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Beat wave cyclotron heating of rippled density plasma

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Abstract

Laser beat wave heating of magnetized plasma *via* electron cyclotron damping is proposed and analyzed. A plasma density ripple is presumed to exist across the magnetic field. Two collinear lasers propagating along the magnetic field exert a beat frequency ponderomotive force on electrons, driving a large amplitude Bernstein quasi-mode which suffers cyclotron damping on electrons. Finite Larmor radius effects play an important role in the heating. Electron temperature initially rises linearly with time. As the temperature rises cyclotron damping becomes stronger and temperature rises rapidly. The process, however, requires ripple wavelength shorter than the wavelength of the beat wave.

Introduction

The excitation of collective modes in plasmas by beating intense electromagnetic beams has been studied extensively over the years (Tajima and Dawson, 1979; Joshi *et al.*, 1984; Clayton *et al.*, 1993; Krall *et al.*, 1993). The plasma waves excited by such schemes have proven useful for electron acceleration up to ultra-relativistic energies (Liu and Tripathi, 1994; Nakajima *et al.*, 1995; Esarey *et al.*, 1996; Ting *et al.*, 1997; Jarwal *et al.*, 1999). In tokomak, radio frequency wave driven space charge modes are used for current drive, plasma heating, and diagnostics (Liu and Tripathi, 1986).

The efficiency of mode coupling processes is significantly influenced by the presence of plasma density ripple. Experimental and analytical studies have demonstrated resonant enhancement in the efficiency of second and third harmonic generation and terahertz generation due to density ripples (Parashar and Pandey, 1992; Liu and Tripathi, 2008; Kumar and Tripathi, 2012. Vijay and Tripathi (2016) found density ripple to be effective in laser beat frequency heating of unmagnetized plasma. Malik *et al.* (2017) have reported resonant enhancement in two color laser excitation of terahertz radiation due to a density ripple.

In this paper, we study the beat frequency heating of a magnetized plasma in the presence of a density ripple. The magnetic field introduces cyclotron damping as an effective route to energy deposition where finite Larmor radius effects could also play a role. We employ two collinear lasers, with frequency difference near the electron cyclotron frequency, propagating along the static magnetic field but transverse to ripple wave vector. The beat frequency ponderomotive force by the laser drives an electrostatic Bernstein quasi-mode (Kumar and Tripathi, 2010). The mode is cyclotron damped on electrons and gives rise to strong electron heating. We employ fluid theory to obtain electron response to lasers and Vlasov theory to obtain the response of magnetized electrons at the beat frequency, including finite Larmor radius effects.

In Section "Excitation of Bernstein quasi-mode", we deduce the beat frequency electric field produced by two collinear lasers in a rippled density plasma. In Section "Anomalous heating", we obtain the anomalous heating rate of electrons and study the rise in electron temperature. In Section "Discussion", we discuss the results.

Excitation of Bernstein quasi-mode

Consider a magnetized plasma of electron density n_0, electron temperature T_e, and ambient magnetic field $B_s\,\hat{z}$. The plasma has a density ripple,

$$n_0 = n_0^0 + n_q$$

$$n_q = n_{q_0} e^{iqx} \tag{1}$$

Two collinear lasers propagate through the plasma along the magnetic field (Fig. 1),

$$\vec{E}_j = \hat{x} A_j e^{-i(\omega_j t - k_j z)},\ j = 1, 2 \tag{2}$$

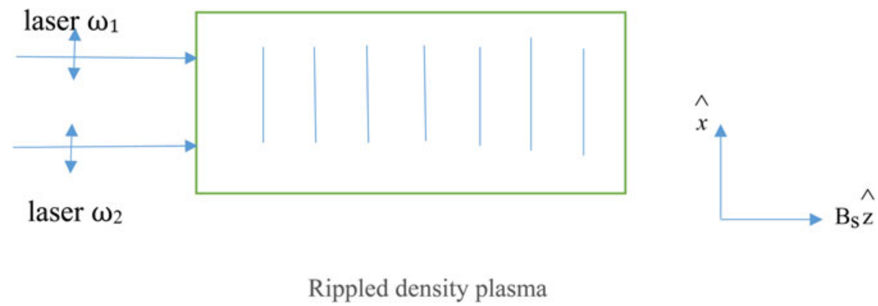

Fig. 1. Schematic of laser beat wave heating of electrons in rippled density plasma in the presence of a parallel static magnetic field.

where $\omega_1 - \omega_2 \approx \omega_c$, $\omega_1 - \omega_2 \gg \omega_p$, ω_c $k_1 \approx (\omega_1/c)$, $k_2 \approx (\omega_2/c)$, $\omega_p = (n_0 e^2/m\varepsilon_0)^{1/2}$ is the plasma frequency, $\omega_c = eB_s/m$ is the electron cyclotron frequency, $-e$ and m are the electron charge and mass, and ε_0 is the free space permittivity. The lasers impart oscillatory velocities to electrons $\vec{v}_1 = ((e\vec{E}_1)/(im\omega_1))$, $\vec{v}_2 = ((e\vec{E}_2)/(im\omega_2))$ and exert a ponderomotive force on them at $\omega = \omega_1 - \omega_2$, $(k_1 - k_2)\hat{z}$,

$$F_{pz} = \frac{e\partial\phi_p}{\partial z} \tag{3}$$

where

$$\phi_p = \frac{A_1 A_2 e}{m\omega_1\,\omega_2} e^{-i[(\omega_1-\omega_2)t-(k_1-k_2)z]} \tag{4}$$

The ponderomotive force creates a velocity perturbation

$$v_{\omega z}^{NL} = -\frac{F_{pz}}{m_i(\omega_1 - \omega_2)} \tag{5}$$

which beats with the density ripple n_q to produce nonlinear density perturbation $n_{\omega,k}^{NL}$ at $\omega = \omega_1 - \omega_2$, $\vec{k} = (k_1 - k_2)\hat{z} + q\hat{x}$. Following Vijay and Tripathi (2016), one may write

$$n_{\omega,k}^{NL} = -\frac{k_z n_q e k_z \phi_p}{2\omega m^2} \tag{6}$$

The nonlinear density perturbation $n_{\omega,k}^{NL}$, produces space charge potential $\phi_{\omega,k}$. This potential produces self-consistent electron density perturbation $n_{\omega,k}^{L}$, called linear density perturbation. Following Vlasov theory one may write $n_{\omega,k}^{L}$ in terms of $\phi_{\omega,k}$ and electron susceptibility χ_e as

$$\chi_e = \frac{2\omega_p^2}{k^2 v_{th}^2}\left[1 + \frac{\omega}{k_z v_{th}} \sum_l Z\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right) I_l(b) e^{-b}\right] \tag{7}$$

where I_l is the modified Bessel function of order l and argument $b = k_\perp^2 v_{th}^2/2\omega_c^2$, $v_{th} = (2T_e/m)^{1/2}$ is the electron thermal velocity, and $\perp$ refers to the component perpendicular to magnetic field. From the Poisson's equation (assuming ions to be immobile),

$$\nabla^2\phi_{\omega,k} = \frac{e}{\varepsilon_0}(n_{\omega,k}^{L} + n_{\omega,k}^{NL}) \tag{8}$$

we obtain

$$\phi_{\omega,k} = -\frac{e}{\varepsilon_0 k}\left[\frac{k\varepsilon_0\chi_e}{e}\phi_{\omega,k} - \frac{k_z n_q e}{2m\omega^2}\phi_p\right] \tag{9}$$

$$\phi_{\omega,k} = \phi_p \frac{\chi_e}{\varepsilon}, \tag{10}$$

where $\varepsilon = 1 + \chi_e$ is the plasma permittivity.

The electric field at ω, k produced by the beating of lasers, is

$$\vec{E}_{\omega,k} = -\nabla\phi_{\omega,k}$$

$$\vec{E}_{\omega,k} = -\frac{ie}{\epsilon_0\,(1 + \chi_e)}\left(\frac{k_z n_q e}{2m\omega^2}\right)\phi_p \tag{11}$$

This field is large when permittivity $\varepsilon(\omega, k) = 1 + \chi_e$ is small, i.e., the ω, k mode is a quasi-Bernstein mode.

Anomalous heating

The beat wave driven space charge field heats the electrons. The time average heating rate per unit volume is given by

$$H = \mathrm{Re}[-(1/2)n_0^0 e\vec{E}_{\omega,k}^{*} \cdot \vec{v}_{\omega,k}] \tag{12}$$

where $\vec{v}_{\omega,k}$ is the electron velocity due to $\vec{E}_{\omega,k}$.

To obtain $\vec{v}_{\omega,k}$ we solve the Vlasov equation for electrons,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f - e(\vec{E}_{\omega,k} + \vec{v} \times B_s\,\hat{z}) \cdot \frac{\partial f}{\partial \vec{v}} = 0 \tag{13}$$

In equilibrium $f = f_0$ which we take to be Maxwellian

$$f_0 = n_0\left(\frac{m}{2\pi T_e}\right)^{3/2} e^{-v_\perp^2/v_{th}^2} e^{-v_z^2/v_{th}^2}$$

and

$$(J_{l+1}(x) + J_{l-1}(x)) = \frac{2l}{x} J_l(x)$$

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Following Kumar and Tripathi (2010), we expand f as $f_0 + f_1$ and linearize the Vlasov equation, we obtain

$$f_1 = -\frac{e\phi}{T_e} f_0 \sum_{l'} J_l\left(\frac{k_\perp v_\perp}{\omega_c}\right) J_{l'}\left(\frac{k_\perp v_\perp}{\omega_c}\right) \frac{l\omega_c + k_z v_z}{\omega - l\omega_c - k_z v_z} e^{-i(l-l')\theta} \tag{14}$$

Using the perturbed distribution function we obtain the drift velocity,

$$\vec{v}_{\omega,k} = \frac{1}{n_0} \int_{-\infty}^{\infty} \int_0^{2\pi} \int_0^{\infty} f_1 \vec{v} dv_\perp d\theta dv_z \tag{15}$$

$$v_{x\,\omega,k} = \frac{e\phi l v_{th}}{T_e \pi \beta}\left[1 + \frac{\omega}{k_z v_{th}} z\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right)\right][I_l(\beta^2/2)e^{-\beta^2/2}] \tag{16}$$

$$v_{z\,\omega,k} = \frac{e\phi 2\pi\omega}{T_e \pi k_z v_{th}}\left[1 + \frac{\omega - l\omega_c}{k_z v_{th}} z\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right)\right][I_l(\beta^2/2)e^{-\beta^2/2}] \tag{17}$$

Thus the heating rate is:

$$\begin{aligned} H_e = &-\frac{n_0^0 e}{2}\left[\frac{ie}{\varepsilon_0}\left(\frac{n_q e}{2m\omega^2}\right)\right]\phi_p^*\left\{(I_l(\beta^2/2)e^{-\beta^2/2})\frac{e}{T_e\pi}\right\} \\ &\left[(k_x + q)\frac{l v_{th}}{\beta}\left(1 + \frac{\omega}{k_z v_{th}} z\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right)\right)\right] \\ &+ \frac{2\pi\omega}{k_z}\left[1 + \frac{\omega - l\omega_c}{k_z v_{th}} z\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right)\right]\phi \end{aligned} \tag{18}$$

We may write $\chi_e = \chi_{er} + i\chi_{ei}$

where

$$\chi_{er} = \frac{2\omega_p^2}{k^2 v_{th}^2}\left[1 - \frac{\omega}{\omega - l\omega_c} I_l(b)e^{-b}\right] \tag{19}$$

As

$$\frac{\chi_e}{\varepsilon} \cong \frac{\chi_{er}}{1 + \chi_{er}}$$

where

$$\frac{\chi_{er}}{1 + \chi_{er}} = \left[1 - \frac{1}{1 + \frac{2\,\omega_p^2}{k^2\,v_{th}^2}\left[1 - \frac{\omega}{\omega - l\omega_c} I_l(b)e^{-b}\right]}\right] \tag{20}$$

The heating rate per electron per second is

$$\begin{aligned} \frac{H}{n_0^0} = &\left(\frac{\omega_p^2}{\omega^2}\right) n q_0 \left(\frac{eA_1}{m\omega_1 v_{th}} \cdot \frac{eA_2}{m\omega_2 v_{th}}\right)^2 \frac{T_e}{\pi} \\ &\left[I_l\left(\frac{1}{2}\frac{k_\perp v_{th}}{\omega_c}\right)^2 e^{-\left(\frac{k_\perp v_{th}}{2\omega_c}\right)^2}\right](k_x + q) \\ &\left[\frac{l\omega_c}{k_\perp v_{th}} \cdot v_{th}\sqrt{\pi}\frac{\omega}{k_z v_{th}} e^{-(\omega - l\omega_c)2/k_z^2 v_{th}^2}\right. \\ &\left. + \frac{\omega}{k_z}\sqrt{\pi}\left(\frac{\omega - l\omega_c}{k_z v_{th}}\right) e^{-(\omega - l\omega_c)^2/k_z^2 v_{th}^2}\right] \\ &\left[1 - \frac{1}{1 + ((2\,\omega_p^2)/(k^2\,v_{th}^2))[1 - (\omega/(\omega - l\omega_c))I_l(b)e^{-b}]}\right] \end{aligned}$$

$$\begin{aligned} \frac{H}{n_0^0} = &\left(\frac{\omega_p^2}{\omega^2}\right) N q_0 \left(\frac{v_1 v_2}{c^2}\right)^2 \frac{mc^2}{8\pi}\left[I_l\left(\frac{\beta^2}{2}\right)e^{-\beta^2/2}\right] \\ &(k_x + q)\left[\left(\frac{l\omega_c}{k_\perp v_{th}} + \frac{\omega - l\omega_c}{k_z v_{th}}\right)\frac{\omega}{k_z}\sqrt{\pi} e^{-(\omega - l\omega_c)2/k_z^2 v_{th}^2}\right] \end{aligned}$$

$$\frac{H}{n_0^0} = hmc^2\omega$$

where

$$\begin{aligned} h = &\left(\frac{\omega_p^2}{\omega^2}\right)\frac{N_{q_0}}{8\pi}\left(\frac{v_1 v_2}{c^2}\right)^2 \frac{k_\perp}{k_z}\sqrt{\pi}\left[I_l\left(\frac{\beta^2}{2}\right)e^{-\beta^2/2}\right] \\ &\left[\left(\frac{l\omega_c}{k_\perp v_{th}} + \frac{\omega - l\omega_c}{k_z v_{th}}\right)\frac{\omega}{k_z}\sqrt{\pi} e^{-(\omega - l\omega_c)2/k_z^2 v_{th}^2}\right] \end{aligned}$$

The equation governing the evolution of electron temperature can be written as

$$\begin{aligned} &\frac{d}{dt}\left(\frac{3}{2}T_e\right) = hmc^2\omega = C_1\frac{\chi_{ei}}{|\varepsilon|^2} \\ &= C_1\left[I_l\left(\frac{\beta^2}{2}\right)e^{-(\beta^2/2)}\right](k_x + q) \\ &\left[\left(\frac{l\omega_c}{k_\perp v_{th}} + \frac{\omega - l\omega_c}{k_z v_{th}}\right)\frac{\omega}{k_z}\sqrt{\pi} e^{-(\omega - l\omega_c)2/k_z^2 v_{th}^2}\right] \\ &\left[1 - \frac{1}{1 + ((2\,\omega_p^2)/(k^2\,v_{th}^2))[1 - (\omega/(\omega - l\omega_c))I_l(b)e^{-b}]}\right] \end{aligned} \tag{21}$$

$$C_1 = \frac{N_{q_0}}{8\pi}\left|\frac{v_1 v_2^*}{c^2}\right|^2 \frac{\omega_p^2}{6\omega^2} \tag{22}$$

For pulsed lasers, e.g., $A_1^2 A_2^2 = A_{10}^2 A_{20}^2 e^{-2t^2/\tau^2}$, C_1 is a function of time and one may solve Eq. (22) numerically to obtain T_e as a function of time. As the electron temperature rises the cyclotron damping of electrons (manifested through the rise in χ_{ei}) increases, hence heating rate becomes stronger. We introduce

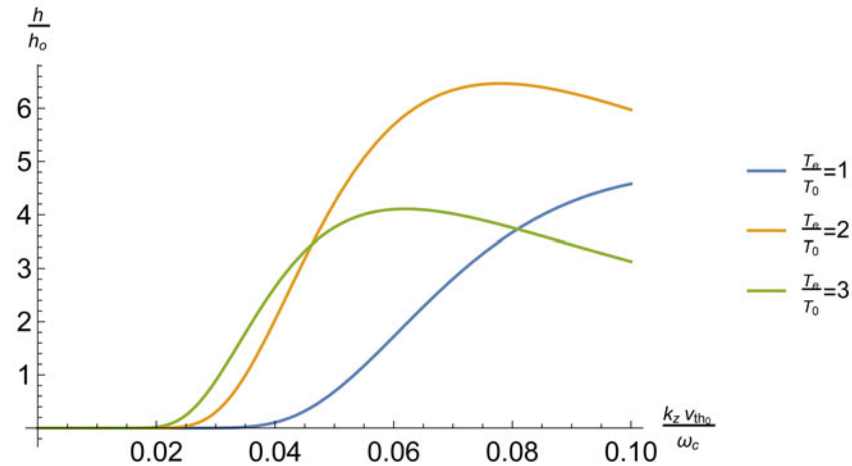

Fig. 2. Normalized heating rate of electrons as a function of normalized wave number $k_z v_{\mathrm{th}_0}/\omega_c$ of the ripple assuming the electron temperature to be clamped for different T_e/T_0.

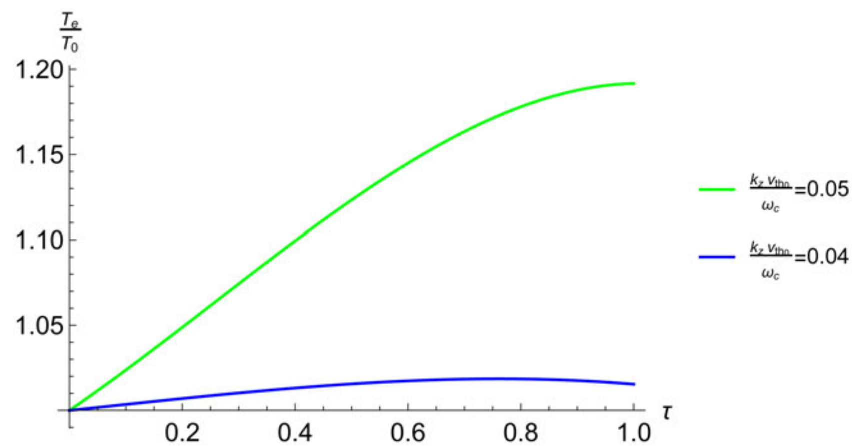

Fig. 3. Variation of electron temperature as a function of normalized time.

normalized laser amplitudes $a_{10} = eA_{10}/m\omega_1 c$, $a_{20} = eA_{20}/m\omega_2 c$ and dimensionalize the normalized heating rate h by a quantity h_0.

$$h_0 = \left(\frac{\omega_p^2}{\omega^2}\right)\frac{N_{q_0}}{8\pi}\left(\frac{v_1 v_2}{c^2}\right)^2\sqrt{\pi}. \tag{23}$$

In Fig. 2, we have plotted the normalized heating rate of electrons h/h_0 as a function of $k_z v_{\mathrm{th}_0}/\omega_c$, the normalized parallel wave number of the quasi-mode (or beat wave number of the lasers), assuming electron temperature to be clamped. The parameters are $(\omega - \omega_c) = 0.1$, $qv_{\mathrm{th}}/\omega_c = \sqrt{2}$.

The heating rate increases with $k_z v_{\mathrm{th}_0}/\omega_c$, attains a maximum when $(\omega - \omega_c)/k_z v_{\mathrm{th}} \approx 1$ and then falls off gradually due to the k_z dependence of the factor outside the exponential. Fig. 3 shows the variation of electron temperature as a function of normalized time.

The electron temperature initially rises with time more than linearly. As the temperature increases cyclotron damping becomes stronger. Beyond the laser intensity peak the rise ion electron temperature is slow and temperature saturates. gradually and eventually saturates with time.

Discussion

A density ripple of 10% having wavelength few times the laser wavelength is effective for beat wave cyclotron heating of electrons. The ripple could be created by a machining laser beam as done by Milchberg *et al.* (2001). The driven beat mode heats the electrons *via* cyclotron damping. Initially the heating is slow. However, as the electron temperature rises the damping becomes more severe and the heating becomes quite efficient. For Gaussian laser beams, the heating rate rises rapidly as the electron temperature increases. After attaining $(\omega - \omega_c)/k_z v_{\mathrm{th}} \approx 1$, the heating rate slows down. The electron temperature rises with time and saturates in the rear phase of the laser pulse.

Heating rate is a function of wave number of the ripple due to finite Larmor radius effects. The temperature is enhanced on increasing the normalized temperature and the less normalized time is required on increasing the normalized temperature. However, the peak for $T_e/T_0 = 2$ is higher than $T_e/T_0 = 3$ due to the term $[I_l(\beta^2/2)e^{-(\beta^2/2)}]$ and exponential term and in Eq. (21).

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