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IMPROVED INSPECTION SCHEMES FOR DETERIORATING EQUIPMENT

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In this paper, we introduce and analyze simple, new inspection policies for maintaining deteriorating equipment with non-self-announcing failures. These policies utilize information from the inspection/repair history as well as the system lifetime distribution to schedule future inspections. Numerical results indicate that the policies can significantly outperform standard (periodic) maintenance strategies by providing higher availability for a given inspection rate.

1. INTRODUCTION

Consider a piece of equipment (hereafter referred to as a *system*) that is placed into service, deteriorates as a response to its usage and its operating environment, and eventually fails. Suppose that failure of the system can be detected only by inspection and that inspections can determine neither the level of deterioration nor the remaining equipment life. Such is the case in many protective systems such as circuit breakers, alarms, and protective relays, as well as in spare or standby systems. If at an inspection the system is found to be failed, it is replaced immediately with a statistically independent and identical (new) system, and the process repeats. A maintenance policy for such equipment (also referred to as an *inspection/repair/replacement* policy) consists of preassigned times at which the equipment is inspected. If an inspection finds the system to be working, the maintenance policy may direct that it be left undisturbed or that it be preventively replaced. To be effective, a maintenance policy should guarantee a high level of availability at any time. Moreover, it should do so at low cost.

Several authors, dating back to Barlow, Hunter, and Proschan [1], have studied the problem of developing an effective maintenance policy for such a system. The work of Valdez-Flores and Feldman [8] provides an excellent reference for research on maintained systems between 1975 and 1987.

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More recently, Yeh [11] studied a similar system that could be repaired (to a working state, but as bad as the old state) and determined a maintenance policy that minimizes long-run expected cost per unit time while ensuring that the time-dependent availability exceeds a given lower bound. Her model considers costs of inspection, repair, and replacement, as well as a fixed "penalty" cost that depends on the lower bound on availability. Unfortunately, even for relatively simple lifetime distributions, it is difficult to explicitly determine an "optimal" (i.e., minimum cost) sequence of inspection times. For more realistic assumptions on the lifetime distribution, the model quickly becomes intractable.

Wortman and Klutke [9] and Klutke, Wortman, and Ayhan [4] derived expressions for availability for a system where failures are not self-announcing and inspections were performed according to a stochastic counting process. In their models, the rate of deterioration was described by a random process governed by an exogenous environment. Both of these papers report qualitative results that lead to lower bounds on availability. Wortman, Klutke, and Ayhan [10] considered a similar model in which the deterioration is caused by random shocks. When shocks occur according to a Poisson process, they showed that, among renewal inspection processes for a given inspection rate, availability is maximized when time between inspections is constant. Castaño-Pardo and Shivani [2] considered inspection policies that are driven by nonhomogeneous Poisson processes for systems with non-self-announcing failures; their results indicate that there is some advantage to nonperiodic inspections.

Our model differs from previous work on several counts. Unlike Yeh, we do not consider "partial" repair. Our interest is in determining simple, implementable maintenance policies with easily computable performance measures. Rather than develop a cost model, we focus on computing the "competing" performance measures of availability and inspection rate. Our belief is that, in practice, costs associated with inspections, replacements, and downtimes are not only difficult to obtain, but are rather meaningless in the current context. For protective equipment, the overiding goal is to ensure that the system will work when needed; availability is of primary interest, and cost is secondary. To achieve a high level of availability requires a certain inspection capacity (which we take to be the long-term inspection rate). Our interest in this paper is in trade-offs between these measures; that is, we wish to determine how much inspection capacity is required to guarantee a specified level of availability and, conversely, the level of availability that can be achieved given a fixed inspection rate. If inspection, replacement, and downtime costs are available, it is not difficult to develop an expression for long-term expected cost from the relationships we derive in this paper.

The paper is organized as follows. Section 2 describes the notation and assumptions used in the paper and defines the performance measures of interest. Section 3 analyzes a common inspection policy that inspects at periodic intervals. This policy is a special case of the more general class of *renewal* inspection policies. In Section 4, we describe and analyze a new inspection policy that is easy to implement and significantly outperforms periodic inspections. Section 5 describes a simple hybrid policy which trades off availability with decreased inspection rate compared

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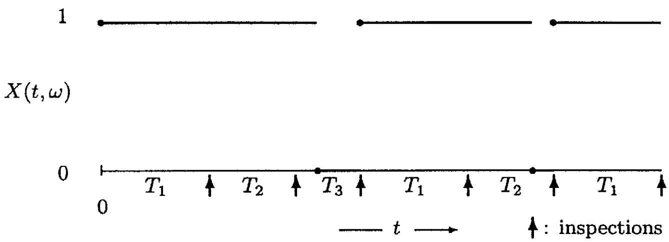

FIGURE 1. Sample path of system state.

to the policy described in Section 4. Section 6 provides numerical results that compare the policies. Finally, Section 7 presents some concluding comments, including some suggestions for obtaining lifetime data.

2. THE MODEL

Let $(\Omega, \mathcal{F}, P)$ be a probability space, and let $\{\mathcal{F}(t), t \geq 0\}$ be a filtration of $\mathcal{F}$, where $\mathcal{F}(t)$ represents the history of the system up to time t. All random variables are defined on this (filtered) probability space. The state of the system is described by a right-continuous random process $\mathcal{X} = \{X(t), t \geq 0\}$, where $X(t)$ is 0 if the system is down at time t and 1 if the system is up at time t. A *maintenance* policy consists of a sequence of inspections ages $\{\tau_1, \tau_2, \ldots\}$ and a replacement age τ_r with $0 < \tau_1 < \cdots < \tau_r$. For simplicity, we will also define interinspection ages $T_1 = \tau_1$ and $T_k = \tau_k - \tau_{k-1}$, $k = 2,3,\ldots$. Beginning with a new system, the system is replaced at the first inspection age at which it is found failed, or at age τ_r. Upon replacement, the process repeats; that is, the new system is again inspected at ages $\tau_1, \tau_2, \ldots$ and replaced when found failed or at age τ_r. Let the sequence of successive lifetimes of new systems be $\{L_1, L_2, \ldots\}$ and the sequence of successive "up" times be $\{U_1, U_2, \ldots\}$, where $U_i = \min(L_i, \tau_r)$. Henceforth, we will suppress the index on the lifetime and use L (respectively U) to designate a random variable independent of and with the same distribution as L_1 (respectively U_1). In this paper, we will assume that L has a known, continuous, strictly increasing cumulative distribution function F with mean μ. Let $\{N_1, N_2, \ldots\}$ denote the successive number of inspections between replacements and let $\{C_1, C_2 \ldots\}$ denote the sequence of replacement cycle lengths (times between replacements). Because of our assumptions about the successive new system lifetimes, $\{N_1, N_2, \ldots\}$ and $\{C_1, C_2 \ldots\}$ are both i.i.d. sequences. Again, we will suppress the subscript and take N and C to be random variables independent of and with the same distribution as N_1 and C_1, respectively. A sample path of system state is shown in Figure 1.

In this paper, we are interested in two performance measures. We define the *limiting average availability* as

$$A_{\text{av}} = \lim_{t\to\infty} \frac{\int_0^t E[X(s)]\,ds}{t}$$

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and the *long-run inspection rate* as

$$\beta = \lim_{t\to\infty} \frac{E[N(t)]}{t},$$

where $N(t)$ is defined as the number of inspections up to time t. Because of the regenerative nature of the system at replacement times, the limiting average availability can be determined by the ratio of expected "up" time in a replacement cycle to expected cycle length,

$$A_{\text{av}} = \frac{E[U]}{E[C]},$$

and the long-run inspection rate by the ratio of expected number of inspections in a cycle to expected cycle length,

$$\beta = \frac{E[N]}{E[C]}.$$

These results follow from basic regenerative process theory (cf. Ross [7, Chap. 3]) and are stated here without proof. In what follows, we will restrict ourselves to the case where τ_r is infinite (i.e., there is no preventive replacement) and hence $E[U] = E[L]$. Analysis of the age replacement case follows similarly but with slightly messier algebra.

3. PERIODIC INSPECTIONS

Perhaps the most widely used maintenance policy in practice is to schedule inspections periodically (i.e., at constant interinspection times) starting at each time of replacement. Such a policy has the advantage of being simple to implement and relatively easy to analyze. Suppose the interinspection time is τ. Then, inspections occur at times $\tau, 2\tau, 3\tau, \ldots$ (i.e., $\tau_n = n\tau$). We will henceforth refer to this policy as PI.

3.1. Availability

To compute the limiting average availability for periodic inspections, we must compute $E[C]$. Now,

$$\begin{aligned} E[C] &= \tau E[N] \\ &= \tau \left(\sum_{m=0}^{\infty} P\{N > m\} \right) \\ &= \tau \left(\sum_{m=0}^{\infty} P\{L > m\tau\} \right) \\ &= \tau \sum_{m=0}^{\infty} \bar{F}(m\tau). \end{aligned}$$

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The limiting average availability is then given by

$$A_{\text{av}} = \frac{\mu}{\tau \sum_{m=0}^{\infty} \bar{F}(m\tau)}.$$

3.2. Inspection Rate

The inspection rate for periodic inspections is easily shown to be τ^{-1}.

4. QUANTILE-BASED INSPECTIONS

The weakness of a periodic inspection strategy is clear: Because it employs no information about the time since the last replacement, it tends to "overinspect" at less likely failure times and "underinspect" at more likely failure times. Thus, it ignores information about the remaining life that is inherent in the sequence of previous inspection times. Consider the following inspection/replacement policy. Fix $0 < \alpha < 1$ and set

$$\tau_1 = \sup\{t > 0\colon P\{L > t\} \ge \alpha\},$$
$$\tau_n = \sup\{t > \tau_{n-1}\colon P\{L > t \mid L > \tau_{n-1}\} \ge \alpha\}, \qquad n \ge 2.$$

If F is continuous and strictly increasing, as we have assumed, then

$$\tau_n = \bar{F}^{-1}(\alpha^n) \quad \text{for } n = 0,1,\ldots.$$

For obvious reasons, we call this inspection policy "quantile-based inspections" with parameter α and denote it by QBI(α). A similar strategy was considered independently by Castaño-Pardo and Shivani [2], although they provide few details about its performance. It has the following structural property.

LEMMA 1: *If the lifetime distribution of L is* IFR (DFR), *then the interinspection times of* QBI(α) *are nonincreasing* (*nondecreasing*).

PROOF: We will prove the lemma for the IFR case; the DFR case follows similarly. If F is IFR, then $\bar{F}(x+t)/\bar{F}(t)$ is nonincreasing in t for all $x > 0$. Therefore, for all n,

$$\frac{P\{L > (\tau_{n+1} - \tau_n) + \tau_n\}}{P\{L > \tau_n\}} \le \frac{P\{L > (\tau_{n+1} - \tau_n) + \tau_{n-1}\}}{P\{L > \tau_{n-1}\}};$$

that is,

$$P\{L > \tau_{n+1} \mid L > \tau_n\} \le P\{L > (\tau_{n+1} - \tau_n) + \tau_{n-1} \mid L > \tau_{n-1}\}.$$

Now, by definition,

$$\tau_{n+1} = \sup\{t > \tau_n : P\{L > t \mid L > \tau_n\} \ge \alpha\}$$

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and, thus,

$$P\{L > (\tau_{n+1} - \tau_n) + \tau_{n-1} | L > \tau_{n-1}\} \geq \alpha.$$

However,

$$\tau_n = \sup\{t > \tau_{n-1}\colon P\{L > t | L > \tau_{n-1}\} \geq \alpha\}.$$

This implies that

$$(\tau_{n+1} - \tau_n) + \tau_{n-1} \leq \tau_n$$

and, therefore,

$$\tau_{n+1} - \tau_n \leq \tau_n - \tau_{n-1};$$

hence, the interinspection times form a nonincreasing sequence. ■

Note that if the lifetime distribution is exponential, then interinspection times under QBI are constant, and QBI reduces to PI.

4.1. Availability

The expected cycle length for the QBI(α) policy is given by

$$\begin{aligned} E[C] &= \sum_{m=1}^{\infty} \tau_m P\{\tau_{m-1} < L \leq \tau_m\} \\ &= \sum_{m=1}^{\infty} \tau_m (\bar{F}(\tau_{m-1}) - \bar{F}(\tau_m)) \\ &= \sum_{m=1}^{\infty} \tau_m (\alpha^{m-1} - \alpha^m) \\ &= (1-\alpha) \sum_{m=1}^{\infty} \bar{F}^{-1}(\alpha^m) \alpha^{m-1}. \end{aligned}$$

The limiting average availability is then given by

$$A_{\text{av}} = \frac{\mu}{(1-\alpha) \sum_{m=1}^{\infty} \bar{F}^{-1}(\alpha^m) \alpha^{m-1}}.$$

4.2. Inspection Rate

Note that the QBI policy has the property that the conditional probability that an inspection finds the system failed, given that the system is working at the previous inspection, is a constant, $1 - \alpha$. Thus, the number of inspections required to find a

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failure on each cycle has a geometric distribution, and the long-run inspection rate is then given by

$$\beta = \frac{1/(1-\alpha)}{(1-\alpha)\sum_{m=1}^{\infty}\tau_m \alpha^{m-1}} = \frac{1}{(1-\alpha)^2\sum_{m=1}^{\infty}\tau_m \alpha^{m-1}}.$$

5. HYBRID INSPECTION POLICIES

As the numerical results in Section 6 will indicate, for a fixed inspection rate QBI can offer significantly higher availability than PI. There are, however, some potential weaknesses to QBI. When the failure rate of the lifetime distribution is strictly increasing and the system experiences a particularly long lifetime, the inspection sequence becomes a "death watch"; the inspections become so frequent that, in effect, the system is monitored continuously. Similarly, if the lifetime distribution is DFR, QBI might call for almost continuous monitoring of the system in the early phases of operation. In practice, we might not have the resources to monitor the system continuously. In the IFR case, for example, we might consider preventively replacing the system when some minimum interinspection time is reached. Alternatively, we can consider maintenance policies that avoid continuous monitoring. In this section, we develop and analyze two hybrid inspection strategies that offer availability comparable to QBI but at a (sometimes considerably) smaller inspection rate. These strategies are tailored to lifetime distributions that can be classified as IFR or DFR.

5.1. A Hybrid Inspection Policy for IFR Lifetime Distributions

For IFR lifetime distributions, consider the following maintenance policy:

$$\tilde{\tau}_n = \tau_n, \qquad n \le M,$$

$$\tilde{\tau}_n = \tilde{\tau}_M + (n-M)\tilde{\tau}, \qquad n > M,$$

where τ_n is the nth inspection age determined by QBI(α), M is a positive integer, and $\tilde{\tau} = \tau_M - \tau_{M-1}$. We will denote this policy by HYBI(α, M). Note that HYBI(α, M) behaves like QBI until the Mth inspection; thereafter, it behaves like PI. Figure 2 presents an example of inspection ages for QBI and HYBI with $M = 3$.

Note that HYBI(α, M) trades off availability for a smaller inspection rate in the tail of the system lifetime distribution. The following lemma compares the expected number of inspections per cycle for QBI(α) and HYBI(α, M).

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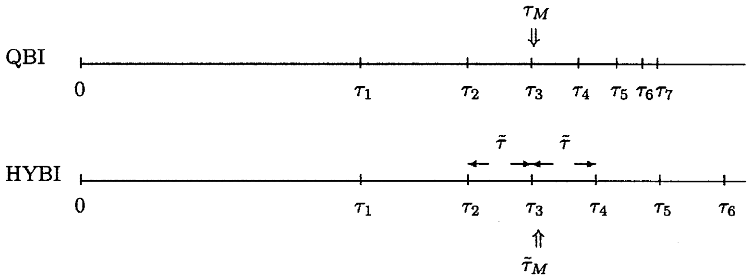

FIGURE 2. Inspection ages for QBI and HYBI with $M = 3$.

LEMMA 2: *For a given* $0 < \alpha < 1$ *and integer* M, *let* N_{QBI} *and* N_{HYBI} *denote the number of inspections per cycle for* $\text{QBI}(\alpha)$ *and* $\text{HYBI}(\alpha, M)$, *respectively. Then,*

$$E[N_{\text{HYBI}}] \le E[N_{\text{QBI}}].$$

PROOF: For $\text{HYBI}(\alpha, M)$, the expected number of inspections per cycle is

$$\begin{aligned} E[N_{\text{HYBI}}] &= \sum_{m=1}^{M} mP\{\tau_{m-1} < L \le \tau_m\} \\ &\quad + \sum_{m=1}^{\infty} (M+m)P\{\tau_M + (m-1)\tilde{\tau} < L \le \tau_M + m\tilde{\tau}\} \\ &= (1-\alpha)\sum_{m=1}^{M} m\alpha^{m-1} + \alpha^M M + \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau}) \\ &= \frac{1-\alpha^M}{1-\alpha} + \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau}). \end{aligned}$$

Thus, we have

$$\begin{aligned} E[N_{\text{QBI}}] - E[N_{\text{HYBI}}] &= \frac{\alpha^n}{1-\alpha} - \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau}) \\ &= \sum_{m=0}^{\infty} \bar{F}(\tau_{M+m}) - \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau}). \end{aligned}$$

For IFR lifetime distributions, $\{\tau_n - \tau_{n-1}, n = 1,2,\ldots\}$ is decreasing and, hence, for $m = 1,2,\ldots,$

$$\begin{aligned} \tau_{M+m} &= \tau_M + \sum_{k=1}^{m} (\tau_{M+k} - \tau_{M+k-1}) \\ &\le \tau_M + m(\tau_M - \tau_{M-1}) \\ &= \tau_M + m\tilde{\tau}. \end{aligned}$$

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Because $\bar{F}$ is a decreasing function, for each $m = 1, 2, \ldots,$

$$\bar{F}(\tau_{M+m}) \geq \bar{F}(\tau_M + m\tilde{\tau}),$$

and the result follows. ■

5.1.1. HYBI availability. The expected cycle length for $\mathrm{HYBI}(\alpha, M)$ is given by

$$\begin{aligned} E[C] &= \sum_{m=1}^{M} \tau_m P\{\tau_{m-1} < L \leq \tau_m\} \\ &\quad + \sum_{m=1}^{\infty} (\tau_M + m\tilde{\tau}) P\{\tau_M + (m-1)\tilde{\tau} < L \leq \tau_M + m\tilde{\tau}\} \\ &= (1-\alpha) \sum_{m=1}^{M} \tau_m \alpha^{m-1} + \alpha^M \tau_M + \tilde{\tau} \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau}) \end{aligned}$$

and, hence, the availability can be computed as

$$A_{\mathrm{av}} = \frac{\mu}{(1-\alpha) \sum_{m=1}^{M} \tau_m \alpha^{m-1} + \alpha^M \tau_M + \tilde{\tau} \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau})}.$$

5.1.2. HYBI inspection rate. From Lemma 2, we have

$$E[N_{\mathrm{HYBI}}] = \frac{1-\alpha^M}{1-\alpha} + \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau})$$

and, thus, the long-run inspection rate is

$$\beta = \frac{\dfrac{1-\alpha^M}{(1-\alpha)} + \displaystyle\sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau})}{(1-\alpha) \displaystyle\sum_{m=1}^{M} \tau_m \alpha^{m-1} + \alpha^M \tau_M + \tilde{\tau} \sum_{m=0}^{\infty} \bar{F}(\tau_M + m\tilde{\tau})}.$$

5.2. A Hybrid Inspection Policy for DFR Lifetime Distributions

If the lifetime distribution is DFR, interinspection times under QBI are increasing and hence, our hybrid inspection policy is constructed in a different way. Again, let $\{\tau_1, \tau_2, \tau_3, \ldots\}$ denote inspection ages determined by $\mathrm{QBI}(\alpha)$. For a fixed positive integer M, let

$$M' = \min\left\{n \in \mathcal{N};\, n \geq \frac{\tau_M}{\tau_{M+1} - \tau_M}\right\},$$

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where $\mathcal{N} = \{1,2,3,\ldots\}$, and let

$$\tau' = \frac{\tau_M}{M'}.$$

Note that

$$M = \frac{M(\tau_{M+1} - \tau_M)}{\tau_{M+1} - \tau_M} \geq \frac{\sum_{m=1}^{M} (\tau_m - \tau_{m-1})}{\tau_{M+1} - \tau_M} = \frac{\tau_M}{\tau_{M+1} - \tau_M}$$

and, hence, $M' \leq M$.

Now consider the following inspection policy:

$$\tau'_n = n\tau', \qquad n \leq M',$$
$$\tau'_n = \tau_{K+n}, \qquad n > M',$$

where $K = M - M'$. We will refer to this inspection policy as HYBD. Figure 3 presents an example of HYBD with $M = 4$ (in this case, $M' = 2$).

The following lemma shows that the same relationship between the expected number of inspections holds for HYBD as for HYBI.

LEMMA 3: *For a given* $0 < \alpha < 1$ *and integer* M, *let* N_{HYBD} *denote the number of inspections per cycle for* HYBD(α, M), *respectively*. *Then*,

$$E[N_{\text{HYBD}}] \leq E[N_{\text{QBI}}].$$

PROOF: For HYBD(α, M), the expected number of inspections per cycle is given by

$$\begin{aligned} E[N_{\text{HYBD}}] &= \sum_{m=1}^{M'} mP\{(m-1)\tau' < L \leq m\tau'\} + \sum_{m=M'+1}^{\infty} mP\{\tau'_{m-1} < L \leq \tau'_m\} \\ &= \sum_{m=0}^{M'-1} \bar{F}(m\tau') - M'\alpha^M + \alpha^{M-M'}(1-\alpha) \sum_{m=M'+1}^{\infty} m\alpha^{m-1} \\ &= \sum_{m=0}^{M'-1} \bar{F}(m\tau') + \frac{\alpha^M}{1-\alpha}. \end{aligned}$$

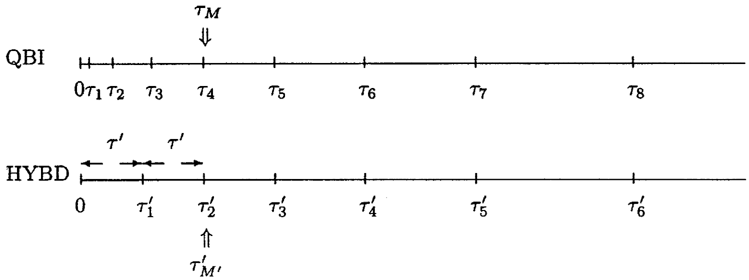

FIGURE 3. Inspection ages for QBI and HYBD with $M = 4$.

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Then,

$$E[N_{\mathrm{QBI}}] - E[N_{\mathrm{HYBD}}] = \frac{1-\alpha^M}{1-\alpha} - \sum_{m=0}^{M'-1} \bar{F}(m\tau')$$

$$= \sum_{m=0}^{M-1} \bar{F}(\tau_m) - \sum_{m=0}^{M'-1} \bar{F}(m\tau')$$

$$= \sum_{m=0}^{M'-1} (\bar{F}(\tau_m) - \bar{F}(m\tau')) + \sum_{m=M'}^{M-1} \bar{F}(\tau_m).$$

Recall that

$$\tau' = \frac{\tau_M}{M'} \geq \frac{\tau_M}{M}.$$

Because the lifetime is DFR, $\tau_M/M \geq \tau_m/m$ for $m = 0,1,\ldots,M$. Thus, for $m = 0,1,\ldots,M'-1$,

$$\tau_m \leq m\tau'$$

and, hence,

$$\bar{F}(\tau_m) \geq \bar{F}(m\tau').$$

Therefore,

$$E[N_{\mathrm{HYBD}}] \leq E[N_{\mathrm{QBI}}]. \qquad \blacksquare$$

5.2.1. HYBD availability. The expected cycle length for $\mathrm{HYBD}(\alpha, M)$ is given by

$$E[C] = \sum_{m=1}^{M'} m\tau' P\{(m-1)\tau' < L \leq m\tau'\}$$

$$+ \sum_{m=M'+1}^{\infty} \tau_{K+m} P\{\tau_{K+(m-1)} < L \leq \tau_{K+m}\}$$

$$= \tau' \sum_{m=0}^{M'-1} \bar{F}(m\tau') - \tau_M \alpha^M + (1-\alpha) \sum_{m=M+1}^{\infty} \tau_m \alpha^{m-1}.$$

The limiting average availability is computed as

$$A_{\mathrm{av}} = \frac{\mu}{\tau' \sum_{m=0}^{M'-1} \bar{F}(m\tau') - \tau_M \alpha^M + (1-\alpha) \sum_{m=M+1}^{\infty} \tau_m \alpha^{m-1}}.$$

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5.2.2. HYBD inspection rate. The expected number of inspections per cycle is given by

$$E[N_{\mathrm{HYBD}}] = \sum_{m=0}^{M'-1} \bar{F}(m\tau') + \frac{\alpha^M}{1-\alpha}$$

and the long-run inspection rate is computed by

$$\beta = \frac{\displaystyle\sum_{m=0}^{M'-1} \bar{F}(m\tau') + \frac{\alpha^M}{1-\alpha}}{\displaystyle\tau' \sum_{m=0}^{M'-1} \bar{F}(m\tau') - \tau_M \alpha^M + (1-\alpha) \sum_{m=M+1}^{\infty} \tau_m \alpha^{m-1}}.$$

6. NUMERICAL RESULTS

Because the QBI and hybrid inspection policies involve the evaluation of quantile functions, it is difficult to compare them analytically with PI. However, for a given lifetime distribution, the availability and inspection rate for each policy are easy to compute numerically. Tables 1–6 present numerical results that illustrate the advantages afforded by QBI and HYBI or HYBD over PI. Tables 1–5 show examples from the Weibull family of distributions, as this family is widely used for modeling both IFR and DFR lifetime distributions. Recall that the Weibull distribution function is given by

$$F(x) = 1 - \exp\left\{-\left(\frac{x}{\theta}\right)^{\gamma}\right\}, \quad x \geq 0, \theta, \gamma > 0.$$

For values of the shape parameter γ less than 1, the distribution is DFR. For values of γ greater than 1, the distribution is IFR. For $\gamma = 2$, the failure rate is linearly increasing, and for $\gamma > 2$, the failure rate is convex increasing.

Finally, to further illustrate the advantages of QBI and HYBD over PI for DFR distributions, Table 6 presents results for a mixed exponential lifetime distribution. The mixed exponential distribution is often used to model system burn-in (Kuo, Chien, and Kim [5]). Its distribution function is given by

$$F(x) = \sum_{i=1}^{k} \lambda_i e^{-x/\mu_i}, \quad \text{where } \sum_{i=1}^{k} \lambda_i = 1.$$

For Table 6, we used $k = 4$ and

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.2, \quad \lambda_3 = 0.1, \quad \lambda_4 = 0.5,$$
$$\mu_1 = 1.0, \quad \mu_2 = 3.0, \quad \mu_3 = 7.0, \quad \mu_4 = 100.0.$$

To compare PI and QBI in the tables, we have fixed the inspection rate β and computed the corresponding availability. Note that as α increases, the performance of all policies converges, but at smaller values of α, QBI can significantly outperform PI. It is clear that in many situations, HYBI (HYBD) can achieve an almost identical availability as QBI with a much smaller inspection rate.

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6.1. Comparison of Inspection Policies for IFR Lifetimes

Table 1. $L \sim$ Weibull(2,10)

		PI	QBI	HYB					
				$M=2$	$M=4$	$M=6$	$M=8$	$M=10$	$M=30$
$\alpha = 0.50$	A_{av}	0.760	0.790	0.778	0.789	0.790	0.790	0.790	0.790
	β	0.178	0.178	0.166	0.177	0.178	0.178	0.178	0.178
$\alpha = 0.60$	A_{av}	0.806	0.833	0.816	0.830	0.832	0.833	0.833	0.833
	β	0.235	0.235	0.208	0.229	0.233	0.235	0.235	0.235
$\alpha = 0.80$	A_{Av}	0.901	0.915	0.886	0.907	0.911	0.913	0.914	0.915
	β	0.516	0.516	0.370	0.454	0.485	0.500	0.507	0.516
$\alpha = 0.85$	A_{av}	0.926	0.935	0.905	0.925	0.930	0.933	0.934	0.935
	β	0.703	0.703	0.454	0.578	0.632	0.660	0.677	0.703
$\alpha = 0.90$	A_{av}	0.950	0.956	0.924	0.943	0.949	0.952	0.953	0.956
	β	1.079	1.079	0.596	0.791	0.888	0.947	0.985	1.073
$\alpha = 0.95$	A_{av}	0.975	0.977	0.948	0.963	0.969	0.971	0.973	0.977
	β	2.205	2.205	0.915	1.276	1.487	1.631	1.738	2.115
$\alpha = 0.99$	A_{av}	0.995	0.995	0.977	0.985	0.988	0.989	0.990	0.994
	β	11.230	11.230	2.252	3.339	4.067	4.632	5.099	7.669

Table 2. $L \sim$ Weibull(4,10)

		PI	QBI	HYB					
				$M=2$	$M=4$	$M=6$	$M=8$	$M=10$	$M=30$
$\alpha = 0.50$	A_{av}	0.776	0.866	0.856	0.865	0.866	0.866	0.866	0.866
	β	0.191	0.191	0.175	0.189	0.191	0.191	0.191	0.191
$\alpha = 0.60$	A_{av}	0.817	0.891	0.878	0.890	0.891	0.891	0.891	0.891
	β	0.246	0.246	0.210	0.238	0.244	0.245	0.246	0.246
$\alpha = 0.80$	A_{av}	0.904	0.942	0.917	0.936	0.940	0.941	0.941	0.942
	β	0.520	0.520	0.336	0.438	0.478	0.498	0.508	0.520
$\alpha = 0.85$	A_{av}	0.927	0.955	0.927	0.947	0.952	0.953	0.954	0.955
	β	0.702	0.702	0.394	0.540	0.608	0.645	0.666	0.702
$\alpha = 0.90$	A_{av}	0.951	0.968	0.937	0.958	0.963	0.966	0.967	0.968
	β	1.068	1.068	0.485	0.704	0.822	0.895	0.944	1.061
$\alpha = 0.95$	A_{av}	0.975	0.983	0.950	0.970	0.975	0.978	0.980	0.983
	β	2.169	2.169	0.662	1.035	1.271	1.440	1.568	2.046
$\alpha = 0.99$	A_{av}	0.995	0.996	0.968	0.982	0.987	0.989	0.991	0.995
	β	10.990	10.990	1.212	2.091	2.744	3.281	3.741	6.522

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Table 3. $L \sim \text{Weibull}(8,10)$

		PI	QBI	HYB					
				$M=2$	$M=4$	$M=6$	$M=8$	$M=10$	$M=30$
$\alpha=0.50$	A_{av}	0.801	0.923	0.917	0.922	0.923	0.923	0.923	0.923
	β	0.196	0.196	0.178	0.193	0.196	0.196	0.196	0.196
$\alpha=0.60$	A_{av}	0.822	0.937	0.929	0.936	0.937	0.937	0.937	0.937
	β	0.249	0.249	0.211	0.240	0.247	0.248	0.249	0.249
$\alpha=0.80$	A_{av}	0.906	0.966	0.949	0.962	0.964	0.965	0.965	0.966
	β	0.513	0.513	0.320	0.425	0.468	0.489	0.499	0.513
$\alpha=0.85$	A_{av}	0.928	0.973	0.954	0.968	0.971	0.972	0.972	0.973
	β	0.689	0.689	0.368	0.516	0.587	0.626	0.649	0.688
$\alpha=0.90$	A_{av}	0.951	0.981	0.960	0.974	0.978	0.979	0.980	0.981
	β	1.041	1.041	0.440	0.658	0.779	0.855	0.907	1.033
$\alpha=0.95$	A_{av}	0.975	0.989	0.965	0.981	0.985	0.986	0.987	0.989
	β	2.101	2.101	0.571	0.929	1.163	1.333	1.464	1.967
$\alpha=0.99$	A_{av}	0.995	0.997	0.973	0.987	0.991	0.993	0.994	0.997
	β	10.591	10.591	0.925	1.685	2.273	2.769	3.201	5.922

6.2. Comparison of Inspection Policies for DFR Lifetimes

Table 4. $L \sim \text{Weibull}(0.3,1)$

		PI	QBI	HYBD					
				$M=2$	$M=4$	$M=6$	$M=10$	$M=30$	$M=50$
$\alpha=0.50$	A_{av}	0.628	0.693	0.630	0.249	0.146	0.043	0.003	0.001
	β	0.150	0.150	0.102	0.030	0.107	0.005	0.000	0.000
$\alpha=0.60$	A_{av}	0.705	0.766	0.742	0.471	0.325	0.114	0.009	0.003
	β	0.207	0.207	0.152	0.067	0.042	0.013	0.001	0.000
$\alpha=0.80$	A_{av}	0.855	0.893	0.892	0.868	0.839	0.679	0.137	0.044
	β	0.482	0.482	0.404	0.286	0.240	0.138	0.016	0.005
$\alpha=0.85$	A_{av}	0.892	0.921	0.921	0.914	0.904	0.841	0.324	0.121
	β	0.663	0.663	0.578	0.442	0.387	0.260	0.045	0.014
$\alpha=0.90$	A_{av}	0.929	0.948	0.948	0.947	0.945	0.932	0.684	0.385
	β	1.024	1.024	0.932	0.773	0.706	0.538	0.160	0.058
$\alpha=0.95$	A_{av}	0.966	0.975	0.975	0.975	0.974	0.974	0.951	0.883
	β	2.105	2.105	2.005	1.819	1.734	1.504	0.796	0.439
$\alpha=0.99$	A_{av}	0.994	0.995	0.995	0.995	0.995	0.995	0.995	0.995
	β	10.744	10.744	10.638	10.428	10.325	10.023	8.734	7.627

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TABLE 5. $L \sim$ Weibull(0.6,1)

		PI	QBI	HYBD					
				$M=2$	$M=4$	$M=6$	$M=10$	$M=30$	$M=50$
$\alpha=0.50$	A_{av}	0.680	0.697	0.674	0.534	0.431	0.314	0.167	0.121
	β	0.926	0.926	0.851	0.524	0.369	0.238	0.114	0.081
$\alpha=0.60$	A_{av}	0.752	0.768	0.756	0.661	0.572	0.449	0.262	0.195
	β	1.276	1.276	1.211	0.823	0.604	0.395	0.190	0.135
$\alpha=0.80$	A_{av}	0.884	0.893	0.891	0.872	0.846	0.789	0.629	0.536
	β	2.967	2.967	2.930	2.421	2.027	1.499	0.757	0.539
$\alpha=0.85$	A_{av}	0.915	0.921	0.920	0.911	0.898	0.865	0.754	0.677
	β	4.081	4.081	4.052	3.510	3.063	2.400	1.281	0.913
$\alpha=0.90$	A_{av}	0.945	0.948	0.948	0.945	0.940	0.927	0.870	0.823
	β	6.303	6.303	6.283	5.705	5.197	4.367	2.596	1.878
$\alpha=0.95$	A_{av}	0.973	0.975	0.975	0.974	0.973	0.971	0.957	0.943
	β	12.955	12.955	12.945	12.327	11.749	10.707	7.748	6.015
$\alpha=0.99$	A_{av}	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.994
	β	66.131	66.131	66.128	65.474	64.829	63.566	58.878	54.721

TABLE 6. Mixed Exponential Life Distribution

		PI	QBI	HYBD					
				$M=2$	$M=4$	$M=6$	$M=10$	$M=30$	$M=50$
$\alpha=0.50$	A_{av}	0.659	0.700	0.631	0.567	0.495	0.495	0.495	0.495
	β	0.027	0.027	0.023	0.019	0.014	0.014	0.014	0.014
$\alpha=0.60$	A_{av}	0.730	0.766	0.652	0.605	0.595	0.587	0.579	0.578
	β	0.037	0.037	0.024	0.022	0.021	0.020	0.020	0.020
$\alpha=0.80$	A_{av}	0.872	0.893	0.887	0.846	0.774	0.771	0.769	0.769
	β	0.087	0.087	0.072	0.058	0.046	0.045	0.045	0.045
$\alpha=0.85$	A_{av}	0.907	0.921	0.919	0.889	0.838	0.830	0.826	0.825
	β	0.119	0.119	0.104	0.077	0.064	0.063	0.062	0.062
$\alpha=0.90$	A_{av}	0.940	0.948	0.948	0.946	0.936	0.895	0.886	0.885
	β	0.184	0.184	0.184	0.153	0.126	0.101	0.097	0.096
$\alpha=0.95$	A_{av}	0.972	0.975	0.975	0.974	0.974	0.972	0.945	0.945
	β	0.378	0.378	0.378	0.361	0.345	0.285	0.199	0.197
$\alpha=0.99$	A_{av}	0.995	0.995	0.995	0.995	0.995	0.995	0.995	0.995
	β	1.932	1.932	1.932	1.932	1.932	1.913	1.775	1.500

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7. CONCLUSIONS

We have developed and analyzed maintenance policies for deteriorating systems that use information about the inspection history and system lifetime distribution to schedule future inspections. The inspection schedule for each of these policies is very simple to compute. Moreover, the policies are easy to implement and can afford significant advantages over periodic maintenance schedules.

All of the policies we consider in this paper require some information about the distribution of system lifetime. Because system lifetimes are determined both by nominal life and by environmental factors in which the system operates, even if the distribution of nominal life is known, it may be difficult to derive the system lifetime distribution analytically. Several authors (cf. Çinlar [3] and Özekici [6]) have directly investigated the deterioration process, but to our knowledge, no one has developed analytical expressions of the lifetime distribution for general nominal initial life and deterioration processes. In some cases, it may be appropriate to estimate the distribution of nominal life separately from degradation caused by operating conditions. Because nominal life is determined primarily by the manufacturing process, it can be estimated, for example, in controlled laboratory experiments. The environmental process can then be modeled separately, and lifetime data generated via simulation. Such an approach, coupled with the strategies suggested in this paper, should lead to reasonable maintenance strategies for systems that operate in a wide range of environmental conditions.

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