Robotica (2017) volume 35, pp. 1365–1377.
doi:10.1017/S0263574716000102

Formation control of networked mobile robots with guaranteed obstacle and collision avoidance

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(Accepted February 8, 2016. First published online: March 8, 2016)

SUMMARY

This note presents a novel method to design a controller for the formation of networked mobile robots, where the communication between the members of the group is affected by variable time-delay. The control objective is twofold: to maintain the formation during the motion along a desired path and guarantee no collisions with obstacles or adjacent robots. Initially, an innovative dynamical model is formulated for the system; afterwards, the notion of model predictive control is employed to ensure collision avoidance with guaranteed stability. Simulation results are provided to demonstrate the applicability and effectiveness of the suggested method.

KEYWORDS: Multi-agent formation; Communication delays; Model predictive controller; collision and obstacle avoidance.

1. Introduction

Employing several vehicles in a formation to accomplish a task, instead of a single one, increases the flexibility and reliability of the system, as well as provides redundancy and possibility of reconfiguration. Therefore, formation of autonomous agents has been extensively used in many real-world applications such as transportation of heavy loads, search and rescue missions, surveillance and reconnaissance operations.

Formation control of mobile agents has recently become an active research field in the control community.[1–7] The main strategies to organize formation among the bunch of mobile agents are the leader–follower approach,[1–4] the virtual leader method,[5,6] and the behavioral scheme.[7] Various control techniques based on the potential-field idea,[8–15] model predictive control,[16–18] sliding-mode scheme[19] and adaptive controllers[3] were used to implement these strategies in the centralized or decentralized structure. In the centralized arrangement, all the control actions are generated by a central controller that is realized on a remote processor or one of the robots in the formation. In contrast, in the decentralized framework, each agent makes its own decision based on the information received from its neighbors. The design and implementation of the centralized control system is simple; also, less communication burden is required in the centralized configuration compared to the decentralized one.

Recently, by the development of communication technology, autonomous agents are connected together via communication networks. The presence of a network brings numerous advantages such as low cost and easy maintenance; but, some imperfections such as communication delays arise.[20] Consequently, the issue of delayed communication needs to be considered in the synthesis of formation controller for networked vehicles.

The compensation of effects of time-delay in the data exchange has been studied in the literature of multi-agent system.[21–28] To the best of the authors' knowledge, only a few papers have investigated the challenge of delayed communications in the formation control problem.[23–28] In ref. [23], the formation

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control of unmanned aerial vehicles (UAVs) was explored in the presence of time-delay; utilizing the Lyapunov–Krasovskii theorem, delay-dependent and delay-independent conditions were obtained assuming full-state information. Moreover, sufficient conditions were derived in the case that the velocity of the vehicle is not measured. In ref. [24], a decentralized adaptive controller was designed for the formation of spacecrafts subject to communication delay and model uncertainty. A low-pass filter was used to extract the velocity information of the agents; afterwards, a Lyapunov function is used to prove that the proposed control law ensures stability and global convergence to the desired position and velocity. In ref. [25], the formation problem of autonomous underwater vehicles (AUVs) with constant communication delay was solved. To simplify the AUV dynamic equation, the inner–outer loop approach was used to decouple the orientation and translation dynamics; subsequently, the controller was derived using the Lyapunov–Krosovkii functional. In ref. [26], the formation control of agents in the presence of the time-varying delay and stochastic switching topology was considered. Controller was synthesized using the potential function, and then, the stability of the system was analyzed by the Lyapunov–Krasovskii argument. In ref. [28], a formation controller based on the virtual structure approach was designed. To overcome the problem of uncertain attitude measurements and external disturbances, cross sliding-mode controllers were introduced independent of the communication topology.

On the other hand, collision and obstacle avoidance in the formation control of mobile vehicles is an important concern. In general, the controllers are derived based on potential function idea to assure collision and obstacle avoidance. Collision avoidance issue in the formation control of mobile agents with single-integrator dynamic equation was investigated in ref. [8], wherein the employed potential function takes its minimum value when the desired formation is achieved and tends to infinity when a collision is about to occur. A non-linear controller was developed using the proposed potential function and the stability of the system was proved. This approach was applied for the mobile robots in refs. [9, 10] and under-actuated ships in refs. [11, 12]. Swarm formation with obstacle avoidance was studied in ref. [13], where elliptical surfaces generated from normal and sigmoid potential functions with limiting functions were utilized to provide tighter swarm formation. In addition, the potential function method to design controller for formation with guaranteed obstacle avoidance were employed in refs. [14, 15]. Since the controllers derived based on the potential functions need the instantaneous information of the neighbor agents, these types of approaches cannot be employed in the presence of communication delays.

In this study, formation control problem for a group of networked unicycle-like mobile robots with guaranteed collision and obstacle avoidance is tackled where the data transmission between agents is subject to variable time-delay. A simple kinematic model of the robots is considered to specifically focus on the controller design issue in the presence of delayed communication between the agents. The main novelty of the proposed method is to reformulate the closed-loop system as a linear time-delay differential equation with tunable parameters, which are determined online by the control algorithm. The concept of model predictive control is utilized to include collision and obstacle avoidance criteria as constraints and handle the data arrival latency. Stability of the movement is assured in the proposed method.

The rest of the paper is organized as follows: In Section 2, the formation control problem is described in detail. Section 3 presents the main results where the control algorithm is explained. In Section 4, simulation results are shown to verify the applicability and efficiency of the suggested approach. Finally, Section 5 concludes the paper.

2. Problem Formulation and Preliminaries

Consider unicycle-like mobile robot actuated by two driving rear wheels mounted on the same axis as shown in Fig. 1. The kinematics of the ith mobile robot is described as follows:[29]

$$\begin{aligned} \dot{r}_{x_i} &= v_i \cos(\theta_i) \\ \dot{r}_{y_i} &= v_i \sin(\theta_i) \\ \dot{\theta}_i &= \omega_i \end{aligned} \tag{1}$$

where r_{x_i} and r_{y_i} are the Cartesian positions of the center of wheels axis, v_i and ω_i are the linear and angular velocity, and θ_i is the orientation of the ith robot. The agent kinematic equation in Eq. (1) is

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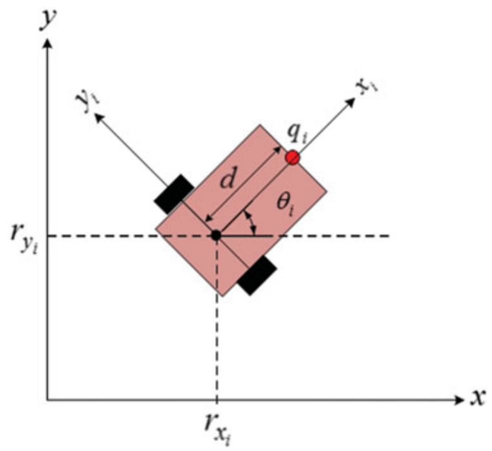

Fig. 1. Schematic of the ith mobile robot in the Cartesian frame.

linearized at $q_i = [x_i,\ y_i]^{\mathrm{T}}$ as the following:

$$\begin{aligned} x_i &= r_{x_i} + \mathrm{d}\ cos\,(\theta_i) \\ y_i &= r_{y_i} + \mathrm{d}\ \sin\,(\theta_i) \end{aligned} \tag{2}$$

where d is depicted in Fig. 1. By defining

$$\begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos\,(\theta_i) & \sin\,(\theta_i) \\ -\frac{1}{\mathrm{d}}\sin\,(\theta_i) & \frac{1}{\mathrm{d}}\cos\,(\theta_i) \end{bmatrix} \begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix}, \tag{3}$$

the following simplified kinematic equation is obtained for the ith mobile robot:

$$\underbrace{\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix}}_{\dot{q}_i} = \underbrace{\begin{bmatrix} u_{x_i} \\ u_{y_i} \end{bmatrix}}_{u_i}, \tag{4}$$

which is in the form of a single integrator.

N mobile robots modeled as in Eq. (4) send their position information to the leader and receive the control actions from it through a shared packet-delaying network. The exact map of the motion environment is not known previously for the robots; so, the obstacles may be encountered in the planned route. The path planner can correct the desired motion trajectory online as described in ref. [30] to round safely the detected obstacle; however, avoidance from the obstacle or other adjacent robots is not assured yet. The controller will assure obstacle and collision avoidance during movement.

The collision avoidance condition between agents i and j is described as the following constraint:

$$\|q_i - q_j\| \geq R_{c_1} \quad \forall i \neq j, \tag{5}$$

where R_{c_1} is the collision radius and the $\|\|$ stands for Euclidean norm. Similarly, the obstacle avoidance criterion is formulated as follows:

$$\|q_i - q_{o_k}\| \geq R_{c_2} \quad \forall i, \tag{6}$$

where q_{o_k} is the position of the kth obstacle that can be detected in the meantime of the motion and R_{c_2} is the collision radius.

Figure 2 depicts the configuration of communication between robots and the controller. The value of variable transmission delay between the agents and the controller (shown by $\tau_{ac}(t)$) can be calculated using the time-stamp of the data packets and is consequently known for the controller at

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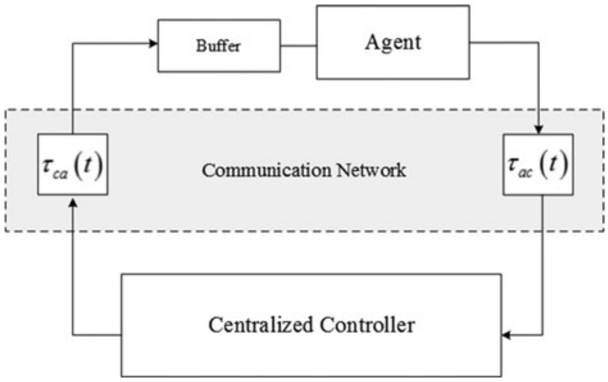

Fig. 2. Configuration of formation control system over a network with buffer used for compensation of varying actuation delay.

time t . However, at the time of computation of control signal in the controller, the control algorithm does not have any information about the future transmission delay in the controller-agent channel (shown by $\tau_{ca}(t)$). Similar to ref. [31], the variable delay induced by the communication link between the controller and the robots is set to be fixed. To this end, the control data received by the agents were saved in the buffer which is located in the actuator side, until the delay value becomes equal to the upper bound of the varying delay, $\bar{\tau}_{ca}$. Briefly, by the mentioned policy, the varying delay that occurs in the data exchange between the controller and the robots is set to a known constant value $\bar{\tau}_{ca}$ for all t . Since the designed controller will be static, these two values can be combined as $\tau = \tau_{ac} + \bar{\tau}_{ca}$. It is worth noting that the overall delay, τ is variable yet.

In brief, the problem of interest is to design a controller that moves each robot in its desired path despite communication delay, additionally ensures the collision and obstacle avoidance.

3. Formation Controller Design

The proposed control action applied to the ith robot consists of two parts as follows:

$$u_i = u_{c_i} + u_{a_i} \tag{7}$$

u_{c_i} is computed by the central controller and sent through the network to the$i$th robot with delay. u_{a_i} is the auxiliary input calculated in the ith robot based on its own information. Auxiliary controller tries to keep the stability of agent motion in the failure of communication link between the robot and the main controller. The central controller is implemented on the leader robot.

The following structures are introduced for u_{c_i} and u_{a_i}:

$$\begin{aligned} u_{c_i} &= \alpha_i e_i(t-\tau) + \sum_{\substack{j=1\\ j\neq i}}^{N} k_{ij}(e_i(t-\tau) - e_j(t-\tau)) \\ u_{a_i} &= \lambda_i e_i(t) + \dot{q}_{id}(t) \end{aligned}, \tag{8}$$

where $e_i = q_i - q_{id}$ and τ is the total delay between the controller and the ith agent. Constants λ_i, k_{ij} and α_i are design parameters. q_{id} represents the desired trajectory of the ith robot and is determined by a path planner based on the latest acquired map of the environment. It is assumed that the path planner can correct online the desired path when the robot sensors detect any obstacle in the current reference trajectory as in ref. [30].

Combining Eqs. (4) and (8) results in the following closed-loop system:

$$\dot{q}_i = \alpha_i e_i(t-\tau) + \sum_{\substack{j=1\\ j\neq i}}^{N} k_{ij}(e_i(t-\tau) - e_j(t-\tau)) + \lambda_i e_i(t) + \dot{q}_{id}(t) \tag{9}$$

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So, the dynamic equation of the tracking error for the ith agent will be as follows:

$$\dot{e}_i = \sum_{\substack{j=1\\ j\neq i}}^{N} k_{ij}(e_i\,(t-\tau) - e_j\,(t-\tau)) + \alpha_i e_i\,(t-\tau) + \lambda_i\, e_i\,(t) \tag{10}$$

Note that when the connection between the central controller and the ith agent fails, i.e., $u_{c_i} \equiv 0$, the system equation and corresponding error dynamics are as the following:

$$\begin{aligned} \dot{q}_i &= \lambda_i e_i\,(t) + \dot{q}_{id}\,(t)\ \ i \\ \dot{e}_i\,(t) &= \lambda_i e_i\,(t) \end{aligned} \tag{11}$$

So, assigning appropriate value for λ_i renders the th agent asymptotically stable.

Regarding Eq. (10), the coefficients k_{ij} and α_i must be determined such that the tracking error tends to zero and at the same time, collision and obstacle avoidance is ensured. The framework of model predictive control is utilized to compute these parameters.

To formulate the finite-horizon optimal control problem of the predictive controller, the augmented state vector E is defined as the following:

$$E = \begin{bmatrix} e_1 \\ \vdots \\ e_N \end{bmatrix} \tag{12}$$

which involves all the error vectors of the agents. The corresponding error dynamics is obtained for the augmented closed-loop system as follows:

$$\dot{E}\,(t) = K\,E\,(t-\tau) + \Lambda\,E\,(t)\,, \tag{13}$$

where the matrices K and Λ are defined in the following:

$$\begin{aligned} K &= \begin{bmatrix} \alpha_1 + \sum\limits_{j=2}^{N} k_{1j} & -k_{12} & \cdots & -k_{1N} \\ -k_{21} & \alpha_2 + \sum\limits_{j=1, j\neq 2}^{N} k_{2j} & \cdots & -k_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -k_{N1} & -k_{N2} & \cdots & \alpha_N + \sum\limits_{j=1}^{N-1} k_{Nj} \end{bmatrix}. \\ \Lambda &= \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_N \end{bmatrix} \end{aligned} \tag{14}$$

As stated before, the matrix Λ is design parameter and is selected such that the following inequality is satisfied:

$$\frac{1}{2\,\|\Lambda\|}\left(\mathrm{e}^{-2\|\Lambda\|(\delta-\tau)} + \mathrm{e}^{2\|\Lambda\|\delta}\right) < 1 \tag{15}$$

where δ is the sampling interval. The reference path, q_{id}, is updated by the path planner during movement to round detected obstacles; however, collision avoidance to the obstacle and neighboring robots is not guaranteed yet. The control procedure is developed such that obstacle and collision avoidance is assured.

The collision avoidance constraint in Eq. (5) can be rewritten in terms of error variables as follows:

$$\|e_i\,(t) - e_j\,(t) + q_{ij\,d}\| \geq R_{c_1}\,, \tag{16}$$

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where $q_{ijd} = q_{id} - q_{jd}$. Inequality of Eq. (16) is restated with respect to the vector E:

$$\|a_{ij} E(t) + q_{ijd}\| \geq R_{c_1} \ , \tag{17}$$

where a_{ij} is a column vector such that

$$a_{ij} E(t) = e_i(t) - e_j(t) \tag{18}$$

Similarly, the obstacle avoidance constraint in Eq. (6) is reformulated as follows:

$$\|a_i E(t) - \left(q_{o_k} - q_{i\,d}\right)\| \geq R_{c_2} \tag{19}$$

wherein a_i is a column vector such that:

$$a_i E(t) = e_i(t) \tag{20}$$

Regarding the above-mentioned notations, the finite-horizon optimal control problem at time t is formulated as follows:

$$\begin{aligned} &\min_K \int_t^{t+\mathrm{T}_p} E(s)^{\mathrm{T}} Q\ E(s)\,\mathrm{d}s \ + \ \mu \|E_{t+\mathrm{T}_p}\|_{l_2}^2 \\ &s.t. \end{aligned} \tag{21}$$

$$\dot{E}(t) = K\, E(t-\tau) + \Lambda\, E(t), \quad E_t = \bar{E}_t \tag{22}$$

$$\|a_i E(s) - \left(q_{o_k} - q_{id}\right)\| \geq R_{c_2}, \quad i = 1, 2, \ldots., N \tag{23}$$

$$\|a_{ij} E(s) + q_{ijd}\| \geq R_{c_1}, \quad i \neq j \tag{24}$$

$$\|E_{t+\mathrm{T}_p}\|_{l_2} \leq \gamma \tag{25}$$

where $E(s)$ is the predicted trajectory starting from real value $\bar{E}_t = \bar{E}(t-\eta)$ for $\eta \in [0, \tau]$, Q is the weight matrix, T_p denotes prediction horizon, $\|\|_{l_2}$ stand for the l_2 norm of the function, $E_{t+\mathrm{T}_p} = E(t + \mathrm{T}_p - \eta)$ for $\eta \in [0, \tau]$, and μ and γ are determined later to ensure the stability of the system.

Remark 1. Standard optimization algorithms presented in ref. [32] can be used to compute the gain matrix K after zero-order-hold discretization of Eqs. (21)–(25). Then, the control action in Eq. (7) can be calculated simply and applied to the robots.

Remark 2. The terminal constraint in Eq. (25) and terminal cost in Eq. (21) are introduced to guarantee the stability of the closed-loop time-delay system. Sufficient conditions to determine μ and γ will be given in what follows.

Inspired by ref. [17] in Lemma 1, the upper bound of γ is derived. It will be proven that in the region defined by the terminal inequality constraint in Eq. (25), obstacle and collision avoidance constraints in Eqs. (23)–(24) are satisfied.

Lemma 1. If the parameter γ in Eq. (25) satisfies the inequalities in Eqs. (26) and (27), then the constraints in Eqs. (23)–(24) hold in the terminal region defined by Eq. (25).

$$\gamma \leq \frac{\|q_{id}(t) - q_{jd}(t)\| - R_{c_1}}{2} \tag{26}$$

$$\gamma \leq \|q_{id}(t) - q_{o_k}(t)\| - R_{c_2} \tag{27}$$

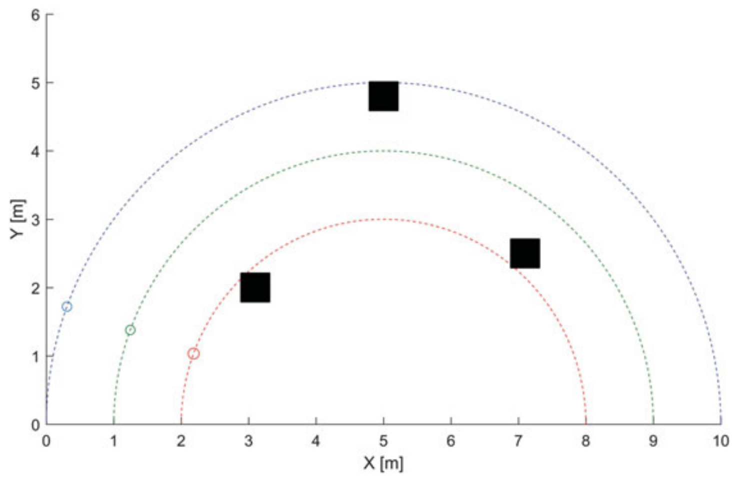

Fig. 3. Desired motion path (dashed lines) and static obstacles (filled rectangles).

Proof: see the appendix.

Remark 3. Equations (26) and (27) can be combined as the following:

$$\gamma \leq \min\left\{\min_t\left(\frac{\left\|q_{id}(t)-q_{jd}(t)\right\|_2-R_{c_1}}{2}\right),\ \min_t\left(\left\|q_{id}(t)-q_{o_k}(t)\right\|_2-R_{c_2}\right)\right\}\quad \forall t\in\left[t,t+\mathrm{T}_p\right) \tag{28}$$

As seen, γ is a time varying parameter that is specified in each optimization step using Eq. (28). Theorem 1 summarizes the main results of the paper.

Theorem 1. If the optimal control problem in Eqs. (21)–(25) with γ satisfying Eq. (28) and μ satisfying Eq. (29) is feasible at the initial time t_0, all the robots converge to their desired positions without any collisions.

$$\mu > \frac{\lambda_{\max}(Q)\,\delta}{\left(1-\frac{1}{2\|\Lambda\|}\left(\mathrm{e}^{-2\|\Lambda\|(\delta-\tau)}+\mathrm{e}^{2\|\Lambda\|\delta}\right)\right)}, \tag{29}$$

where, δ is the sampling interval.

Proof. see the appendix.

4. Simulation Results

Simulation results are presented in this section to demonstrate the effectiveness and applicability of the proposed scheme. In the simulation scenario, three mobile robots track semicircles in formation and encounter suddenly some obstacles in their predefined path. The robots are connected via a network with communication delay equal to $\tau = 0.05$ second. In Fig. 3, the desired path is depicted by the dashed lines, the obstacles are shown by the filled rectangles and the robots are exposed by the circles.

The matrix of auxiliary controller, Λ is set to be $-10\,\mathbf{I}$, where $\mathbf{I}$ denotes identity matrix. The control actions are determined by the solution of the optimization problem in Eqs. (21)–(25) with prediction horizon $\mathrm{T}_p = 0.1$, weight matrix $Q = \mathbf{I}$ and collision radius $R_{c_1} = 0.9$ and $R_{c_2} = 0.75$. To illustrate the performance of the formation controller clearly, the snapshots of motion trajectories are shown in Fig. 4. As seen, when one of the robots approaches close to the obstacle, the controller reduces its velocity to avoid collision with its neighbor robots as long as it is around the obstacle. Another interesting phenomenon is observed when two robots encounter with the same obstacle simultaneously; the controller steers the robots from two opposite sides of the obstacle while deviating

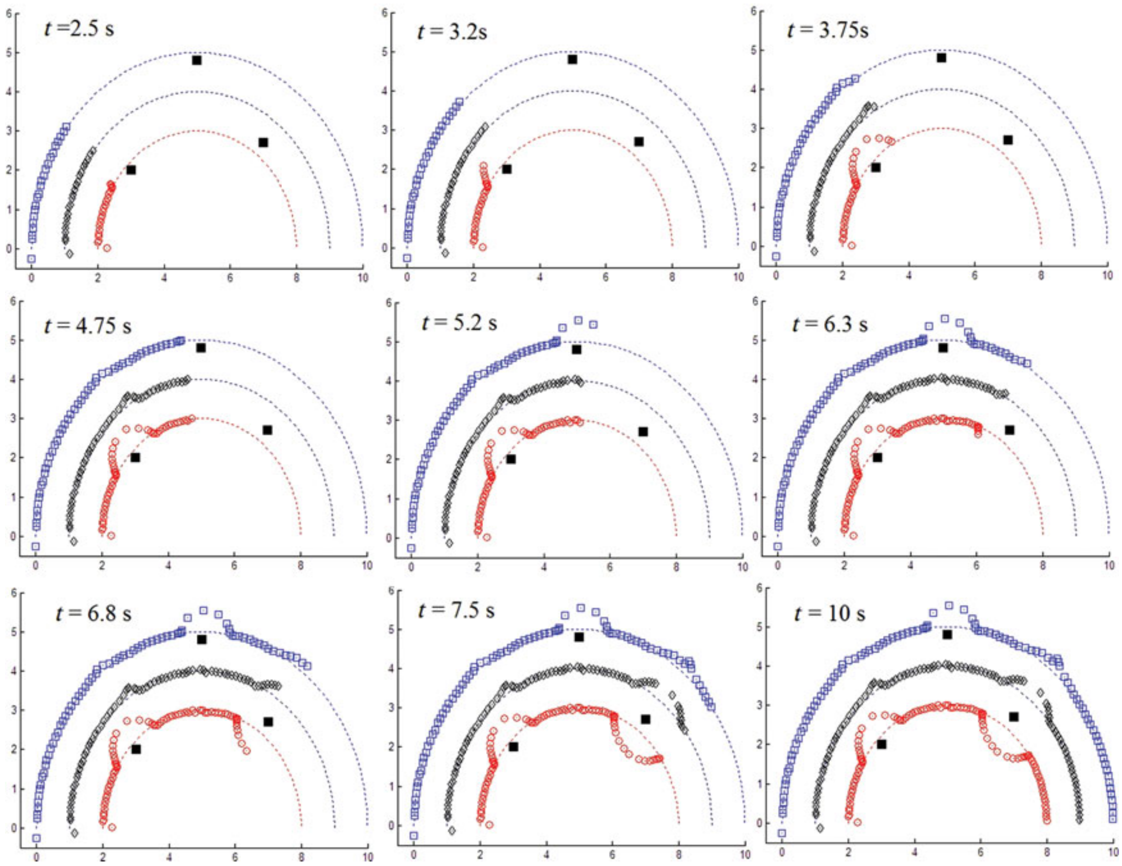

Fig. 4. Snapshots from the trajectories of robots which deviate from their initial desired paths to avoid obstacles without any collision to each other.

the path of the third robot to satisfy the collision constraint. It is obvious that all of the robots track their desired paths without any collision to each other or obstacles.

To provide more evidence for the real-world applicability of the suggested method, the aforementioned simulation scenario was implemented again in the Webots software package, which is a professional toll to prototype mobile robots. The Webots simulation program can be easily ported to the existing real robots. Three Khepera$^{\text{TM}}$ III robots are employed to move in formation by the proposed control scheme. As the snapshots of Fig. 5 show, the outcomes of the Webots are in good agreement with the results of the corresponding simulation of simple models in Matlab$^{®}$.

To demonstrate the merits of the proposed control scheme, a conventional LMI-based design method for time-delay systems, which is based on negating the time-derivative of an appropriate Lyapunov–Krasovskii functional over the closed-loop dynamics of the system,[33] is utilized to tune offline the controller gains in Eq. (8). Although, the robots track their desired paths by this simple controller, the collision avoidance constraint is violated during motion. Figure 6 depicts relative distances of the robots during motion. As seen, the relative distance between adjacent robots is less than desired $R_{c_1} = 0.9$.

5. Conclusion

In this study, a novel approach has been developed to control the formation of mobile robots with guaranteed collision and obstacle avoidance when the information is exchanged between agents with variable time-delay. A new form for the control signal has been introduced and then, the closed-loop dynamic equation of the system has been formulated as a delay differential equation with tunable parameters; subsequently, centralized predictive controller, which includes collision and obstacle avoidance as constraints, has been employed to determine online the controller gains. Simulation

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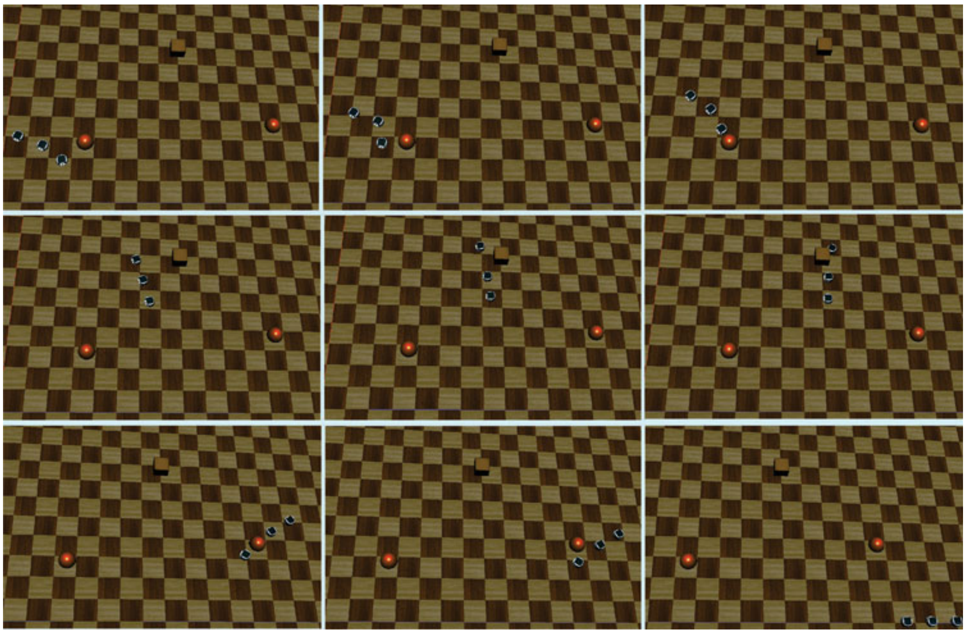

Fig. 5. Snapshots from simulation in Webots Software. Three obstacles are shown by balls and cube.

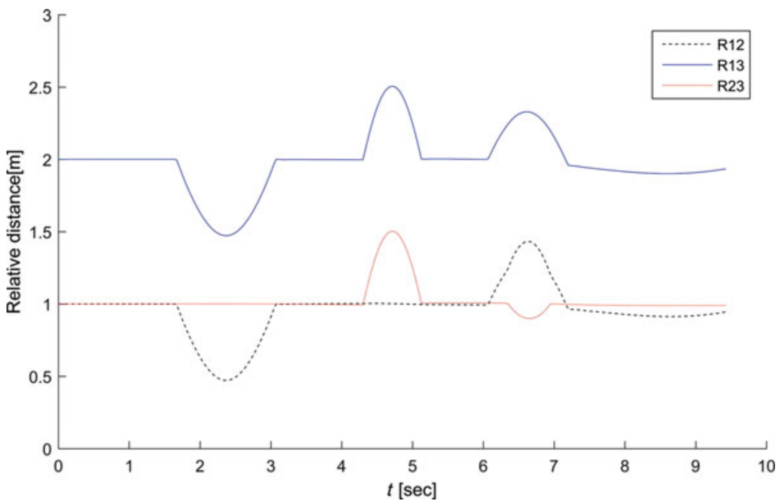

Fig. 6. Relative distance between the robots using the conventional control scheme.

results have been presented to illustrate the efficiency and applicability of the method. Considering more practical requirements like communication topology between agents defines future research line.

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Appendix

Proof of Lemma 1:
If $\|E_{t+\mathrm{T}_p}\|_{l_2} \leq \gamma$, then $\|E(t)\| \leq \gamma$ which results in

$$\|e_i(t)\| \leq \gamma, \quad \|e_j(t)\| \leq \gamma, \quad i \neq j \tag{A.1}$$

and from Eq. (26), the following holds:

$$\|q_{id}(t) - q_{jd}(t)\| - 2\gamma \geq R_{c_1} \tag{A.2}$$

Regarding Eq. (A.1), Eq. (A.2) can be rewritten as follows:

$$\|q_{id}(t) - q_{jd}(t)\| - \|e_i\| - \|e_j\| \geq R_{c_1}$$

which implies that

$$\|e_i - e_j + q_{ijd}\| \geq R_{c_1}$$

This inequality can be written as Eq. (23).

On the other hand, combining Eqs. (27) and (A.1) yields to

$$\|q_{id} - q_{o_k}(t)\| - \|e_i\| \geq R_{c_2} \tag{A.3}$$

using the following relation

$$\|q_{id} - q_{o_k}(t)\| - \|e_i\| \leq \|q_{id}(t) - q_{o_k}(t) + e_i\|$$

Equation (A.3) results in Eq. (24).

Proof of Theorem 1:
To prove the stability of the motions, it will be shown that the optimal cost of the problem in Eqs. (21)–(25) is non-increasing. The non-increasing property of the optimal cost is sufficient to prove the asymptotic stability of the predictive controller.[17] The optimal cost at time t_i starting from the initial condition E_{t_i} is denoted by

$$J^*\left(t_i, E_{t_i}\right) = \int_{t_i}^{t_i+\mathrm{T}_p} E^*(s)^{\mathrm{T}} Q\, E^*(s)\, \mathrm{d}s \;+\; \mu \left\| {E^*}_{t_i+\mathrm{T}_p} \right\|_{l_2}^2$$

The cost at time instant $t_i + \sigma$ with $\sigma \in (0, t_{i+1} - t_i]$ is as follows:

$$J\left(t_i + \sigma, E_{t_i+\sigma}\right) = \int_{t_i+\sigma}^{t_i+\sigma+\mathrm{T}_p} \hat{E}(s)^{\mathrm{T}} Q\, \hat{E}(s)\, \mathrm{d}s \;+\; \mu \left\| \hat{E}_{t_i+\sigma+\mathrm{T}_p} \right\|_{l_2}^2$$

A feasible solution for the time interval $[t_i + \sigma, t_i + \sigma + \mathrm{T}_p]$ is given by the following:

$$\hat{K}(s) = \begin{cases} K^*(s) & \forall s \in \left[t_i + \sigma, t_i + \mathrm{T}_p\right] \\ 0 & \forall s \in \left[t_i + \mathrm{T}_p, t_i + \sigma + \mathrm{T}_p\right] \end{cases} \tag{A.5}$$

Regarding $\hat{K}(s) = 0$ for $s \in [t_i + \mathrm{T}_p, t_i + \sigma + \mathrm{T}_p]$ and using Eq. (13), we have

$$\dot{\hat{E}}(t) = \Lambda \hat{E}(t) \tag{A.6}$$

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and therefore $\hat{E}(s) = e^{\Lambda(s-t_i-T_p)} E^*(t_i + T_p)$ for $s \in [t_i + T_p, t_i + \sigma + T_p]$. Since Λ is a stable matrix, we can conclude that

$$\left\| \hat{E}\,(s) \right\| \le \left\| E^*\left(t_i + T_p\right) \right\| \qquad \forall s \in \left[t_i + T_p, t_i + \sigma + T_p\right]$$

and therefore

$$\hat{E}(s)^{\mathrm{T}} Q\, \hat{E}\,(s) \le \lambda_{\max}(Q) \; \left\| \hat{E}\,(s) \right\|^2 \le \lambda_{\max}(Q) \; \left\| E^*\left(t_i + T_p\right) \right\|^2 \quad \forall s \in \left[t_i + T_p, t_i + \sigma + T_p\right] \tag{A.7}$$

Now, we compute $\Delta \tilde{J}$ as follows:

$$\begin{aligned}
\Delta \tilde{J} &= J\left(t_i + \sigma, E_{t_i+\sigma}\right) - J^*\left(t_i, E_{t_i}\right) \\
&= \int_{t_i+\sigma}^{t_i+\sigma+T_p} \hat{E}(s)^{\mathrm{T}} Q\, \hat{E}\,(s)\,\mathrm{d}s - \int_{t_i}^{t_i+T_p} E^*(s)^{\mathrm{T}} Q\, E^*\,(s)\,\mathrm{d}s + \mu\left(\left\| \hat{E}_{t_i+\sigma+T_p} \right\|_{l_2}^2 - \left\| E^*{}_{t_i+T_p} \right\|_{l_2}^2 \right) \\
&\le \int_{t_i+T_p}^{t_i+\sigma+T_p} \hat{E}(s)^{\mathrm{T}} Q\, \hat{E}\,(s)\,\mathrm{d}s - \int_{t_i}^{t_i+\sigma} E^*(s)^{\mathrm{T}} Q\, E^*\,(s)\,\mathrm{d}s \\
&\quad + \mu\left(\int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left\| e^{\Lambda(s-t_i-T_p)} E^*(t_i + T_p) \right\|^2 \mathrm{d}s - \left\| E^*(t_i + T_p) \right\|^2 \right)
\end{aligned}$$

Using Eq. (A.7), we have

$$\begin{aligned}
\Delta \tilde{J} &\le \lambda_{\max}(Q)\,\sigma \left\| E^*\left(t_i + T_p\right) \right\|^2 - \int_{t_i}^{t_i+\sigma} E^*(s)^{\mathrm{T}} Q E^*\,(s)\;\mathrm{d}s \\
&\quad + \mu\left(\int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left\| e^{\Lambda\left(s-t_i-T_p\right)} \right\|^2 \mathrm{d}s \left\| E^*\left(t_i + T_p\right) \right\|^2 - \left\| E^*\left(t_i + T_p\right) \right\|^2 \right) \\
&\le - \int_{t_i}^{t_i+\sigma} E^*(s)^{\mathrm{T}} Q E^*\,(s)\;\mathrm{d}s \\
&\quad + \left(\lambda_{\max}(Q)\,\sigma + \mu\left(\int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left\| e^{\Lambda\left(s-t_i-T_p\right)} \right\|^2 \mathrm{d}s - 1 \right) \right) \left\| E^*\left(t_i + T_p\right) \right\|^2
\end{aligned} \tag{A.8}$$

Using the upper-bound computed for the norm of exponential function of a matrix in ref. [34], yields to

$$\begin{aligned}
&\int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left\| e^{\Lambda\left(s-t_i-T_p\right)} \right\|^2 \mathrm{d}s \le \int_{t_i+\sigma+T_p-\tau}^{t_i+T_p} \left(e^{\|\Lambda\|\left(T_p+t_i-s\right)} \right)^2 \mathrm{d}s + \int_{t_i+T_p}^{t_i+\sigma+T_p} \left(e^{\|\Lambda\|\left(s-t_i-T_p\right)} \right)^2 \mathrm{d}s \quad \forall \sigma < \tau \\
&\int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left\| e^{\Lambda\left(s-t_i-T_p\right)} \right\|^2 \mathrm{d}s \le \int_{t_i+\sigma+T_p-\tau}^{t_i+\sigma+T_p} \left(e^{\|\Lambda\|\left(s-t_i-T_p\right)} \right)^2 \mathrm{d}s \qquad \forall \sigma \ge \tau
\end{aligned}$$

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and finally, we can obtain

$$\int_{t_i+\sigma+\mathrm{T}_p-\tau}^{t_i+\sigma+\mathrm{T}_p} \left\| \mathrm{e}^{\Lambda\left(s-t_i-\mathrm{T}_p\right)} \right\|^2 \mathrm{d}s \leq \frac{1}{2\,\|\Lambda\|}\left(\mathrm{e}^{-2\|\Lambda\|(\sigma-\tau)} + \mathrm{e}^{2\|\Lambda\|(\sigma)}\right) \qquad \forall\sigma \in (0, \delta]$$

Using Eq. (A.8) and the above result, we have

$$\Delta\tilde{J} \leq -\int_{t_i}^{t_i+\sigma} E^*(s)^{\mathrm{T}} Q E^*(s)\,\mathrm{d}s$$
$$+\left(\lambda_{\max}(Q)\,\sigma + \mu\left(\frac{1}{2\,\|\Lambda\|}\left(\mathrm{e}^{-2\|\Lambda\|(\sigma-\tau)} + \mathrm{e}^{2\|\Lambda\|(\sigma)}\right) - 1\right)\right)\left\| E^*\left(t_i + \mathrm{T}_p\right)\right\|^2$$

Regarding inequality in Eq. (15), for every $\sigma \in (0, \delta]$, choosing $\mu > \frac{\lambda_{\max}(Q)\sigma}{(1-\frac{1}{2\|\Lambda\|}(\mathrm{e}^{-2\|\Lambda\|(\sigma-\tau)}+\mathrm{e}^{2\|\Lambda\|(\sigma)}))}$, the term $\lambda_{\max}(Q)\sigma + \mu(\frac{1}{2\|\Lambda\|}(\mathrm{e}^{-2\|\Lambda\|(\sigma-\tau)} + \mathrm{e}^{2\|\Lambda\|(\sigma)}) - 1)$ will be negative which results in $\Delta\tilde{J} \leq 0$. Not that for the ease of computations, μ is chosen such that to obey the following inequality:$\mu > \frac{\lambda_{\max}(Q)\delta}{(1-\frac{1}{2\|\Lambda\|}(\mathrm{e}^{-2\|\Lambda\|(\delta-\tau)}+\mathrm{e}^{2\|\Lambda\|\delta}))}$ Therefore,

$$J\left(t_i + \sigma, E_{t_i+\sigma}\right) \leq J^*\left(t_i, E_{t_i}\right) \tag{A.9}$$

On the other hand, the optimality of $E^*(s)$ results in

$$J^*\left(t_i + \sigma, E_{t_i+\sigma}\right) \leq J\left(t_{i+\sigma}, E_{t_i+\sigma}\right) \tag{A.10}$$

From Eqs. (A.9) and (A.10), it is concluded that the optimal cost is non-increasing, i.e.:

$$J^*\left(t_i + \sigma, E_{t_i+\sigma}\right) \leq J^*\left(t_i, E_{t_i}\right)$$

By straightforward reasoning as in ref. [17], one can conclude that

$$\lim_{t\to\infty} E(t) = 0$$

This implies that there exist a time instant t_1 such that

$$\|E_t\|_{l_2} \leq \gamma \quad \forall t \geq t_1$$

Therefore, regarding Lemma 1, the constraints in Eqs. (23)–(25) are satisfied.

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