% per annum, which corresponds to TFP growth of about 2.9%. This is within the range of estimates provided by the literature on TFP growth in China (see, for example, Zhu (2012) and Young (2003)).

ϕ_1, ϕ_2, and ϕ_3 capture the fixed cost of raising children in the sixth, seventh, and eighth period of a parent's life (the first, second, and third period of a child's life). Using the figures reported in Figure 5, I calculate the average share of income spent on children aged between 1 and 5 years, 6 and 10 years, and 11 and 15 years. I use values from 1992 as the pre-intervention values. Since one period in the model corresponds to five years, I then calculate averages for 1992 to 1995, 1996 to 2000, 2001 to 2005, and 2006 to 2009. I form the time series for the cost of education by using each average in the corresponding period.[14] The values used are $\{\phi_1, \phi_2, \phi_3\}_{pre-intervention} = 0.030, 0.034, 0.034$; $\{\phi_1, \phi_2, \phi_3\}_{1991 to 1995} = 0.036, 0.038, 0.04$; $\{\phi_1, \phi_2, \phi_3\}_{1996 to 2000} = 0.032, 0.040, 0.058$; $\{\phi_1, \phi_2, \phi_3\}_{2001 to 2005} = 0.030; 0.062; 0.070$, and $\{\phi_1, \phi_2, \phi_3\}_{2006 to 2010} = 0.029; 0.063; 0.063$.

As shown in Figure 5, discretionary education spending has increased. However, during this time period, the nominal price of education also increased significantly. As a proxy for the price of education, I calculate high school and college fees per person using data from NBSC.[15] The fee per person is calculated as total fees divided by enrollment. Table 4 summarizes the changes during this period. The average annual growth rate for college fees is 12.9%, and the growth rate for high school fees is 13.9%.

I obtain the real price of education by dividing the nominal fees in Table 4 by nominal wage income. The nominal wage income in column (1) and column (4) of Table 5 are the average income of families with children aged between 16 and 20 years and 21 and 25 years. Column (2) and column (5) of Table 5 show the corresponding real price of education. Column (2) is a weighted sum of education fees, with the high school fee given a weight of 0.6 and the college fee a weight of 0.4. Column (5) is constructed using college fees only. In both column (2) and

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TABLE 4. Education fees per person

Year	College fee per person		High school fee per person	
	Level	Growth rate	Level	Growth rate
1996	1477.12		307.49	
1997	1823.75	0.23	382.63	0.24
1998	2144.72	0.18	419.11	0.10
1999	2921.71	0.36	484.34	0.16
2000	3463.60	0.19	576.41	0.19
2001	3927.71	0.13	673.50	0.17
2002	4324.25	0.10	737.93	0.10
2003	4561.89	0.05	810.66	0.10
2004	4857.09	0.06	898.23	0.11
2005	5070.67	0.04	994.13	0.11
2006	4931.45	−0.03	1008.33	0.01
2007	6489.44	0.32	1470.71	0.46
2008	7016.87	0.08	1585.81	0.08
2009	7182.25	0.02	1672.75	0.05
Average		0.129		0.139

Notes: Data are from NBSC and Educational Statistics Yearbook of China. I use the total fees divided by the corresponding enrolment number. The average is calculated as $\left(\frac{7182}{1477}\right)^{1/13} - 1 = 0.129$ and $\left(\frac{1672}{307}\right)^{1/13} - 1 = 0.139$

TABLE 5. Wage income and real education price

	Families with child age 16 to 20			Families with child age 21 to 25		
	Wage income	Real education price		Wage income	Real education price	
	(1)	(2)	(3)	(4)	(5)	(6)
1996	13230.93	1		8221.824	1	
1997	15971.45	1.024725		10986.34	0.923984	
1998	17170.08	1.102544		11154.26	1.070243	
1999	17219.23	1.446194		11043.99	1.472532	
2000	20058.12	1.472911	1.209275	12706.3	1.517268	1.196805
2001	20434.87	1.649432		15030.02	1.454566	
2002	21608.03	1.715674		17406.03	1.382815	
2003	21513.29	1.833241		18750.54	1.354204	
2004	25250.19	1.677244		23141.78	1.168243	
2005	27924.03	1.60401	1.69592	27127.98	1.040401	1.280046
2006	30692.47	1.433107		31537.58	0.870362	
2007	36535.99	1.624545		35692.54	1.012006	
2008	37599.82	1.705676		32064.26	1.21808	
2009	44811.22	1.476241	1.602154	37596.78	1.063318	1.097801

Notes: Family wage income is calculated from UHS.

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column (5), I normalize the real price in 1996 to be 1. Columns (3) and (6) contain five-year averages, which correspond to $p_{E_4,t}$ and $p_{E_5,t}$ in the model.

5.1.3. Matching Pre-policy Fertility. The only parameter that remains to be determined is υ, the love of children parameter. I use the 1992 fixed cost of child raising as the pre-intervention fixed cost. With all other parameters set to their values above, I find the value of υ such that the unconstrained steady-state fertility would be 1.46.

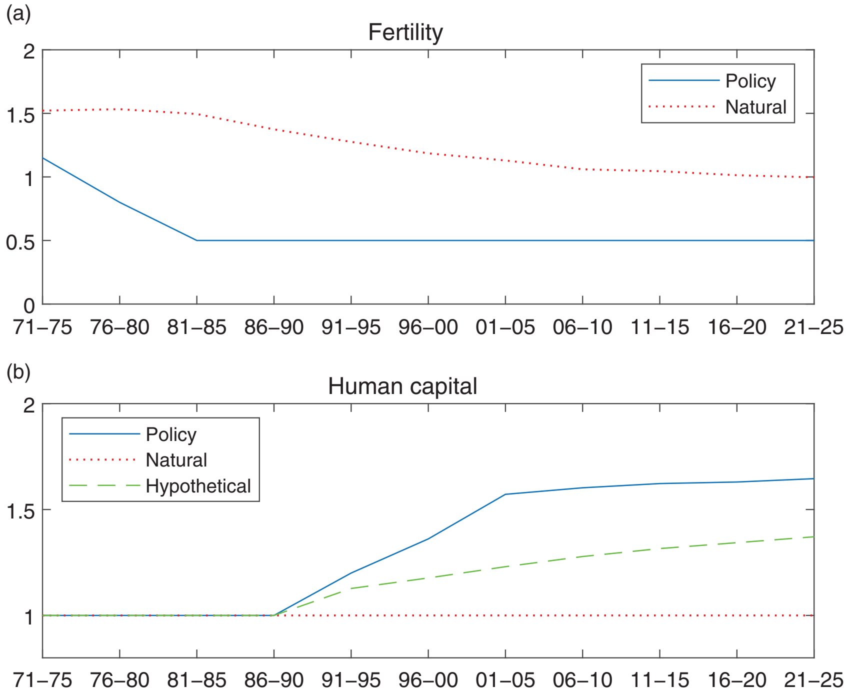
Solving the quantitative model also requires initial conditions. In particular, the initial age structure of the population, the initial human capital level, and the initial wealth distribution. I use the reported urban population distribution for 1982 to back out the distribution for 1970 and use this as the initial distribution. I assume that all agents born before 1970 have a human capital level equal to the unconstrained steady-state level predicted by the model in the pre-1970 period. The initial asset distribution is similarly set to that of the pre-1970 unconstrained steady state.

5.2. Results

This section presents the main results when the model is calibrated to match moments from data on urban households. The model is solved with and without fertility restrictions. The results without fertility restrictions are then used as a counterfactual. The policy fed into the model is a 2.3 children limit from 1971 to 1975, a 1.6 children policy from 1976 to 1980, and a one-child policy from 1981 onward. The same cost of education series is used in both cases.

Figure 7 presents the model's predictions for fertility and individual human capital. Recall that the model is calibrated such that a single period corresponds to five years. Therefore, the period labeled 71–75 corresponds to the years 1971 through to 1975. The labels for other period are analogous. The top panel of Figure 7 shows, for each period, the fertility decision of agents who give birth in that period. In the absence of fertility restrictions, the model does not generate a fall in fertility rates in the 1970s. In fact, the model predicts that the fertility rate would have increased slightly in that decade. Agents have perfect foresight and the cost of compulsory education does not start to increase until the 91–95 period, so those who give birth in the 1970s are not affected. Moreover, those giving birth during the 71–75 and 76–80 periods anticipate the increase in productivity that starts in the 81–85 period, and increase fertility in response.

With no policy intervention, fertility stabilizes at $n = 0.98$, just below two children per family. This is higher than the observed current level of fertility. The one-child policy was relaxed to a two-children policy at the end of 2015. However, according to World Bank data, total fertility in China in 2017 was 0.8 (1.6 children per family). Fertility in urban areas may be even lower. The difference between the model's prediction and the observed outcome suggests that there are factors outside the model affecting China's fertility. One possibility is housing costs.

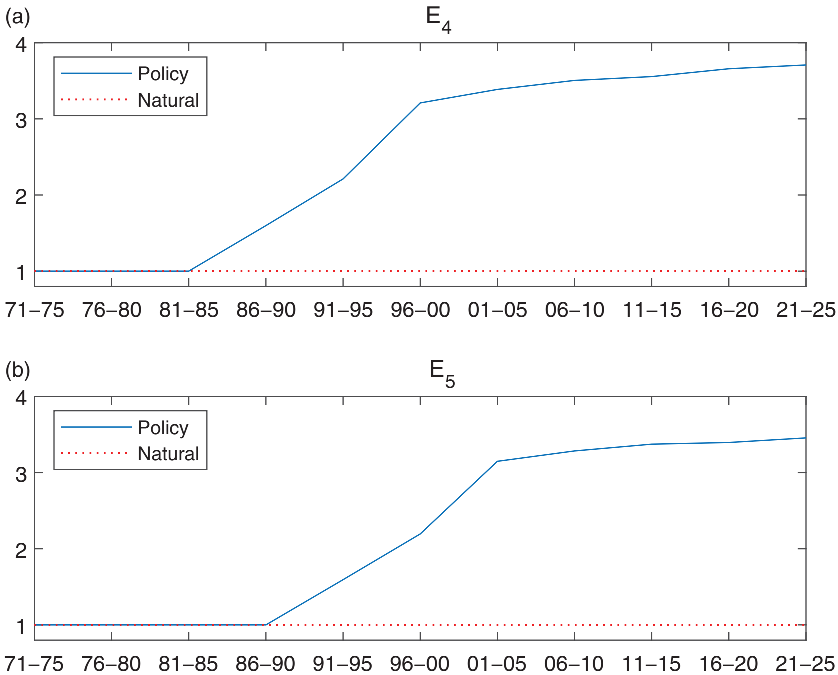


Note: The period labelled as 71-75 corresponds to years 1971 to 1975. Human capital is normalised to be one for all generations when there are no fertility restrictions.

FIGURE 7. Fertility and individual human capital: urban.

In 1998, China moved to a market system for housing. Since then, housing prices have surged. Yi and Zhang (2010) studied Hong Kong and showed that rising house prices reduce fertility. Based on both the model results, and the observed outcome, it is reasonable to conclude that the current two-children policy is not a binding constraint in urban China.

The bottom panel shows human capital for agents entering the labor market in the corresponding period. For example, the first generation born under fertility restrictions is born in period 71–75 under the 2.3 children limit. They receive educational investment in periods 71–75 through to 91–95 and join the labor market in 91–95. Thus, the difference in human capital between the restricted and unrestricted models becomes evident in 91–95. In all periods, I normalize human capital to be 1 in the unrestricted model. Hence, the time path under intervention is the level relative to the unrestricted model. The average human capital of generations born during the periods 71–75 through to 91–95 is about 47% higher under fertility restrictions. From 01–05 onward, relative human capital under fertility restrictions is stable at about 68% higher than the unrestricted model. Figure 8 shows the corresponding quantities of human capital goods. The quantity is again normalized to be 1 in all periods in the unrestricted model.



Note: The period labelled as 71–75 corresponds to years 1971 to 1975. The quantity is normalised to be one for all periods when there are no fertility restrictions.

FIGURE 8. Human capital goods: urban.

Compared to the outcome of the model, some empirical studies have reported a more modest effect of the one-child policy on human capital. For example, using data on twins, Rosenzweig and Zhang (2009) found that the one-child policy had, at most, a moderately positive effect on human capital. Li and Zhang (2017) also found a modest effect using prefecture-level data. However, much of the difference can be explained by general equilibrium effects. I decompose the difference in human capital into a fertility effect and a general equilibrium effect. I construct a hypothetical series for human capital using the fertility series from the unrestricted model and the interest rate series from the model with fertility restrictions. The resulting human capital series is labeled 'Hypothetical' in Panel (B) of Figure 7.

The difference in human capital level between the hypothetical series and the series from the model with fertility restrictions represents the fertility effect. Here the interest rate is the same in both cases, but fertility levels differ. The difference between the hypothetical series and the unrestricted case represents the general equilibrium effect. Here fertility is the same but the interest rate differs. With fertility restrictions, the interest rate is lower as there is a higher ratio of capital to efficient units of labor. Intuitively, the capital used by younger generations

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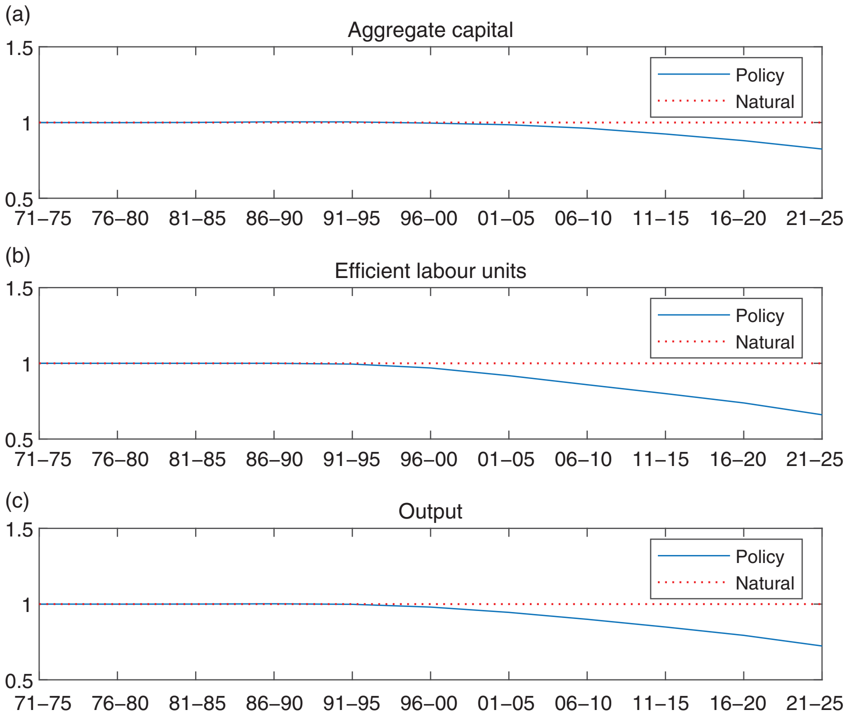


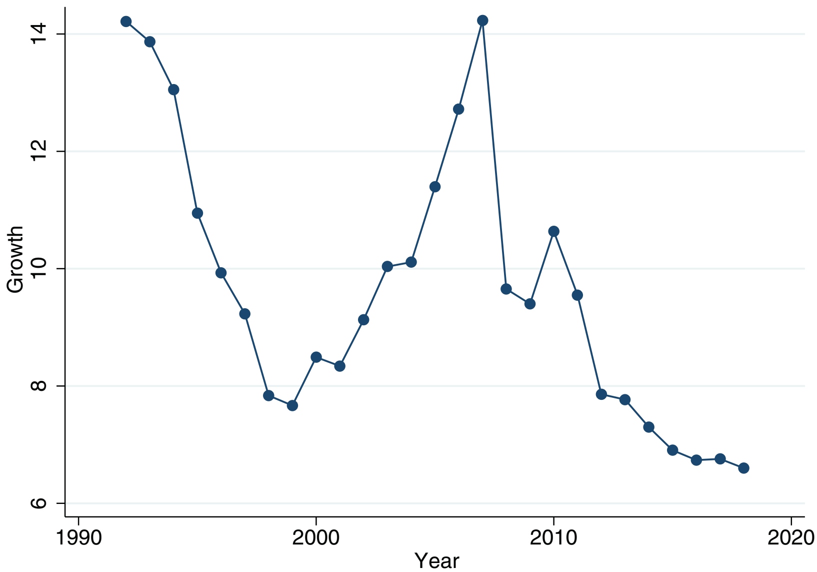
FIGURE 9. Aggregate changes: urban.

is provided by older generations. Lower fertility makes older generations larger relative to younger ones, so there is more capital per young worker. This increases the ratio of capital to efficiency units of labor and lowers the interest rate. This effect is only present when fertility is constrained. The lower interest rate makes saving less profitable and further increases investment in human capital.

For generations born in periods 71–75 through to 91–95, half of the increase in human capital is due to fertility effect, and the other half is due to the general equilibrium effect. After the model stabilizes in period 16–20, 65% of the increase in human capital is accounted for by the general equilibrium effect and 35% by the fertility effect.

The quantitative model also generates predictions for aggregate variables. Figure 9 presents the series for the aggregate capital stock, efficiency units of labor and units. Again, I normalize values in the unrestricted model to be 1 in all periods, so the series for the restricted model consists of relative values.

Panel (A) shows the time series for the capital stock. Before 96-00, the capital stock is slightly higher with fertility restrictions. This is due to the saving decisions of generations born before the policy intervention. With fertility restrictions, their total spending on education for their children (both compulsory and discretionary) is smaller. This leads to increased saving and a higher capital stock. However, the magnitude of this effect is small.[16]



Note: Data from World Bank.

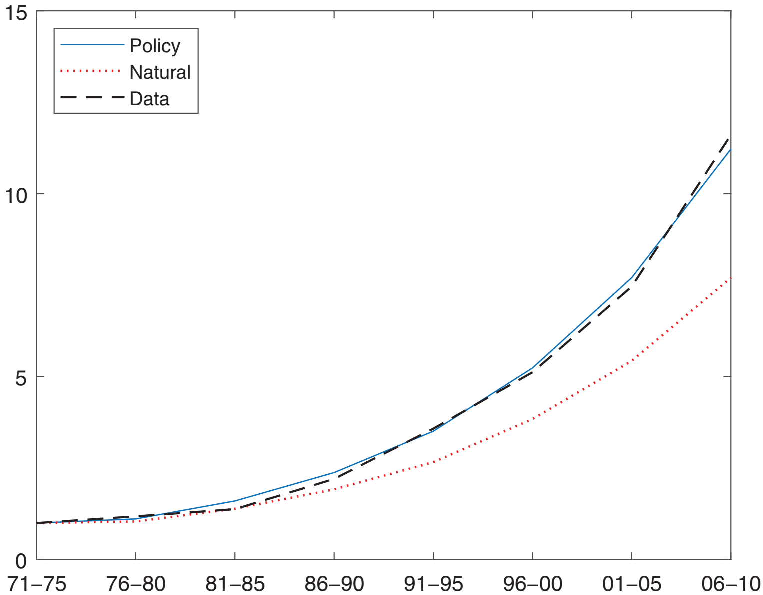
FIGURE 10. Real growth rate of aggregate GDP.

As the generations born under fertility restrictions join the labor market, the fall in population size leads both the capital stock and supply of efficiency units of labor to fall. Panel (B) of Figure 9 plots the time series for efficiency units of labor L_t. After 91–95, the number of efficiency units of labor is smaller with fertility restrictions. In other words, even though capital per worker is higher under the policy intervention, aggregate human capital is lower. Hence, output will eventually fall below the unrestricted case, as shown in panel (C). The increase in individual human capital does not overturn the effect of the decline in fertility.

The model suggests that, relative to the counterfactual case, the negative effect on aggregate GDP became evident during the 96-00 period. This does not mean we would observe a decline in the data in the corresponding for two reasons. One is that this decline is relative to the counterfactual, and we do not observe the counterfactual in real life. Second, the model does not account for factors such as rural–urban migration and the privatization of state-owned enterprises. Figure 10 presents the growth rate of China's real GDP since 1990. We see that deceleration of growth is a phenomena only after 2010.

5.3. Comparison with Data

This section compares the model's prediction for GDP per capital with observed urban GDP per capita. This comparison is appropriate as the model does not account for rural–urban migration. Neither urban GDP nor urban GDP per capita data are directly available, so I construct a proxy for urban GDP per capita.



Note: Data refers to the urban GDP constructed using income information from NBSC and PWT8.0.

FIGURE 11. GDP per capita: urban.

The NBSC yearbook reports disposable income per capita for urban households and net income per capita for rural households from 1978 onward. Using the urban and rural population share as weights, I calculate national disposable income per capita as a weighted average. Then I construct:

$$s_t = \frac{urban\ disposable\ income\ per\ capita_t}{national\ disposable\ income\ per\ capita_t}$$

$$urban\ GDP\ per\ capita_t = s_t * GDP\ per\ capita_t,$$

where GDP per capita is obtained from the PWT 8.0. Before 1978, s_t is not available, so I set the pre-1978 values equal to the 1978 value. I then calculate the average of s_t for every five years and compare it to the model's prediction. Figure 11 shows this. All three series are normalized to one for the period 71–75. The model's predicted GDP per capita with fertility restrictions closely matches the constructed series for urban GDP per capita. On the other hand, GDP per capita from the unrestricted model fails to match the constructed series.

The time-varying parameters in the calibration help the model predictions match the data, but they are not the only reason. The two time-varying parameters are productivity growth and the cost of education. Of the two, productivity growth has the strongest effect on output. As described earlier, it is set to zero before 1980 and 5% per year after that. These values are close to estimates from data, estimates which themselves capture the effect of distortions to China's economy on productivity. With this productivity growth, the model with fertility restrictions matches the data well. However, the same productivity growth series is used in the natural case, where the model's predictions fail to match the data. The difference between the two cases mostly reflects difference in human capital, and the close match between policy case and the data indicates that the model's prediction for human capital is reasonable.

5.4. Welfare

This section analyzes the welfare implications of fertility restrictions. There are several channels through which fertility restrictions affect welfare. Most obviously, it affects the utility derived from having children. In addition, we have seen that those born under fertility restrictions will have higher human capital, and this increases their wages and welfare.

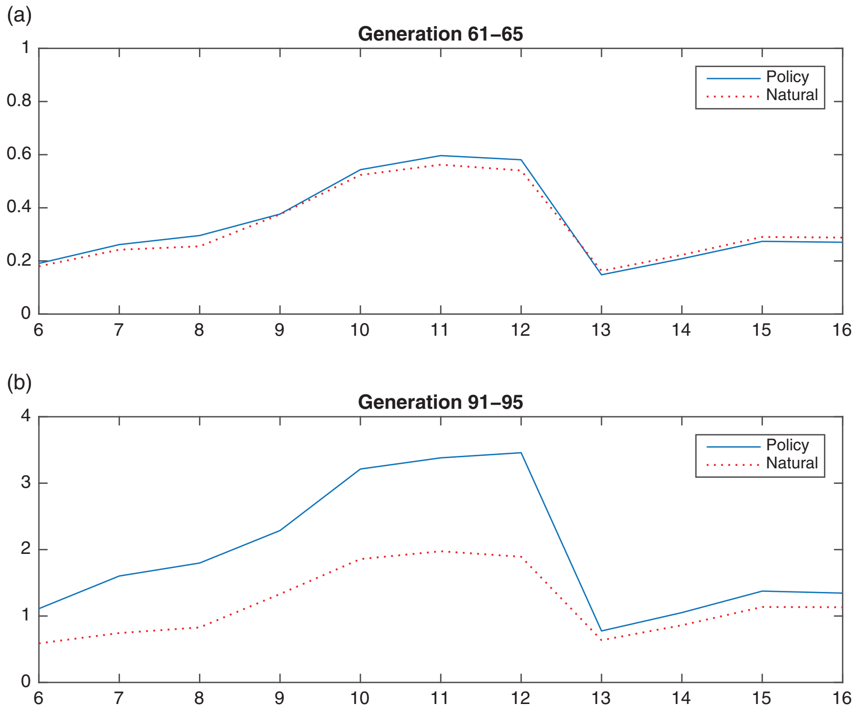
A third channel operates through the ratio of capital to efficiency units of labor. As discussed earlier, lower fertility leads to more capital per worker. This increases welfare for all agents under fertility restrictions. This effect is emphasized in Liao (2013), who finds that removing fertility restrictions will lead to 'capital dilution' with each individual working with less capital.

The remaining channels work through transfers. Both transfers given to parents and received from children may be affected. The share of wage income an adult child transfers to their parents is $\psi \frac{n_t^{\omega-1}}{\omega}$, which increases when fertility declines. This channel is irrelevant for generations born before 71–75, because their parents' fertility decisions are not affected by the fertility restrictions. The share of children's income received as a transfer by their parents is $\psi \frac{n_t^{\omega}}{\omega}$, which declines when fertility falls. Note that this does not mean that the total transfer $\psi \frac{n_t^{\omega}}{\omega} w_{y,t+1}$ is smaller, because lower fertility increases human capital and wage income.

As an illustration, Figure 12 plots disposable income by age for generation 61–65 and generation 91–95. Generation 61–65 refers to those born in 61–65, before fertility restrictions are imposed. I define disposable income to be income available for consumption. It excludes expenditure on children's education or transfers to parents, but does include transfers received from children.

Periods 6 through 12 are the periods during which agents work. Income for both generations in these periods is higher under fertility restrictions. For generation 61–65, this is due to the ratio of capital to efficiency units of labor being higher. For generation 91–95, an extra factor is higher human capital. In old age, which corresponds to periods 13 through 16, generation 61–65 receives less in transfers due to having fewer children.[17] For generation 91–95, the difference in human capital of their children with and without fertility restrictions is larger than that for generation 61–65, and they actually receive a larger transfer with fertility restrictions.[18]

Table 6 shows the welfare of each generation relative to their welfare with no fertility restrictions. Generations 61–65 and 66–70 have higher welfare when



Note: Generation 61-65 refers to those born in 61-65. They themselves give birth in 86-90.

FIGURE 12. Disposable income: urban.

TABLE 6. Relative welfare by generation: urban

61–65	66–70	71–75	76–80	81–85	86–90	91–95	96–00	01–05	06–10
0.75	0.81	0.99	1.16	1.35	1.42	1.49	1.54	1.59	1.63

Note: Generation 61-65 refers to those born in 1961 to 1965. I normalize the welfare under the natural transition to one for all generations.

there are no fertility restrictions, while generation 71–75 has roughly the same welfare. Those born after the restriction is imposed are better off.

6. CALIBRATION: THE WHOLE OF CHINA

Section 5 calibrates the model using data on urban households. In this section, I calibrate the model using data for all of China.

One challenge of generalizing to the whole of China is the lack of time series data on rural education spending at the household level. Education spending in rural areas may be different from urban areas. To investigate, I use the China

Household Income Project (CHIP) 2002, which covers both rural and urban households.[19] The fraction of income spent on education in Section 4 was calculated using wage income. However, many rural households are self-employed and do not receive a regular wage. Hence, I calculate the income for rural families as the summation of wage income reported by family members and household income from operations.[20]

The upper panel of Figure 13 presents education spending shares for two parent one child households. The main difference between educational spending for rural and urban children occurs during early childhood. I thus modify the cost of compulsory education to take this into account. The lower panel calculates the average by age cohort. Based on this, the rural ϕ_1 is 0.07 times the urban ϕ_1, rural ϕ_2 is 0.7 times the urban ϕ_2, and rural ϕ_3 is 0.94 times the urban ϕ_3. I assume that these relative rations apply in all years and multiply the fixed cost derived in Section 4 to get the rural ϕ_s. Then, using weights equal to urban and rural population shares, I calculate a national ϕ_s.

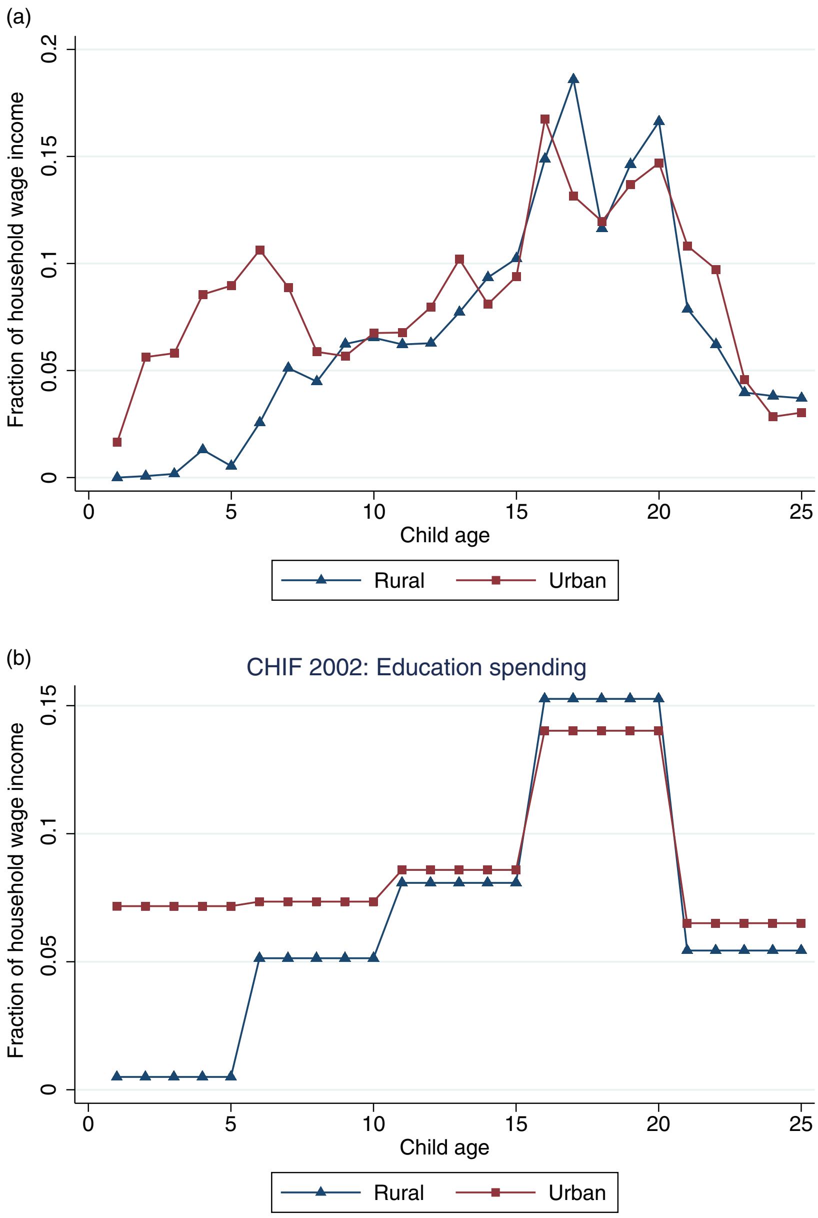
I leave the values of γ and τ unchanged, but I alter the love of children parameter to be 4.17 to match the national 1970 fertility of 2.72 (5.47 children per family).

Since I am considering the whole economy, the relevant policy outcome is the observed nationwide fertility rate. I calculate the five-year average of the total fertility reported by World Bank and insert it into the corresponding model periods. The results for fertility and human capital are presented in Figure A.3 and Figure A.4 in the appendix. With no fertility restrictions, the model predicts that fertility falls to about 1.5 (three children per family). As discussed earlier, observed total fertility in 2017, under the two-children policy, is 0.8 (1.6 children per family). The counterfactual fertility generated by the model is thus greater than the observed fertility.

I also compare the model's predicted GDP to the data. I use GDP data from PWT8.0. Starting with 1971 to 1975, I calculate the average for each five-year period and normalize by the first average. Results are presented in Figure 14. The upper panel of Figure 14 shows GDP per capita, and the lower panel shows aggregate GDP. Both series generated by the model fall below the observed data. This may be because the model omits changes such as rural–urban migration and privatization which improved the allocation of resources. Given these results, it appears that the model is more successful at matching data for urban households.

There are a few caveats to applying this model to the whole of China. First, while urban data generally support the quality–quantity trade-off, evidence from rural data does not always do so. For example, Qian (2009) found that having an additional child improves the probability that the first child is enrolled in education in rural area.

Second, I do not address gender issues.[21] The implementation of the one-child policy in rural China was gender contingent. Rural couples were allowed a second child if their first born was female (Zhang (2017)). As China has a traditional



Note: Data from CHIP 2002.

FIGURE 13. Education spending: rural and urban.

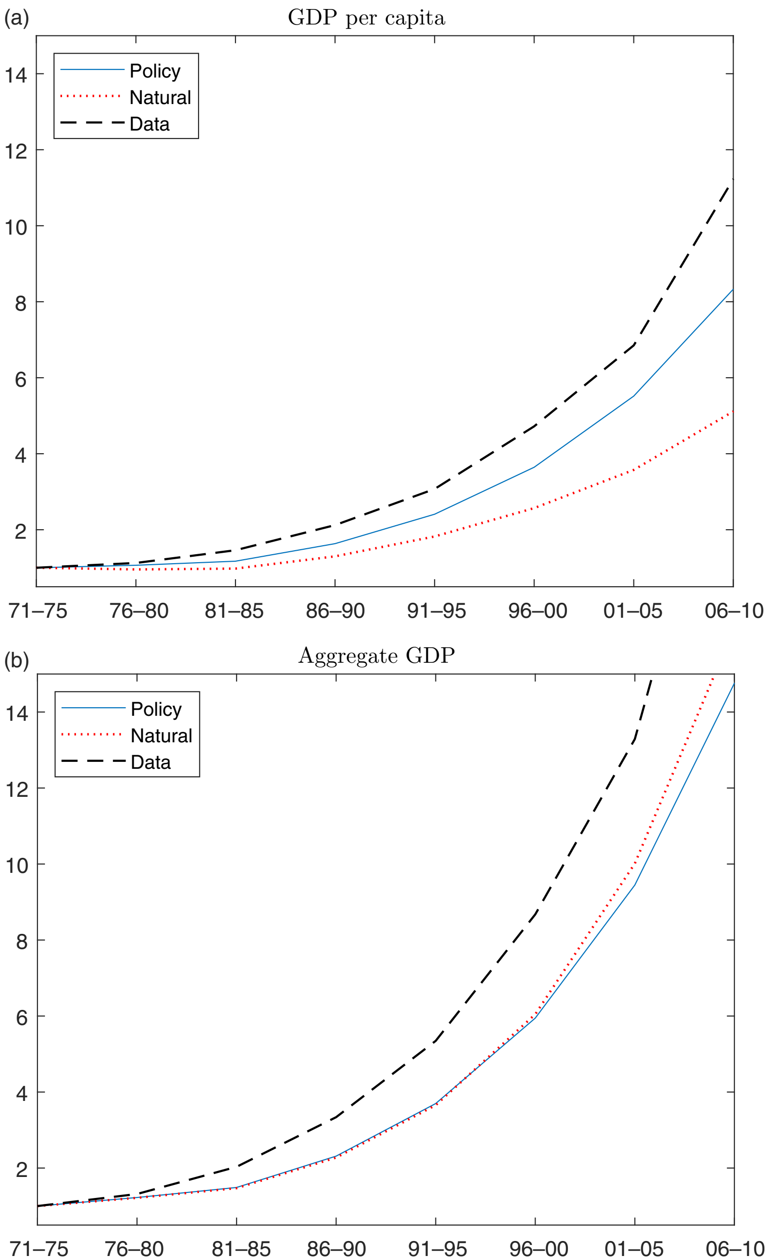


FIGURE 14. GDP: model and data.

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preference for sons, the fertility intervention generated an unbalanced gender ratio and educational differences in families with more than one child.[22]

Third, there exists a rural–urban gap in the return to education. In particular, the return to human capital is lower in rural areas (see Fu and Ren (2010) for an analysis). In terms of the model, this could be due to a lower A_h, a lower γ, or both. A lower A_h and a lower γ both imply less investment in human capital for a given level of fertility, but they have different implications for quantity–quality trade-off. A lower A_h does not affect the quantity–quality trade-off, but a smaller γ reduces the difference in investment in human capital for two given fertility rates.[23] Hence, the change in human capital induced by a policy that imposes fertility restrictions can be larger or smaller depending on the unconstrained fertility in rural areas and the difference between constrained and unconstrained fertility rates.

Fourth, I abstract from stochastic mortality. Data for China show that life expectancy has been increasing. At the national level, life expectancy has increased from 67.77 in 1981 to 76.36 in 2016 (according to the 2018 Statistical Yearbook). However, life expectancy for urban households has been consistently higher than rural households. Hongyuan and Yongliang (2004) reported that life expectancy in rural areas increased from 67.5 in 1990 to 69.9 in 2002, whereas in urban areas it increased from 71.1 in 1990 to 76.1 in 2002. A 2016 government report estimates that urban life expectancy has reached 80 (Li et al., 2016). The model's assumptions are closer to urban data and do not capture changes in life expectancy or the rural–urban gap. Increasing life expectancy does not affect the quantity–quality trade-off.[24] With respect to the rural–urban gap, differences in life expectancy will affect relative fertility levels, and hence the corresponding human capital. A two sector model is required to investigate this. I leave this for future research.

7. CONCLUSION

This paper presents an OLG model of fertility and human capital choices in the presence of intergenerational transfers. When a binding fertility restriction is imposed, agents choose the maximum fertility allowed. This increases the human capital and wage income of the generations born under the restriction. To assess the policy's effect on total income, I extend the model to 16 periods and calibrate.

To focus on the one-child policy, the paper assumes a single region closed economy. Given differences between urban and rural areas, it is unsurprising that the model is unable to match data for the whole of China. When the model is calibrated using data for urban China, I find that, compared to the counterfactual with no fertility restrictions, the human capital of generations born under fertility restrictions is about 47% higher. These generations enjoy higher wage income and welfare. In addition, the model is able to closely match the growth of GDP per capita for urban China in the relevant periods.

The calibrated model predicts current fertility of 0.98 in urban areas (just below two children per family) and 1.5 (three children per family) at the national level. Fertility changes in the model are mainly driven by changes in the cost of education, and predicted fertility is higher than the observed level under the current two-child policy. This implies that factors which are not modeled, such as changes in the cost of housing, could be reducing fertility levels. Given the government's recent efforts to increase fertility, one avenue for future research is identifying the precise cause of low current fertility.

The model predicts that fertility restrictions have had a negative effect on aggregate income, as they have reduced the size of the labor force. This suggests that fertility restrictions may have been a drag on China's growth. However, there are still many distortions in the Chinese economy. For example, rural–urban migration provides abundant amounts of labor to urban China. Restrictions on labor mobility between rural to urban areas have played and will continue to play an important role in China's development. These areas are left for further research.

NOTES

1. This is the first year that I have data available.
2. They also provide evidence supporting the importance of intergenerational transfers in China.
3. See more details of the fertility intervention in Section 5.
4. On the empirical side, few studies consider the interactions between fertility and growth. An exception is Chatterjee and Vogl (2018) in which the author focuses on the response of fertility to growth. In the case of China, Li and Zhang (2007) examines the period 1978 to 1990 and finds that lower fertility contributed to the growth of GDP per capita.
5. The cost of child rearing children is also modeled differently. Zhu et al. (2014) use the estimates from Ye and Ding (1998) on Beijing and Xiamen, whereas here it is endogenous.
6. For example, see Jones (2019).
7. Liao (2013) does not study the effect on aggregate GDP, and Zhu et al. (2014) does not have endogenous fertility.
8. This form for human capital formation abstracts from the years of schooling. Alternatives which incorporate time in education include the Ben-Porath (1967) human capital production function, which is a Cobb–Douglas function of time spent, existing human capital, and resource input, and the function used by Manuelli and Seshadri (2014), which includes both time and resource inputs.
9. Ye and Ding (1998) calculate the cost of raising a child in Beijing in 1996 and in Xiamen in 1995. They calculate the average for children aged 0 to 16 years. They find that in Xiamen, the total cost of each child is 34% of family income. Among this, about 5% is spent on education and the remaining 29% is accounted for by other costs. In Beijing, each child costs 20%. About 3.6% is spent on education and 16.4% on other costs. However, their estimates are for one year only and are not representative of all urban households.
10. Cunha and Heckman (2007) emphasizes that human capital investments in different time periods are not perfect substitutes. In the case of urban China, compulsory education is well

implemented, which corresponds to the first three periods of life in the model. Choukhmane et al. (2014) show that the difference in education spending received between single children and children with twins becomes pronounced after the age of 15. Thus, we can think of A_h as capturing the effect of compulsory education.

11. Zhang (2017) provides a detailed summary of the fertility policy going back to the 1950s and discusses how changes in political leadership shape fertility policy.

12. Bai et al. (2006) report that the average labor share from 1978 to 2003 is 0.52. This is similar to the unadjusted value reported in Guerriero (2019). Young (2003) reports an average of 0.6 for the non-agricultural sector from 1978 to 1998.

13. Zhu (2012) shows that, during the period of government-led industrialization between 1952 and 1978, TFP growth in China was negative.

14. The 1992 to 1995 average is used for the model period corresponding to 1991 to 1995.

15. The NBS website does not have data for 2006. These data are transcribed from the Educational Statistics Yearbook of China.

16. In an environment with an exogenous interest rate, Choukhmane et al. (2014) argue that the one-child policy had a large impact on saving. However, Banerjee et al. (2014) point out that the general equilibrium effect through an endogenous interest rate is important. The small difference between the restricted and unrestricted cases found here suggests that other factors may be responsible for China's high saving rate. For example, İmrohoroğlu and Zhao (2018) argue that the risk faced by the elderly and the lack of family insurance are important factors.

17. Their children have higher human capital, which increases the transfer they receive, but this is dominated by the effect of lower fertility.

18. They have more children with no fertility restrictions, but this effect is dominated by their children having lower human capital channel.

19. The UHS data, used in Section 5, cover only urban households. The corresponding survey on rural household has not been made available by the NBSC.

20. Income from operations includes income from family planting, forest, husbandry, fishery, and non-agricultural operations. I use gross income less production costs.

21. Gender issues are not specific to China. See Mishra et al. (2004), Chamarbagwala (2011), and Rosenblum (2017) for analysis on India.

22. For example, Lee (2012) find no difference in years of schooling between only-child boys and only-child girls, but a significant gap between boys and girls in multiple-child households.

23. To see this, suppose that we have fertility rates n_1 and n_2 and that they must both satisfy equation (10). Then, $\frac{E_2}{E_1} = (\frac{n_1}{n_2})^{\frac{1-\omega}{1-\gamma}}$. This expression depends on γ but not A_h.

24. A lower life expectancy leads parents to receive less in transfers from children. Hence, they invest less in children's human capital at any given fertility. However, this does not affect the difference in human capital between two fertility rates. This can be seen from equation (10). Recall that the left-hand side of (10) is the marginal cost of investing in human capital and the right-hand side the marginal benefit. A term $s < 1$ can be added to the right-hand side to capture a shorter life span resulting in $n_t p_{E,t} E_t = s \frac{\psi n_t^{\omega} \gamma}{\omega} \frac{\alpha A_{t+1} k_{t+1}^{1-\alpha} A_h E_t^{\gamma}}{R_{t+1}}$. The ratio between human capital levels is, however, unchanged $\frac{E_2}{E_1} = (\frac{n_1}{n_2})^{\frac{1-\omega}{1-\gamma}}$. Thus, the quantity–quality trade-off, given fertility rates, is not directly affected.

25. The nine provinces are Beijing, Liaoning, Zhejiang, Anhui, Hubei, Guangdong, Sichuan, Shanxi, and Gansu.

26. http://www.ciidbnu.org/chip/

27. The rural sample is from these 22 provinces: Beijing, Hebei, Shanxi, Liaoning, Jilin, Jiangsu, Zhejing, Anhui, Jiangxi, Shandong, Henan, Hubei, Hunan, Guangdong, Guangxi, Chongqing, Sichuan, Guizhou, Yunan, Shaanxi, Gansu, and Xinjiang The urban sample is from these 12 province: Beijing, Shanxi, Liaoning, Jiangsu, Anhui, Heinan, Hubei, Guangdong, Chongqing, Sichuan, Yunan, and Gansu

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APPENDIX A

A.1. CONSUMPTION AND SAVING IN THE THREE PERIOD MODEL

The consumption decisions of a young agent at time t:

$$c_{y,t} = \frac{1}{1+\beta}\left[\left(1-\psi\frac{n_{t-1}^{\omega-1}}{\omega}-n_t\phi_f\right)w_{y,t}-n_tp_{E,t}E_t+\frac{\psi n_t^{\omega}}{\omega}\frac{w_{y,t+1}}{R_{t+1}}\right] \tag{A.1}$$

$$c_{m,t+1} = \frac{\beta R_{t+1}}{1+\beta}\left[\left(1-\psi\frac{n_{t-1}^{\omega-1}}{\omega}-n_t\phi_f\right)w_{y,t}-n_tp_{E,t}E_t+\frac{\psi n_t^{\omega}}{\omega}\frac{w_{y,t+1}}{R_{t+1}}\right]. \tag{A.2}$$

and the corresponding saving decision is given by

$$a_{y,t} = \frac{\beta}{1+\beta}\left[\left(1-\psi\frac{n_{t-1}^{\omega-1}}{\omega}-n_t\phi_f\right)w_{y,t}-n_tp_{E,t}E_t\right]-\frac{1}{1+\beta}\frac{\psi n_t^{\omega}}{\omega}\frac{w_{y,t+1}}{R_{t+1}}. \tag{A.3}$$

A.2. PROOF OF PROPOSITION 1

Proof. Rewrite (10) as

$$n_tp_{E,t}E_t = \frac{\psi n_t^{\omega}\gamma}{\omega}A_hE_t^{\gamma}\frac{\alpha A_{t+1}k_{t+1}^{1-\alpha}}{(1-\alpha)\,k_{t+1}^{-\alpha}+1-\delta}, \tag{A.4}$$

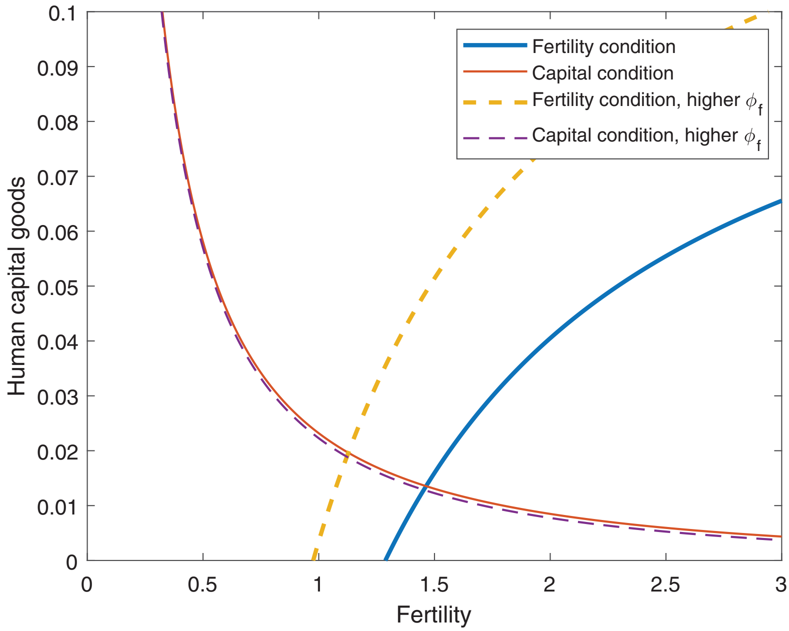
when $\delta = 1$, we have

$$n_tp_{E,t}E_t = \frac{\psi n_t^{\omega}\gamma}{\omega}A_hE_t^{\gamma}\frac{\alpha A_{t+1}k_{t+1}}{(1-\alpha)}. \tag{A.5}$$

Together with fertility condition (13) and physical capital condition (14), we see that an increase in A_{t+1} will lead to a decrease in k_{t+1}, but leave $A_{t+1}k_{t+1}$, n_t and E_t unaffected. ■

A.3. PROOF OF PROPOSITION 2

Proof. First, I simplify the three equations that characterize the equilibrium into two equations. By combining human capital condition (16) and physical capital condition (17),



Note: $\beta = 0.85$; $\upsilon = 0.32$; $\alpha = 0.59$; $g_A = 1.3$; $\phi_f = 0.1$; $A_h = 0.5$; $\gamma = 0.38$; $\psi = 0.09$; $\omega = 0.65$; $\frac{p_E}{w_y} = 0.01$; higher $\phi_f = 0.13$.

FIGURE A.1. Steady state fertility and human capital goods.

we get the following capital condition:

$$\left(\frac{(1+\beta)(1-\alpha)}{\alpha}\frac{\omega}{\gamma\psi}n^{1-\omega} + \left(\beta + \frac{1}{\gamma}\right)\right)\frac{p_E}{w_y}E = \frac{\beta}{n}\left(1 - \psi\frac{n^{\omega-1}}{\omega}\right) - \beta\phi_f. \qquad \textbf{(A.6)}$$

Together with fertility condition (15), they determine the steady state n_{ss} and E_{ss}. They are the mathematical descriptions of two curves shown in Figure 3. Now we examine the movement of the curves when ϕ_f increases. The graphical illustration is shown in Figure A.1.

The upward-sloping fertility curve will shift up. As ϕ_f increases, for a given n, the right-hand side of equation (15) falls. Since $(1-\lambda)$ is negative, E needs to increase to satisfy this equation. For any given n, E needs to be larger, and hence the fertility curve shifts up.

The downward-sloping capital curve will shift down. As ϕ_f increases, the right-hand side of equation (A.6) falls given n. This means that E in the left-hand side needs to decline. This means a downward shift of the capital condition curve. The equilibrium moves from the intersection of the two solid curves to the intersection of the two dashed curves. Together, they imply a lower fertility n_{ss}. With this set of parameter values that I use to draw these curves, we see an increase in E_{ss}, but it does not have to be the case. ■

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A.4. OPTIMALITY CONDITIONS OF THE QUANTITATIVE MODEL

The consumption decision of this agent is standard.

$$c_t(5) = w_t(5)$$
$$c_{t+1}(6) = \frac{1}{\sum\limits_{i=0}^{10} \beta^i} W_{t+1}(6)$$
$$c_{t+i}(5+i) = \beta R_{t+i} c_{t+i-1}(4+i), \forall i = 2, ..., 11$$

where $W_{t+1}(6)$ is given by

$$\begin{aligned}
W_{t+1}(6) = {} & (1 - n_{t+1}\phi_6)w_{t+1}(6) + \frac{(1 - n_{t+1}\phi_7)w_{t+2}(7)}{R_{t+2}} + \frac{\left(1 - n_{t+1}\phi_8 - \frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+3}(8)}{R_{t+2}R_{t+3}} \\
& + \frac{\left(1 - \frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+4}(9) - n_{t+1}p_{t+4}E_{t+4}(4)}{R_{t+2}R_{t+3}R_{t+4}} \\
& + \frac{\left(1 - \frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+5}(10) - n_{t+1}p_{t+5}E_{t+5}(5)}{R_{t+2}R_{t+3}R_{t+4}R_{t+5}} \\
& + \frac{\left(1 - \frac{\psi n_{t-4}^{\omega-1}}{\omega}\right) w_{t+6}(11)}{R_{t+2}R_{t+3}R_{t+4}R_{t+5}R_{t+6}} + \frac{w_{t+7}(12)}{R_{t+2}R_{t+3}R_{t+4}R_{t+5}R_{t+6}R_{t+7}} \\
& + \frac{\frac{\psi n_{t+1}^{\omega}}{\omega} w_{t+8}(8)}{\prod_{i=2}^{8} R_{t+i}} + \frac{\frac{\psi n_{t+1}^{\omega}}{\omega} w_{t+9}(9)}{\prod_{i=2}^{9} R_{t+i}} + \frac{\frac{\psi n_{t+1}^{\omega}}{\omega} w_{t+10}(10)}{\prod_{i=2}^{10} R_{t+i}} + \frac{\frac{\psi n_{t+1}^{\omega}}{\omega} w_{t+11}(11)}{\prod_{i=2}^{11} R_{t+i}}
\end{aligned}$$

The optimal choice of fertility:

$$\begin{aligned}
& \frac{\upsilon}{n_{t+1}} + \frac{\beta^7 \psi n_{t+1}^{\omega-1} w_{t+8}(8)}{c_{t+8}(13)} + \frac{\beta^8 \psi n_{t+1}^{\omega-1} w_{t+9}(9)}{c_{t+9}(14)} + \frac{\beta^9 \psi n_{t+1}^{\omega-1} w_{t+10}(10)}{c_{t+10}(15)} + \frac{\beta^{10} \psi n_{t+1}^{\omega-1} w_{t+11}(11)}{c_{t+11}(16)} \\
& = \frac{\phi_6 w_{t+1}(6)}{c_{t+1}(6)} + \frac{\beta \phi_7 w_{t+2}(7)}{c_{t+2}(7)} + \frac{\beta^2 \phi_8 w_{t+3}(8)}{c_{t+3}(8)} + \frac{\beta^3 p_{E,t+4} E_{t+4}(4)}{c_{t+4}(9)} + \frac{\beta^4 p_{E,t+5} E_{t+5}(5)}{c_{t+5}(10)} \\
& + \frac{1 + \beta + \beta^2 + \beta^3 + \beta^4}{1 + n_{t+1}}
\end{aligned}$$

where the left-hand side is marginal gain from children and the right-hand side is the marginal cost.

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The optimal choice of human capital spending during the fourth and fifth periods of a child's life is given by

$$n_{t+1}p_{E_4,t+4}(4) = \frac{\psi n_{t+1}^{\omega}}{\omega}\frac{\partial h_{t+5}}{\partial E_{t+4}(4)}\left(\frac{1}{R_{t+8}...R_{t+5}}\frac{\partial w_{t+8}(8)}{\partial h_{t+5}} + \frac{1}{R_{t+9}...R_{t+5}}\frac{\partial w_{t+9}(9)}{\partial h_{t+5}} + \frac{1}{R_{t+10}...R_{t+5}}\frac{\partial w_{t+10}(10)}{\partial h_{t+5}} + \frac{1}{R_{t+11}...R_{t+5}}\frac{\partial w_{t+11}(11)}{\partial h_{t+5}}\right)$$

$$n_{t+1}p_{E_5,t+5}(5) = \frac{\psi n_{t+1}^{\omega}}{\omega}\frac{\partial h_{t+5}}{\partial E_{t+5}(5)}\left(\frac{1}{R_{t+8}...R_{t+6}}\frac{\partial w_{t+8}(8)}{\partial h_{t+5}} + \frac{1}{R_{t+9}...R_{t+6}}\frac{\partial w_{t+9}(9)}{\partial h_{t+5}} + \frac{1}{R_{t+10}...R_{t+6}}\frac{\partial w_{t+10}(10)}{\partial h_{t+5}} + \frac{1}{R_{t+11}...R_{t+6}}\frac{\partial w_{t+11}(11)}{\partial h_{t+5}}\right)$$

A.5. DATA

Here I lay out the sources of data this papers uses. The fertility data are from Zhang (2017) and the World Bank (WB).

Urban education spending shares are obtained from the Urban Household Survey (UHS) 1992 to 2009. It is conducted annually by the National Bureau of Statistics of China (NBSC). It has information available on individual income, household income, and household expenditure. Nine provinces are included [26]. There are 5,450 households included the 1992 data and 17,200 households in the 2009 data.

The education fees are downloaded from NBSC website, with the exception of year 2006. The downloaded data does not have information for 2006, so I transcribed it from the Educational Statistics Yearbook of China. The population shares of rural and urban residents are from Statistical Yearbook of China, published by NBSC.

In the calibration to the whole of China, I make use of China Household Income Projects (CHIP) 2002 to calculate the national average spending on education expenditure. The purpose of CHIP is to track income dynamics.[27] It has been conducted for 1988, 1995, 2002, 2007, and 2013. The 1988 data does not report education expenditure, and the 2007 data report culture, education, and entertainment consumption in one category. I decide to use the 2002 data. The 2002 sample has 9,200 rural households from 22 provinces and 6,835 urban households from 12 provinces.[28] I only use the provinces for which both rural and urban samples are available.

For GDP data, I use the real GDP per capita reported in PWT8.0 and real GDP growth data from World Bank.

TABLE A.1. Mincer return: 1992 to 2009

	(1) 2009	(2) 2008	(3) 2007	(4) 2006	(5) 2005	(6) 2004
yos	0.121***	0.123***	0.132***	0.129***	0.132***	0.125***
	(0.00231)	(0.00250)	(0.00249)	(0.00247)	(0.00244)	(0.00260)
exp	0.0991***	0.0916***	−0.000915	0.0817***	0.0820***	0.0870***
	(0.00217)	(0.00237)	(0.000652)	(0.00223)	(0.00225)	(0.00236)
exp2	−0.00169***	−0.00154***	−1.23e−07	−0.00130***	−0.00129***	−0.00138***
	(3.47e−05)	(3.66e−05)	(4.14e−07)	(3.54e−05)	(3.59e−05)	(3.77e−05)
Constant	7.018***	6.895***	7.917***	6.665***	6.525***	6.437***
	(0.0484)	(0.0531)	(0.0452)	(0.0492)	(0.0488)	(0.0512)
Observations	24,940	25,323	26,765	26,341	25,958	25,168
R-squared	0.217	0.189	0.126	0.165	0.167	0.152
	(7)] 2003	(8) 2002	(9) 2001	(10) 2000	(11) 1999	(12) 1998
yos	0.127***	0.127***	0.111***	0.123***	0.0947***	0.0904***
	(0.00265)	(0.00276)	(0.00459)	(0.00413)	(0.00390)	(0.00367)
exp	0.0909***	0.102***	0.124***	0.113***	0.115***	0.117***
	(0.00238)	(0.00249)	(0.00401)	(0.00350)	(0.00327)	(0.00302)
exp2	−0.00139***	−0.00162***	−0.00219***	−0.00187***	−0.00197***	−0.00198***
	(3.84e−05)	(4.01e−05)	(6.76e−05)	(5.90e−05)	(5.45e−05)	(4.96e−05)
Constant	6.169***	6.031***	6.043***	5.895***	6.194***	6.142***
	(0.0509)	(0.0526)	(0.0854)	(0.0752)	(0.0713)	(0.0665)
Observations	23,256	21,332	9,093	9,177	9,710	9,996
R-squared	0.154	0.179	0.180	0.193	0.186	0.200
	(13) 1997	(14) 1996	(15) 1995	(16) 1994	(17) 1993	(18) 1992
yos	0.0845***	0.0703***	0.0659***	0.0803***	0.0584***	0.0524***
	(0.00358)	(0.00328)	(0.00319)	(0.00321)	(0.00272)	(0.00242)
exp	0.119***	0.121***	0.120***	0.119***	0.116***	0.113***
	(0.00304)	(0.00288)	(0.00285)	(0.00278)	(0.00240)	(0.00225)
exp2	−0.00200***	−0.00207***	−0.00208***	−0.00201***	−0.00196***	−0.00187***
	(5.05e−05)	(4.79e−05)	(4.77e−05)	(4.63e−05)	(4.03e−05)	(3.88e−05)
Constant	6.137***	6.212***	6.202***	5.804***	5.832***	5.704***
	(0.0643)	(0.0593)	(0.0576)	(0.0575)	(0.0486)	(0.0435)
Observations	10,246	10,148	10,278	10,243	10,289	10,798
R-squared	0.187	0.200	0.199	0.209	0.221	0.214

Notes: Standard errors in parentheses. ***$p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

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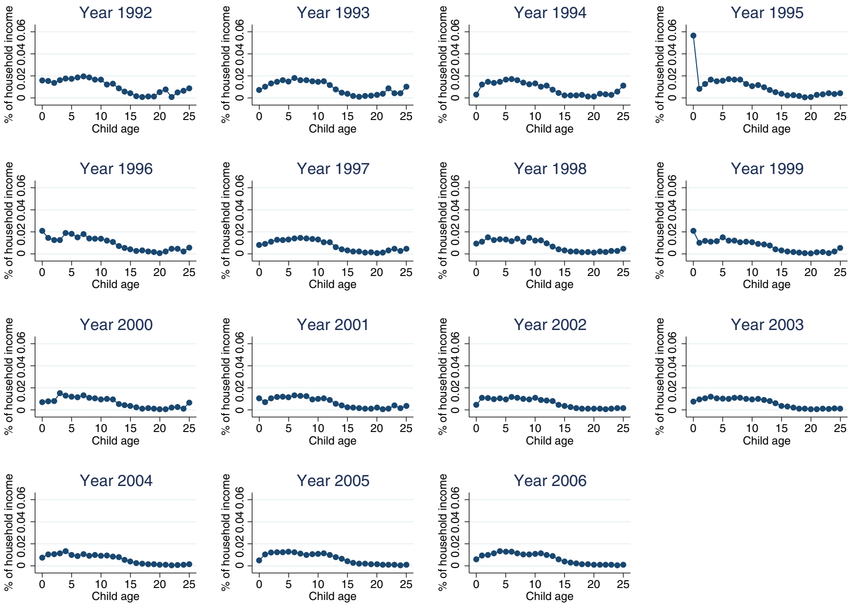

Note: Spending on child clothes is no longer reported from 2007 onwards.

FIGURE A.2. Child clothes spending: 1992 to 2006.

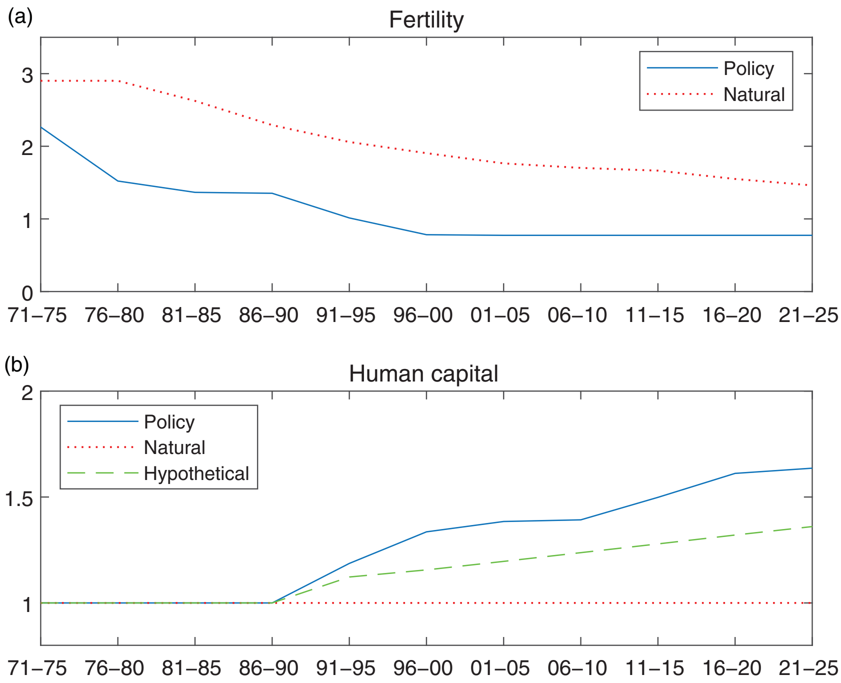

FIGURE A.3. Fertility and human capital: whole China.

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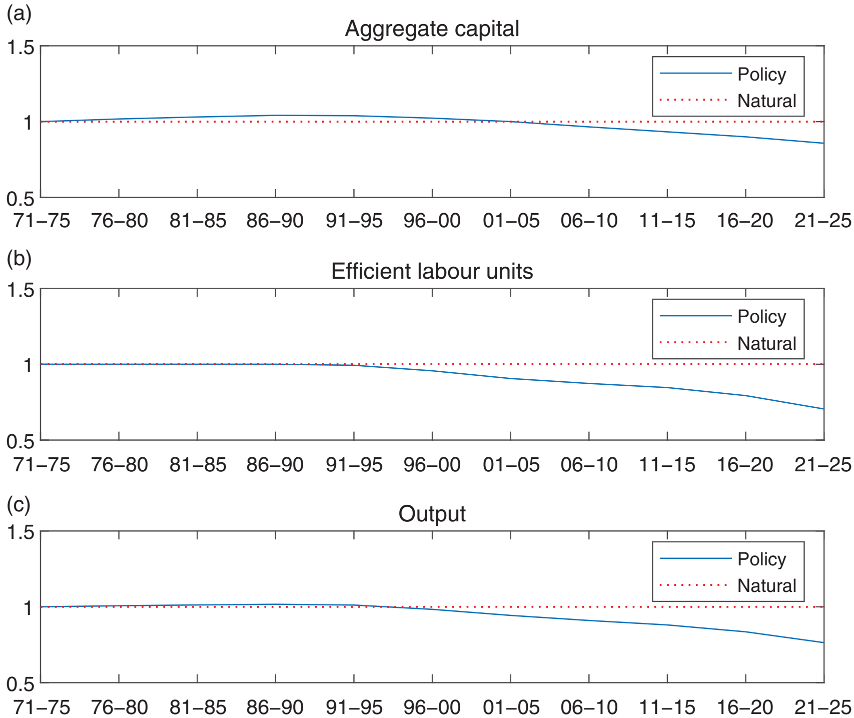

FIGURE A.4. Aggregate outcomes: whole China.