B.A.J. 1, II, 155–330 (1995)

CAPITAL PROJECTS

BY C. G. LEWIN, F.I.A., S. A. CARNE, M.A., F.I.A., M.B.A.E,
N. F. C. DE RIVAZ, M.A., F.I.A., R. E. G. HALL, B.SC., F.I.A.,
K. J. MCKELVEY, B.SC., M.B.A., F.I.A.
AND A. D. WILKIE, M.A., F.F.A., F.I.A.

[Presented to the Faculty of Actuaries, 21 November 1994,
and to the Institute of Actuaries, 28 November 1994]

ABSTRACT

The paper surveys the present state of knowledge about the appraisal and control of capital projects, and identifies the useful contribution which actuaries can make to the process.

Relevant theoretical considerations are discussed, including the choice of appropriate discount rates for use by companies when screening proposals. It is recommended that discount rates should be lower than those commonly used. Some practical working methods are summarised and examples are given of how risk analysis can be carried out, for both simple and complex appraisals. Attention is drawn to the special 'social' considerations which may apply in the case of Government projects. The opportunities now opening up for joint ventures between the U.K. Government and private sector partners are examined, and it is concluded that proper sharing of costs and risks can facilitate arrangements which are acceptable to both sides.

The paper concludes with a discussion of the procedures which companies can use to identify, appraise, control and monitor projects.

KEYWORDS

Appraisal Techniques; Capital Projects; Cost Benefit Analysis; Discounted Cash Flow; Internal Rate of Return; Investment Appraisals; Net Present Value; Private Finance Initiative; Project Appraisals; Project Control/Monitoring; Risk Analysis

Every increase of capital is the result of a past and the cause of a future augmentation of produce.
Jane Marcet, 1816

1. INTRODUCTION AND BACKGROUND

1.1 The purpose of this paper is to survey the present state of knowledge about the appraisal and control of capital projects, and to identify the contribution which actuaries can make.

1.2 Many of the topics discussed in the paper have been extensively debated by the authors, but it has not been possible to achieve complete agreement in every instance. Despite this, we have tried to indicate a definite viewpoint wherever practicable, in order to stimulate discussion. However, it should not be assumed that all the authors necessarily subscribe fully to every viewpoint in the paper.

1.3 This paper brings together a number of results that will be of interest to actuaries (and others) involved in the appraisal of capital projects. Most of the theoretical material appears in textbooks such as those by Brealey & Myers (1991), Levy & Sarnat (1990), Lumby (1991), Samuels, Wilkes & Brayshaw (1990), Weston & Copeland (1988), and many others, to which readers interested in the underlying theory should refer. One result that may be new is the derivation of what we call the 'beta-line' directly from the portfolio selection model (see Appendix C).

1.4 Part of the research for this paper is too long to sit comfortably in an appendix, and it is included as a separate paper by A. D. Wilkie, 'The Risk Premium on Ordinary Shares' which accompanies this paper and is presented along with it. This historical analysis is a useful background against which to assess investors' current expectations of the return from the United Kingdom equity market and hence the returns which companies should be seeking from the projects in which they invest.

1.5 A 'capital project' may be defined in a wide sense as meaning any scheme which involves the investment of resources at the outset, in return for the expectation of a net benefit at a later stage. In this paper, however, we shall use the term in a narrower sense to include only those projects where the investment has significant physical, social or organisational consequences and is not merely to secure a transfer of ownership of an existing asset. The definition therefore includes such schemes as the following:

(a) physical construction, such as building new factories, warehouses, shops, offices, houses, plant, equipment, transport infrastructure, hospitals, schools, prisons, ships, etc;
(b) starting a new business producing goods or services, or a new product line in an existing business;
(c) taking over and modernising an existing business or physical asset;
(d) developing a new asset for an existing business, for example a new computer system or quality control system; and
(e) repairing or renewing an existing asset.

1.6 Although we shall consistently use the narrower definition, it is worth pointing out that some of the considerations we discuss are also relevant to projects falling outside that definition, e.g. portfolio investment.

1.7 Capital projects have been undertaken since the beginning of time. Obvious early examples are the Egyptian pyramids, Stonehenge, and medieval cathedrals and castles. During the Industrial Revolution there was a vast investment in factories, coal mines and plant and machinery. Today we are still deriving benefit from some of the Victorian capital projects, such as the railway system, the Royal Albert Hall, Tower Bridge and the Suez Canal. Some capital projects are ephemeral, whereas others have long-enduring effects. Even those which *appear* to be ephemeral may have hidden long-term effects because they stimulate later projects which endure. The people who put time, effort and expense into the capital project of perfecting the first printing press using movable types in the 15th century started a chain of events which has had incalculable consequences and stimulated innumerable later capital projects.

1.8 How did our predecessors appraise capital projects and avoid wasted investment? We assume they did not use sophisticated discounted cash flow analysis. They were probably guided more by simple concepts, such as the likely rate of return on the investment in the case of a commercial venture, or the social benefit it would create in the case of a public project. They did make some effort to identify the risks and practical difficulties inherent in projects. Many schemes were aborted at the design and discussion stage, for example some of the proposals for rebuilding London after the Great Fire in 1666. Despite its lack of sophistication, the process used resulted in a considerable amount of capital investment which we would regard today as having brought worthwhile benefits.

1.9 Since the Second World War, accountants and financial economists have made attempts to systematise the process of appraising projects, using a variety of modern management tools, including some hitherto used mainly by actuaries. Advances have also been made in the physical and financial control of projects, for example in the introduction of critical path analysis, budgeting, etc. Despite these improvements, there are many examples in recent years where capital investment has been wholly or partially wasted, or where the costs have exceeded expectations, such as tower blocks, railway marshalling yard modernisations in the 1960s, the British Library, and office developments of the late 1980s. Some further examples of cost over-runs are given by Taylor (1994). On the other hand, there are many examples of successful projects, such as London's Victoria Line in the 1960s, the development of Britain's motorway system and Japan's factories for producing consumer goods.

1.10 It seems probable that many projects will be worthwhile in the early stages of development of any country, despite the inevitably high risks for the sponsors of such projects. Although wasteful investment, perhaps for political or prestige reasons, cannot always be ruled out in those early stages, a large project which shows promise when properly appraised will often bring social, as well as financial, benefits which will justify the effort and expense involved. For example, the first bridge over a river will open up opportunities for all kinds of further development on both sides. The second bridge will be more controversial, however — witness the current debate about the desirability or otherwise of building a second road bridge over the River Forth near Edinburgh. Large projects often bring disbenefits to some people in developed countries, and the difficulties need to be balanced against the greater good which the project may bring to the community as a whole; the Channel Tunnel Rail Link is a good example of this. As a country develops, it is more likely that large projects will meet increasing opposition and therefore come under greater political scrutiny. In addition, pressure on the world's finite resources will make wasteful investment less easy to accept.

1.11 It therefore seems likely that more and more attention, on a world-wide basis, will be paid in future to the appraisal and discussion of proposed capital projects. Rigorous analysis of the resource implications and the benefits and disbenefits will include a review of the wider considerations such as pollution, use of scarce resources, disruption for third parties, safety, social benefit, etc. One of the advantages of producing a proper appraisal of a large project is that it may help to secure political acceptability by removing some of the uncertainties in the minds of non-specialists, including, perhaps, the decision-makers themselves. Moreover, a cautious decision-maker may feel more comfortable if he can demonstrate later, should things go wrong, that he took every possible precaution at the outset and obtained the best professional advice available.

1.12 The amount of capital investment required in the world over the next century is immense. Much of the effort of the human race will be used to improve the world's infrastructure and make living conditions easier. In certain Asian countries, it has been estimated that the infrastructure investment required during the comparatively short period 1991-2000 will total U.S.$1,940 billions (see Appendix A). Additional massive investment will be required in Latin America, Africa and Eastern Europe. Some details of the prospects and opportunities for major projects are given in a handbook published by the Major Projects Association (1994). Much of this investment will require choices to be made between alternative projects. The risks and likely returns of each of these projects will need to be weighed up carefully, to reduce the likelihood of wasted investment. Clear thinking will be required, with a long-term perspective. Actuaries are comfortable with these concepts, and could probably make a real difference to some of the decisions which are taken.

2. Summary of Conclusions

2.1 *Actuarial Perspective*

Our main conclusion is that an actuarial perspective can add value to the appraisal of a capital project and can contribute to the overall assessment undertaken by an appraisal team which includes other professionals, such as engineers, economists, planners and accountants. The unique value of the actuary's contribution derives from his training, skills and experience in risk management and finance, both separately and in conjunction. He has developed techniques which enable him to make long-term forecasts with confidence, and he can present results in ways which senior management can easily assimilate as a basis for decisions. Moreover, he has a background in finance and investment, and can assist in finding the most appropriate funding structure for projects, having regard to the risks which each party is prepared to bear.

2.2 *Involvement of Actuaries*

Hence there is considerable scope for greater actuarial involvement, both in the public sector and the private sector, at home and abroad. Moreover, the United Kingdom Government's Private Finance Initiative offers a new field with enormous potential for actuarial advice (see Section 5.7). However, in some quarters appraisals are already carried out with a high degree of sophistication, and there may be reluctance, at least initially, to accept that involving actuaries could add value.

2.3 *Appraisal Principles*

It is suggested that appraisals should normally be based on the following principles (see also ¶5.2.1):

(a) The aim is to see whether the project will leave shareholders better off, and how it ranks against possible alternatives.
(b) There should be clarity about the nature of the information provided.
(c) The probabilistic and systematic risks (see ¶2.6) should be identified and quantified.
(d) The discount rates to be used should be related to investor expectations and should allow for the systematic risk inherent in the particular project.
(e) Account should be taken of the overall environment in which the particular project is to be decided upon and carried out.

2.4 *Actuary's Contribution*

The actuary's main contribution (see Section 5.1) is likely to be in the areas of:

— analysis and management of risk;
— financial model building and choice of assumptions; and
— designing funding structures which enable the risks to be assumed where they can best be handled.

2.5 *Risk Identification and Management*

According to a recent survey by the Confederation of British Industry (1994) (see Section 3.7), only about one quarter of manufacturing companies use quantitative methods to assess project risk, with the majority relying on subjective judgement. The actuary can help companies to build up a capability in this area. He can advise, for example, on how a company could build a portfolio of projects having specified overall risk and return characteristics. With regard to the risk of individual projects (see Section 4.2 and ¶5.2.1), he will be able to assist in:

— identification of the risks which could affect the project;
— assessment of the quantification and distribution of each risk;
— estimation of the likely effect on the project's overall outcome;
— minimisation of risks, both overall and for the actuary's client; and
— control of risks as the project proceeds.

2.6 *Probabilistic and Systematic Risk*

It may sometimes be helpful to consider project risk under two separate headings — the probabilistic risk which can be eliminated by sufficient diversification over a number of projects and the systematic risk which cannot (see ¶4.2.1.2).

2.7 *Insurance of Project Risks*

In particular, the actuary may be able to offer helpful advice on the extent to which the risks inherent in the project may usefully be insured (see Section 5.5).

2.8 *Building Financial Models*

On financial model building, the actuary should be able to develop a suitable framework, including discounted cash flow analysis. The considerations can sometimes be complex if projects with different lengths of life are being considered, or if substantial disposal costs will be incurred at the end of the project's life. Actuarial expertise will be particularly useful in the choice of a suitable discount rate and in making consistent assumptions about the likely course of future inflation of prices, wages and raw materials. The figures chosen should be consistent with investors' current expectations, which is not always the case at present (see Section 4.3 and ¶5.2.2).

2.9 *Advice on Funding Structures*

Advice on suitable funding structures will draw on the actuary's knowledge of the investment needs and risk profiles of long-term financial institutions, particularly life assurance companies and pension funds. The actuary will also know of funding structures which have been found useful in the past, for example the tripartite contracts between local authorities, developers and financial institutions in the 1970s for town centre redevelopment schemes (see ¶5.7.2).

2.10 *Appraisal Techniques Available*

A rich variety of appraisal techniques is available, ranging from very simple calculations to complex computer simulations; in many cases the simpler methods will be adequate to get a good insight into the nature of the project (see Section 5.3).

2.11 *Discount Rates being used by Companies*

There is evidence (see Section 3.7 and ¶¶4.3.13 and 4.3.14) that many commercial companies are still seeking unduly high returns on their projects, up to about 30% p.a. real. (By 'real' rates of return we mean rates which, when used for the appraisal of projects, are employed in conjunction with cash flows expressed at a constant price level.) This discourages sensible investment and prevents companies from exploiting opportunities which are open to them. Moreover, it does not necessarily lead to less risk being incurred — projects with low risks and adequate, but not exciting, returns may be rejected in favour of high-risk projects apparently offering high returns.

2.12 *Lower Discount Rates*

Many of the more sophisticated techniques currently in use for appraising projects derive from the work of financial economists in applying the Capital Asset Pricing Model (CAPM). In Section 3 we give an overview of the appraisal

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

techniques used most often in practice, and in Section 4 we discuss theoretical considerations underlying appraisals and show how our approach can be reconciled with other methods. Generally speaking our approach, which is derived from the current expectations of long-term investors, leads to somewhat lower rates of discount than are currently being used in industry. Other commentators have made a similar point in recent months. A lower rate of discount means that a higher weighting is attached to events in future years, and we believe that this is realistic. Projects which combine a high capital investment with an income for many years into the future are more likely to be judged acceptable than they would be if higher discount rates were used, but only provided they also pass a rigorous analysis of risks.

2.13 *Appropriate Discount Rates in 1994*

We have considered what discount rates would have been appropriate in 1994 for the typical large U.K. company for a typical project (see ¶4.3.8). At one extreme there is an argument for using a discount rate as low as 4.5% p.a. real, whereas at the other extreme there is an argument for using a rate as high as 8% p.a. real (in both cases using cash flows expressed at constant price levels, after allowing for the payment of corporation tax). Even our higher figure is lower than the discount rates of 15% p.a. or 20% p.a. real which appear to be still in common use.

2.14 *Use of Gross Discount Rates*

There is also an argument for using a gross discount rate applied to cash flows which allow for the payment of Corporation Tax, but also allow for the reclaiming of ACT by gross investors (see ¶4.3.11).

2.15 *U.K. Government's Discount Rate*

We suggest that the discount rates used by the U.K. Government for its own purposes should possibly be lower at times of economic recession (see ¶5.6.5).

2.16 *Consequential Benefits of Government Projects*

In the appraisal of Government projects, there should be further study and discussion of the possibility of taking into account any saving to the public purse arising in respect of unemployment benefit, etc. (see ¶5.6.6).

2.17 *Simplified Cost-Benefit Analysis*

We suggest a simple technique of cost-benefit analysis which can be used for Government projects lasting up to about five years (see ¶5.6.8).

2.18 *Private Finance Initiative*

The long-term financial institutions could play an important part in providing long-term funding for projects undertaken under the U.K. Government's Private Finance Initiative, but this is more likely to happen on a large scale if suitable vehicles exist (see ¶5.7.4). Longer concessions might encourage the financial institutions to participate (see ¶5.7.3). Actuarial advice may be useful in negotiations about the size of the Government's contribution to a project (see ¶5.7.6).

2.19 *Control of Risks as Project Proceeds*

The actuary's contribution is not limited to the initial appraisal of a project. During the appraisal process he will have helped to identify the key risks, and he will be in a good position to suggest ways in which these risks should be monitored and controlled at later stages (see Sections 6.6 and 6.7).

2.20 *Reviewing Company Procedures*

There may well be scope for actuaries to assist companies to review their procedures for dealing with projects (see Section 6). This includes the possibility of introducing audits after the project has been put into effect (see ¶¶6.8.1 and 6.8.2), which can provide useful guidance for the future, and can help to limit systematic over-optimism on the part of sponsors.

2.21 *Working as Part of a Team*

The actuary's aim should be to make a contribution alongside other professionals, and to blend his (or her) work with theirs so as to lead to better decisions. Full account needs to be taken of the particular environment in which the project is to take place. The exact nature of the actuary's work in a specific case will depend on the strengths of other team members, and in some circumstances he will need to be prepared to contribute far more than in other cases. For this reason he will need to have at least an acquaintance with aspects of project appraisal which do not derive specifically from his training and experience in more traditional fields of actuarial work (see Section 5.1).

2.22 *Issues for the Profession*

Actuaries unfamiliar with the field should not assume that involvement in capital projects is easy, and that it is just a question of applying standard actuarial techniques. Among other things, a new vocabulary may need to be developed, so that the actuary can communicate more easily with his clients and other team members. If the profession wishes to stimulate actuarial involvement in this field, it needs to consider:

— how the subject should be reflected in the education system or in continuing professional development; and
— how 'quality control' can be ensured, so that actuaries in general come to acquire a good reputation for their work in this field.

3. Overview of Appraisal Techniques

3.1 *Purpose of Appraisals*

The financial appraisal of a project is intended to determine whether the income generated by a project is likely to outweigh its costs, and how the project ranks against possible alternatives. There is a variety of methods in use, ranging from the simplistic to the highly complex.

3.2 *Appraisal Techniques*

3.2.1 *Payback period*

3.2.1.1 The simplest technique is to calculate the 'payback period'. A project is accepted if the number of years of projected cash flow required to return the initial investment is less than a pre-set maximum cut-off period.

3.2.1.2 The disadvantages will be obvious to actuaries. No account is taken of the time value of money. Cash flows beyond the cut-off date are ignored, so that projects with a long maturity will automatically be disfavoured. There is no measure of the quantum of 'value' of the project. The only advantage of this technique is simplicity.

3.2.2 *Internal rate of return*

3.2.2.1 A second technique is to calculate the Internal Rate of Return of the project (IRR: the interest rate that equates the present value of expected future cash flows with initial cost). A project will be accepted if its IRR exceeds the opportunity cost of capital.

3.2.2.2 The IRR approach reflects the time value of money, but it is still to be used with care. Multiple IRR solutions may arise if there is more than one change of sign in the projected cash flows of a project. If used to compare various projects, then IRR gives no measure of the quantum of 'value' of each project: a project with a high 'value' may be rejected in favour of a project of lower 'value' (but higher IRR), although both may generate a return in excess of the opportunity cost of capital.

3.2.2.3 For example, if Project A offers an IRR of 12% p.a., Project B an IRR of 10% p.a., and free cash can be invested to earn 5% p.a., then Project A appears preferable on the basis of IRR alone; but if Project A requires only £100 of invested capital (and cannot be expanded to take any more) while Project B requires £200 of capital, then Project B will earn more profit in total. The decision is easy if £300 of capital is available: undertake both projects. However, if only £200 is available then a pure IRR decision rule will choose Project A, and this choice will not maximise the profit generated from the available capital.

3.2.3 *Net present value*

3.2.3.1 The third widely used technique is to calculate the Net Present Value of a project (NPV: the present value of the expected future cash flows of a project discounted at the opportunity cost of capital). A project is accepted if its

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

NPV is greater than zero. If a choice must be made between different projects, then that with the highest NPV is chosen.

3.2.3.2 Proper application of the NPV approach to project appraisal will more certainly maximise the economic value generated by the capital available to a project sponsor than either of the techniques mentioned earlier, since it largely addresses the limitations of those techniques. The time value of money is reflected. All future cash flows may be taken into account. Projects may be compared on the basis of a measure of value. The measure is additive and may, therefore, be used to determine the optimal allocation of a pool of capital among several alternative projects. In most circumstances NPV is the preferred approach.

3.2.4 *Other methods*

The three techniques described above are those most commonly used, and they are explained fully in the standard text books on the subject. However, other methods are also available — see, for example, Lewin (1967), who points out that if the receipts/costs ratio method were to become widely used, this could result in a better use of the productive resources available in the community as a whole. This method differs from the IRR and NPV methods in that it does not 'net off' costs and receipts each year to get a 'net cash flow' for the year. The numerator of the ratio is the NPV of the gross receipts and the denominator is the NPV of the capital and running costs combined. The method may result in competing projects being ranked in a different order of attractiveness than under the IRR and NPV methods. In general, it is probably more suitable for certain purposes in the public sector than it is for normal use in the private sector.

3.2.5 *Application of NPV technique*

While the case for normally preferring NPV as an appraisal technique is strong, the proper application of NPV in practice is by no means straightforward. The arithmetic of the calculation, given an agreed projection of cash flows and an agreed discount rate, will be well understood by actuaries. However, in the appraisal of projects outside the traditional areas of actuarial expertise, an actuary's familiar tools for determining projected cash flows and an appropriate discount rate may not be appropriate, and new tools may be required. A range of approaches has been developed, mainly outside actuarial circles, which address many of these issues. Some of these approaches have already seen application and discussion in actuarial work and actuarial literature, whilst others have not.

3.3 *Risk and Return*

Regardless of any modern theories linking risk and expected return, it has long been recognised that, other things being equal, a project with a relatively certain outcome is preferable to one with an uncertain outcome. It has therefore long been the practice in assessing projects to make adjustments, either by requiring a higher expected return from a riskier project than a safer project or by deliberately using conservative estimates of the cash flows of a riskier project in making any comparison. The approaches mentioned above and introduced below aim simply to provide a systematic manner for making such adjustments, reflecting how 'the world' actually appears to make the trade-off between risk and return. The approaches may be divided broadly between those dealing with the projection of cash flows, those providing an appropriate interest rate with which to discount projected cash flows, and those dealing with the valuation of contingent or 'optional' cash flows.

3.4 *Projection of Cash Flows*

3.4.1 With regard to projected future cash flows, actuarial analysis has historically tended to use single deterministic point estimates. The same has typically been true in general project appraisal; this probably accurately reflects how many investment decisions will in any event be made; the unassisted human brain is not well equipped to analyse probabilistic propositions and will look for a yes/no basis of decision in any set of information.

3.4.2 A widely-used extension is multi-case sensitivity testing, typically around low/mid/high models of cash flows, with a flexing at the same time of the discount rate assumption. This may give a feeling for the range of likely outcomes, but does not isolate sensitivity to specific individual factors and, if not carefully applied, may treat correlated factors inappropriately.

3.4.3 A complement to sensitivity testing of various 'cases' is to test sensitivity to individual factors, and this is increasingly easy, given spreadsheet-based models. This will, by definition, show sensitivity to individual factors, but the danger of overlooking the effect of correlated variation in key factors remains significant.

3.4.4 A response to the danger of over-simplicity in sensitivity analysis has been to develop formal 'scenario analysis', in which entire potential future scenarios are projected with particular emphasis on the internal consistency of each set of assumptions. There remain the problems of ensuring that the full range of potential outcomes is accurately reflected, and of accurately assessing the probability of occurrence of each scenario. However, where the financial outcome of different scenarios is likely to vary very widely and in a non-linear or discontinuous manner (e.g. the effect on sales volume of over-pricing in a competitive market), then the approach has advantage over a less rigorous sensitivity analysis.

3.4.5 A logical extension of sensitivity or scenario analysis is probabilistic simulation: if the distribution of possible future outcomes of the various individual factors can be simulated, then why not use it as the basis of a probabilistic projection and derive a probability distribution of NPV itself? This has given rise to Monte Carlo simulation techniques and, in the actuarial domain, to stochastic modelling of financial factors. The clear advantage of such an approach is its comprehensive coverage of the full range of possible outcomes for each factor so treated, if properly modelled. In project appraisal the disadvantage is the difficulty of assessing the true distribution of outcomes of many of the most critical factors. For example, reliable projection of the demand for the use of proposed new roads is frequently a problem. Given inaccuracy in such a core factor, a sophisticated probabilistic treatment of the financial factors may give rise to a spurious impression of overall accuracy in the whole model. However, a correct model may serve to show just how uncertain the cash flows, and hence the NPVs, are.

3.5 *Determination of Discount Rate*

3.5.1 There is a widely voiced present concern that U.K. industry is adopting too high a rate of discount in assessing capital investment opportunities, and that this is leading to under-investment (e.g. Dimson & Marsh, 1994). Given recent falls in inflation and interest rates, this would seem to suggest that some people effectively adopt fixed nominal rates of discount in evaluating projects. Logic would suggest that this is a mistake or, at least, leads to an inefficient allocation of capital resources.

3.5.2 A great deal of theoretical work has been done by financial economists over the last few decades in trying to analyse the trade-off between risk and return in a formal manner. Their work has given rise to models which systematically address that trade-off and which can be widely applied where other disciplines have so far failed to develop any similarly systematic alternative of wide application. It is of value, therefore, to mention the key existing model, if only to stimulate further debate.

3.5.3 The theoretical approach of financial economists is encapsulated in the CAPM. One result of the CAPM is that the expected return on any asset is a combination of the risk-free return (as defined, but taken generally as the return on a short-term government security) and a risk-related component varying in direct proportion to the volatility of that asset's return relative to the return on the 'market portfolio' (theoretically a portfolio of all world assets, but taken generally as the traded securities market). The measure of riskiness based on relative volatility of returns is known as 'beta'. The theory of the CAPM is examined in more detail in Section 4.8.

3.5.4 The CAPM provides a theoretical framework to estimate the return which should be required by an investor on any project or on capital investment in any form. However, not all practitioners accept that the CAPM is useful in this context. A raft of practical reasons may be cited why the theoretical requirements of the CAPM seem violated in the real world. Not least in applying the CAPM to a specific project is the problem that a traded market must theoretically exist in capital invested in other similar projects, and this may not be the case.

3.6 *Valuation of Contingent Cash Flows and Optional Courses of Action*

3.6.1 An area of acknowledged weakness in a standard NPV approach is in the treatment of the contingent cash flows and values arising from implicit or explicit 'options' which arise as a project evolves.

3.6.2 As an example, suppose that an organisation is considering Project X which, if effected (but *only* if effected), will allow the organisation later to effect Project Y, if it so wishes (e.g. if sales from Project X turn out to exceed a certain threshold level). Suppose further that Project X, when assessed without regard to Project Y, has a negative NPV and that Project Y, when assessed on the basis of information known today, *also* has a negative NPV. On first view the organisation should not choose to invest in Project X (and will therefore forego Project Y also); but what if a successful outcome to Project X (i.e. better than the 'expected' outcome) would mean that Project Y would be more valuable when instigated than can reasonably be assumed at present? It may be the case that the organisation would be justified in investing in Project X because of the option value of potential subsequent investment in Project Y. Broadly, this counter-intuitive result will arise if the future value of the Project Y cash flows is highly uncertain now, but would be made more certain following the outcome of Project X.

3.6.3 This situation can be approached by the use of decision theory methods, which are well described by French (1988) and by Moore & Thomas (1976). These methods are used also in the evaluation of complex financial options, for example by the use of binomial lattices or trees.

3.6.4 Decision theory methods may require the construction of a model which reflects every possible contingent outcome and assigns a cash flow and probability to each. In practice this may not be feasible. Another key problem, moreover, is that the determination of a single appropriate discount rate to apply in an NPV calculation involving contingent cash flows may be difficult because the risk of each 'option' may change as the project evolves.

3.7 *Appraisal Techniques Currently in Use*

3.7.1 According to a survey of 438 manufacturing companies carried out in May 1994 by the Confederation of British Industry (CBI, 1994), about 90% of companies used a quantitative assessment to determine the required rate of return (RRR). They were asked to rank the appraisal methods they used, and the principal rankings given to each method are shown in Table 3.7.1.

Table 3.7.1 Percentages of companies ranking given methods with rank shown

	Rank		
Method	1 %	2 %	3 %
An accounting rate	0	3	5
A discounted cash flow rate	34	9	8
Simple payback	49	17	6
Return on capital	21	16	10
Return on equity	2	2	3
Other	2	1	0

(Perhaps the fact that the first column of figures totals more than 100% can be explained by some respondents giving a ranking of '1' to more than one method.)

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

3.7.2 Thus it appears that about 49% of the companies using quantitative methods favoured the payback period as their main investment criterion and another 23% used it as a subsidiary criterion. The average payback period required by companies was 2.7 years, with most companies in the range of one year to 5 years. However, the larger companies appeared to rely rather more on discounted cashflow or other more sophisticated techniques. About 56% of companies worked on a pre-tax basis and 41% on a post-tax basis.

3.7.3 About 35% of companies employing quantitative methods, according to the CBI survey, used an RRR measured on a real basis and 63% on a nominal basis. These figures appear to include at least some companies which used simple payback as their main criterion. The RRRs in use are tabulated in Table 3.7.2 (see also ¶4.3.13).

Table 3.7.2. Percentages of companies using real and nominal RRRs of amount shown

RRR % p.a.	Companies using real RRRs %	Companies using nominal RRRs %
0 - 3	2	13
4 - 7	4	8
8 - 11	30	7
12 - 15	12	10
16 - 19	7	6
20 - 23	26	17
24 - 27	9	10
28 and over	7	16
n/a	3	14
	(Mean RRR: 16.4% p.a.)	(Mean RRR: 16.8% p.a.)

4. Theoretical Aspects of Appraisals

4.1 *Introduction*

4.1.1 We believe that the mathematical and analytical approach to investment put forward by many financial economists is compatible with, and indeed should be the same as, the actuarial approach to similar problems. The fact that some actuaries have argued against the conclusions of financial economists is, we believe, because some of the arguments have been misunderstood and misrepresented. This has led to false conclusions being erroneously derived from valid premises. We join those who criticise the false conclusions, but we do not also reject the valid use of the methodology.

4.1.2 In this section we present a methodology for assessing capital projects, or any other investment, without making unwarranted assumptions and drawing erroneous conclusions. In doing so we also expose certain popular fallacies.

4.1.3 This is not a matter purely of academic argument. We believe that the popular misrepresentation of the conclusions of financial economists has had

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

significant practical consequences. In short, companies are currently failing to invest in projects that should be profitable for them, because they are seeking too high a target rate of return on prospective projects. They have possibly been led to this position by the abnormally good returns on shares experienced during the 1980s, and have been led into thinking it correct not to invest unless they can expect the same abnormally good returns again.

4.1.4 This desire for exceptionally good returns is expressed through using a high target rate of discount for assessing capital projects. It is interesting that the rates of interest, and the accompanying assumptions, used by actuaries for the valuation of pension funds are substantially different from the rates of discount used by companies in the assessment of capital projects. This has, on occasions, caused conflict between consulting actuaries and finance directors, the actuary using perhaps a 9% rate of interest for valuing the assets and liabilities of a pension fund, a rate of interest that is (at the time of writing in July 1994) consistent both with the redemption yield on long-term fixed-interest investments and with the expected rate of return on shares, assuming say a 4% dividend yield, 5% inflation and zero real dividend growth (which is perhaps a cautious rather than a best estimate basis). The finance director might have looked at the achieved returns on ordinary share investment of 24% p.a. over the 1980s (December 1979 to December 1989), and the company's current target rate of return of say 20% p.a. (all rates here being nominal, and both the actuary and the finance director assuming, say, 5% p.a. inflation).

4.1.5 It is our intention in this part of the paper to reconcile these two approaches. We first explain, in Section 4.2, what we mean by 'probabilistic risk' and 'systematic risk', and how they should be allowed for in project assessment. We then, in Section 4.3, discuss in general the choice of discount rates, and explain how these can be used to allow for the systematic risk of a project.

4.1.6 The methodology that is used to allow for systematic risk is based on the Markowitz portfolio selection model, which we describe in Section 4.4. We state a useful result, namely the linear relationship between expected return and beta, but without relying on the CAPM. We then go on to show, in Sections 4.5-4.7, how the portfolio selection model can be used by a company for assessing new capital projects. In Section 4.8 we discuss briefly the CAPM.

4.1.7 In Section 4.9 we turn our attention to the historic evidence in relation to fixed-interest and ordinary share investment in the U.K. over the past 70 years. While we argue that a capital project should not be treated as identical with an investment in ordinary shares, share investment can be treated as a class of capital project (using that term in the wider sense referred to in ¶1.5), and the evidence from the stock market is publicly available. On the basis of the evidence described in the accompanying paper, 'The Risk Premium on Ordinary Shares', we conclude that an appropriate 'risk premium' for the assessment of capital projects is not as high as the 7% p.a. or more that has been quoted in a number of articles recently (see, for example, Grubb, 1993 and Dimson, 1993), and we suggest that a figure in the range 2% p.a. to 6% p.a. (gross of ACT) is more appropriate.

4.1.8 Finally, in Sections 4.10 and 4.11, we make some observations about the use of a stochastic investment model, such as the Wilkie model, for the assessment of certain aspects of capital projects.

4.2 *Some Thoughts on Risk*

4.2.1 *Probabilistic and systematic risk*

4.2.1.1 Any attempt at capital project appraisal must inevitably consider questions of 'risk'. All capital projects in the real world face some — and usually many — kinds of risk. Most projects are prone to risk arising out of a downturn in the economy. Many are prone to risk arising out of a change in consumer tastes (i.e. fashions). Some projects are prone to risk arising out of the randomness of the possible outcomes, e.g. insurance related projects. The list of different types of risk is potentially endless, and we shall not enumerate any more here. We use the word 'risk' to include both the possibility of upside gain as well as of downside loss; these may not be symmetrical.

4.2.1.2 What we shall attempt to do here is to identify two quite different types of risk which can be allowed for in an appraisal model, but which, as we shall see, need to be allowed for in two quite different ways. We begin with a brief description of the two types of risk, which we shall call 'probabilistic risk' and 'systematic risk'. *Probabilistic risk* is the risk that can be eliminated (or 'averaged out') by investing in a number of projects. *Systematic risk* is that part of the variability of the return on a project that cannot be eliminated by investing in the same type of project many times over, nor by diversification, because investing in a number of projects cannot reduce this part of the variability to zero. These two types of risk are expanded upon more fully below.

4.2.2 *Probabilistic risk*

4.2.2.1 Probabilistic risk is the risk that can be eliminated (or 'averaged out') by investing in a number of projects. As we shall see below, some types of probabilistic risk can be eliminated by investing in the same project type over and over again (rather like repeatedly tossing a coin). Other types of probabilistic risk can be 'averaged out' by investing in a number of different project types. Diversifying across a range of industries is a familiar strategy for investors averse to probabilistic risk.

4.2.2.2 Underwriting insurance policies is an example of probabilistic risk which is familiar to actuaries, but probabilistic risk applies to all projects. An investment in a construction project, for example, will have uncertain outcomes, each with its own probability. These risks include those arising from the choice of location and the design of the building, etc. By investing in sufficiently many similar construction projects, we can expect the outcome to be close to the mean. This is what we mean by 'averaging out' the risk.

4.2.2.3 However, repeat investment in the same project type will not eliminate

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

all probabilistic risk. A change in mortality levels is a project-type risk for life offices which cannot be eliminated by the life office selling larger numbers of policies, or the investor buying the shares of many life offices. For other industries, project-type risk may arise from changes in the market for its products brought about by, for example, new technology or environmental considerations. This form of probabilistic risk can be eliminated by investing in a range of different project types or different industries, in other words it can be diversified away.

4.2.2.4 It should be noted that the probabilities underlying probabilistic risk may be known at the outset, or they may be completely unknown — or somewhere in between, i.e. capable of some reasonable estimation, but not with complete accuracy. For appraisal purposes the conceptual nature of a risk is not altered from probabilistic to something else just because the probabilities are not known.

4.2.3 *Systematic risk*

4.2.3.1 Systematic risk is the risk that cannot be eliminated by investing in the same project type many times over, nor by diversification, because systematic risk affects all projects. No matter how many investments and how well diversified a portfolio one holds, there is still a risk that one's portfolio may do well or do badly because the performance of 'the market' is uncertain.

4.2.3.2 This definition does not mean that systematic risk is equal for all projects. It is not. For example, construction projects may have a very high level of systematic risk, because the state of the economy will be a major determinant in the levels of rent or the levels of sale prices that can be achieved. Bread projects, or water projects, may have a much lower level of systematic risk, because they are less susceptible to fluctuations in the economy, but there is still some systematic risk.

4.2.3.3 The key point about systematic risk is that it can vary from project type to project type, and one can raise or lower the average level of systematic risk in one's portfolio of projects by selecting projects which have high or low systematic risk, but one cannot eliminate this risk except by investing in totally risk free assets (if such assets exist).

4.2.3.4 The distinction between probabilistic and systematic risk is crucial in what follows. It is also the source of much confusion. The key point to bear in mind is the difference between, on the one hand, risks that can be mitigated either by repeat investments in the same project type or by diversification across different project types and, on the other hand, risks that are incapable of elimination in this way.

4.2.4 *The treatment of probabilistic risk and systematic risk in project appraisal*

4.2.4.1 Having identified two fundamentally quite different types of risk, we now assert that they should be treated quite differently in project appraisals. We submit that probabilistic risk can, and should, be modelled by attaching

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

probabilities to the alternative outcomes, and that systematic risk should be modelled by varying the discount rates used in the model.

4.2.4.2 Actuaries should have little difficulty with the first of these two statements. Applying probabilities to alternative outcomes is second nature. That is exactly how actuaries value the probabilistic risk associated with, for example, mortality and other demographic effects.

4.2.4.3 It is the use of different discount rates to model the systematic risk that may require some argument to convince actuaries. The most convincing justification for that assertion is not one of actuarial theory to be debated academically; it is based on observation of the marketplace and, therefore, turns on whether or not we, and many others before us, are reading the evidence correctly. We observe that investors expect different levels of return from different industries according to their volatility.

4.2.4.4 The evidence for this will be brought forward in a moment; but first we offer some general reasoning by which actuaries might recognise the inherent *plausibility* of the observation before we go into the statistical verification of it.

4.2.4.5 We begin with the well known fact that equities are generally expected to provide a higher total return than gilts, and are priced accordingly. It is generally accepted that the reason for the higher return on equities is to reflect the additional risk inherent in equities.

4.2.4.6 Having observed that investors expect a higher return from equities than from gilts, we now ask whether investors should expect all equities to show the same return and all gilts to show the same return (but a different one from equities)? Or is it the case that investors expect some equities, and some gilts, to show higher returns than others?

4.2.4.7 If we ignore the facts for a moment, and look purely at logical theory, it is perfectly possible to contemplate a theory in which there is but one return on equities and one on gilts. Presumably, in this theory, there would be intermediate returns on local government stocks and on corporate debentures, etc. So each class of investment has its own return.

4.2.4.8 However, we know that, in the real world, returns on gilts do vary from one gilt to another. We know that, in the absence of an expectation that interest rates will fall, returns tend to increase monotonically with the duration of the investment, with overnight money showing the lowest return. We also know that some non-governmental fixed-interest stocks are priced to show near-gilt levels of return — they are the ones issued by the most secure local authorities or government-owned industries, etc. — and that other fixed-interest stocks show much higher expected returns. Within the range of corporate bonds, there are the blue-chip companies that are priced to show relatively low returns and those (e.g. the U.S. junk bonds) which are priced to show much higher, near-equity levels of return.

4.2.4.9 In short (in the absence of an expectation that interest rates will fall), there is a gradation from the lowest returns on overnight money through the various classes of investments to equities. Some of the classes overlap. So, for example, a very blue-chip corporation may be priced to show a lower expected return on its fixed-interest investments than some local authorities at the higher end of the range of returns. So, also, some junk bonds may be priced so as to show a higher expected return than blue-chip equities.

4.2.5 *Getting to grips with the two approaches to risk*

4.2.5.1 So far we have identified two types of risk, probabilistic risk and systematic risk. We have said that probabilistic risk should be dealt with by applying probabilities to the potential cash flow outcomes. We have also said that systematic risk should be dealt with in the discount rate to be applied, arguing that, because the evidence of the market is that investors consistently earn different returns from different investment types according to the observed volatility of the return on the investment versus the return on the market, investors will continue to expect that in future.

4.2.5.2 Interestingly, whilst actuaries have traditionally tended to prefer the use of probabilities to model risk, and have only relatively recently embraced 'risk adjusted discount rates', many non-actuaries have a tendency to place too much reliance on discount rates to model risk. This is understandable, because many businessmen are not comfortable with the use of probabilities.

4.2.5.3 The question arises, however, as to whether the two approaches to treating risk really do need to be kept separate, or whether they can be treated as alternatives to be used according to preference. We say that they need to be kept separate and our reasoning is set out in the following paragraphs.

4.2.5.4 The key point is that the use of higher discount rates to reflect risk means that cash flows further into the future will be affected more than cash flows in the near term. It is appropriate to treat certain risks in this way, but other risks can only be modelled adequately by the use of probabilities.

4.2.5.5 Consider, for example, the technology risk that might face a TV manufacturer, namely that a competitor will come up with a new invention that will render the existing models obsolete. In the long run, that risk is not especially significant, because the manufacturer would expect to renew his plant at intervals anyway, and at each renewal he can adopt the latest methodology at that date; it is not correct to discount successive renewals of the plant at a high discount rate because of a short-term risk. The risk is a more immediate one, and if it occurred it would affect his immediate cash flows; it is therefore necessary to consider the probability of the event occurring, and to adjust the expected cash flows accordingly. It would not, in general, be correct to take the anticipated cash flows if the event did not occur, and to discount them at a higher rate.

4.2.5.6 Similarly, an actuary would not attempt to model the potential development of AIDS by deducting a few percentage points from the valuation rate of interest; he would consider how this affected the expected claims in the portfolio directly.

4.2.6 *A note on terminology*

4.2.6.1 There is a variety of different terminology in use, and this may cause confusion. What we have called *probabilistic risk* is also known in financial economics circles as *unique risk, specific risk, unsystematic risk* or *diversifiable risk.* We prefer the term *probabilistic risk* because this type of risk is made up both of risks which can be eliminated merely by repeating the investment (or more precisely the same investment type) over and over again and also of risks which can be diversified away by investment in a variety of types of project. For example, a gambler in a casino can reduce his probabilistic risk by dividing his stake money up and then either by playing the same game many times over or by moving from table to table and betting his divided stake money on several different games.

4.2.6.2 There is also a variety of terms in use to describe *systematic risk.* It may be called *non-diversifiable risk* or *market risk.* The term *non-diversifiable* may have been coined because one cannot reduce the systematic risk to zero by diversification. However, by diversifying across different project types, one can alter the risk profile of one's portfolio of investments. One can raise it, by going into, say, construction, films and other inherently more risky projects, or lower it by going into projects of the bread and water variety, or any point in between, or outside, that range by investing in a suitable mix of investments selected from the whole market spectrum.

4.2.6.3 The term *market risk* also needs to be used carefully. As used by financial economists, it strictly means the risk associated with the market, but it is popularly used to describe the risk of a particular project type in relation to the rest of the market. So, construction projects have a high 'market risk' (in the jargon) because they are more risky (i.e. more prone to upswings and downswings) versus the rest of the market. Conversely, bread and water projects are said to have a low 'market risk' because they are less prone to upswings and downswings versus the rest of the market.

4.2.6.4 Some authors have distinguished between the words 'risk' and 'uncertainty', applying the former to cases where objective probabilities can be observed, and the latter to cases where there is no information from which objective probabilities can be derived. This distinction seems to have been first made by Knight (1921). Some modern authors find this distinction unnecessary, arguing that the use of subjective probabilities can make the two cases effectively the same. Both sides are discussed by Moore (1983). A similar distinction that can be made in a statistical model is between:

— the probability distributions expressed in the model, which generate the distributions of results;
— uncertainty about the values of the parameters used in the model, because they have been estimated from sample data; and
— uncertainty about whether the correct (or the most satisfactory) model has been used.

4.2.7 *Should a company's target discount rates vary according to the project?*

4.2.7.1 Having identified the systematic risk associated with a company, and

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

thereby estimated its cost of capital, it is tempting to think that the company's management should apply the same discount rate to all projects regardless of the risk associated with each project (we call this 'The Single Discount Rate Hypothesis'). This, however, would be wrong. It would lead to irrational results, as we shall show. The correct theoretical approach is to vary the discount rate according to the systematic risk attached to each project ('The Project Discount Rate Hypothesis').

4.2.7.2 In Appendix B we discuss an example of what would happen if managers adopted the suggestion that they should use the same discount rate for all projects. Two companies have very different systematic risks, and, accordingly, very different rates of return are expected on their projects. It is shown that if these two companies merge, and the merged company uses an average rate of return for assessing all its projects, then it will end up abandoning projects with low systematic risk and adopting only those with high systematic risk, some of which will be accepted at too low an expected rate of return. The correct method of assessment is to adopt different discount rates for projects which have different systematic risk.

4.3 *Choice of Discount Rates*

4.3.1 One of the most complex and keenly debated questions in the theory of appraisals is how to determine the discount rates to be used by companies in NPV calculations.

4.3.2 The natural starting point is to look at the current cost to the company of the capital which will be used in financing the project. If the project has a positive NPV at a discount rate equal to this cost, then the project will be profitable and can be accepted, unless it has an abnormal degree of downside risk or unless projects competing for scarce capital resources show a higher NPV. The sensitivity of the NPV to other discount rates should also be explored.

4.3.3 One method of proceeding is to work out the cash flows each year in monetary terms, allowing for an assumed rate of future inflation, and to discount these cash flows back to the outset using a discount rate equal to the company's 'real' cost of capital compounded with the rate of inflation assumed. This method has the advantage that it allows correctly for items (such as working capital) where the real cost may differ according to the rate of inflation experienced. However, in the discussion which follows we shall assume, for simplicity, that the cash flows will be expressed at constant price levels (with an adjustment for any items where the real cost varies according to the rate of inflation), and then the discount rate used will be equal to the company's 'real' cost of capital. The advantage of this method is that it helps to keep the overall picture in view, since all monetary results obtained in respect of future years have a significance which is easily recognised in terms of the price levels current at the time the appraisal is made, and the need to make a variety of calculations using differing assumptions about the rate of future inflation is avoided. We shall assume that one of the given sets of cash flows to be taken into account

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

consists of the incremental tax payments which will be due from the company if the project goes ahead, allowing for any time lags inherent in the tax system; it will, therefore, be appropriate to use a discount rate which takes account of the after-tax cost of capital.

4.3.4 The historic cost of the company's existing capital is irrelevant. What we normally need to establish is the current cost of raising incremental capital for the company in order to carry out the project. In some cases this capital will come from internal reserves, whereas in other cases it will come from borrowing (by bond or overdraft) or from issuing new equity capital, e.g. a rights issue. Regard may need to be paid, among other things, to the possibility of refinancing the project at a future date, the opportunity cost of using existing reserves for the project, the extent to which particular tranches of capital can be attributable to particular uses, the effect which borrowing for the project may have on the company's gearing and hence its share price, etc. It is also worth remembering that, if the project is undertaken, the company's existing shareholders should be better off than they would be in the absence of the project; the cost of capital can thus be regarded as the rate of return which needs to be achieved if the existing shareholders are to be no better off and no worse off. Hence the following paragraphs should be taken as no more than a broad guide to some, but not all, of the relevant points.

4.3.5 The discount rate to be used will need to be determined as the weighted average of the cost of equity capital and the cost of debt capital. If the company already has an optimal capital structure, this will normally provide the weights to be used. (If the company's existing capital structure is not optimal, the weights used may need to be derived from a consideration of what the optimal structure would be, on the basis that changes to reach an optimal structure could be made independently of whether the project goes ahead or not.) The cost of equity capital and the cost of debt capital used in the calculation should be those appropriate to the degree of systematic risk inherent in the project. In the case of a company carrying out a homogeneous group of projects, these costs of capital may be the same as the costs for the company as a whole. In the case of a company having a heterogeneous group of projects, the costs of capital appropriate for a particular proposed project may need individual consideration, having regard to its level of systematic risk; a pointer may sometimes be obtained from the costs of capital for companies habitually engaging in projects of the type under consideration.

4.3.6 The cost of debt capital should ideally be the cost in real terms of new borrowing for the company, which could be ascertained by taking an appropriate margin over the current yield on index-linked gilts of a suitable duration. In practice, the system of company taxation in the U.K. makes it unattractive for most companies to issue index-linked debt, and it is generally necessary for the company to issue a conventional fixed-interest stock. It is then necessary to make an explicit assumption about the rate of inflation, and it is not possible to work wholly 'in real terms'. It will only be possible to issue a company fixed-interest stock at a redemption yield higher by an appropriate margin over the redemption yield on a corresponding Government fixed-interest stock. If, however, the borrowing is to be done through variable interest rate finance, e.g. by a bank overdraft, the calculation of this margin is more complex — a first approximation might be to take it as the difference between the current overdraft rate payable by the company and the current Interbank three-month rate.

4.3.7 The main problem comes in considering the company's cost of equity capital. Our starting point is investors' current expectations of the rate of total return (gross) 'in real terms', i.e. net of inflation, that they will achieve from a new investment in a portfolio of company shares (mirroring the equity market) which is to be held indefinitely. This can be evaluated using simple methodologies such as that presented by PDFM (1994). At the time of writing (July 1994) the U.K. figure for this real rate of expected total return is about 5% p.a. A simplified explanation of this figure would be to describe it as an initial yield of 4% p.a. plus expected real dividend growth of 1% p.a. on top of inflation. In contrast, the expected 'risk-free' return on index-linked gilts is about 4% p.a. These figures do not seem unrealistic when compared with the actual gross rates of return achieved historically, as can be seen from the accompanying paper, 'The Risk Premium on Ordinary Shares'.

4.3.8 In ¶4.1.7 we suggest that (for projects of average systematic risk) a risk premium in the range of 2% p.a. to 6% p.a. (gross of ACT) is appropriate for assessment purposes. If index-linked gilts are yielding about 4% p.a. real, this implies a total real rate of 6% p.a. to 10% p.a., or about 4.5% p.a. to 8% p.a. after adjustment for tax (see ¶4.3.11), as being an appropriate discount rate in the case of a project with average systematic risk.

4.3.9 In some cases there may be good reasons to think that the current cost of equity capital for the company is above average. This might be the situation, for example, if the company is perceived as engaging in business activities involving large downside risks. Some evidence on whether the market persistently rates the company's shares so as to give an above average expected return might be obtained from a consideration of the earnings yield currently and at (say) quarterly intervals for several years back with the corresponding figures for the market as a whole. If a persistent difference of x% is observed, then it might be appropriate to use a discount rate equal to the figure derived in ¶4.3.8 plus x%. An alternative approach is to use the theoretical result referred to in ¶4.4.8 and apply the company's beta-factor as a multiplier. For example, if the risk-free rate on index-linked gilts were 4% p.a. and the overall expected return on a portfolio of equities were 6% p.a., then the discount rate to be used for a company with a beta-factor of 1.5 would be 4% + 1.5 (6%−4%) = 7%. The theory requires the application of a 'future' beta-factor, whereas the only data available relate to the past experience, but it is not an unusual actuarial procedure to derive estimates of the future from the experience of the past, appropriately adjusted.

4.3.10 Likewise, there may sometimes be good reason to believe that the current cost of equity capital for the company is below average, and this could lead to using a lower discount rate than for the typical company.

4.3.11 The costs of equity and debt capital derived in the manner suggested in ¶¶4.3.6 and 4.3.7 are based on returns to an investor who earns income gross. There are alternative methods of allowing for tax. A common method is to assume that the company's shareholders pay tax at the standard rate, and then to discount post-tax cash flows, in which case it is appropriate to adjust the discount rates described above so that they represent the net-of-tax rates of return expected by shareholders who pay tax at the standard rate. An alternative is to assume that the company's shareholders are gross investors, and then to use a gross rate of interest to discount the post-tax cash flows, suitably grossed up to allow for the recovery of ACT by the shareholders.

4.3.12 Either of these approaches leads to lower discount rates than those usually recommended by financial economists. For example, Dimson & Marsh (1994) suggest that a typical company should use 8% to 9% p.a. real after tax. The main reason their figures are higher than those we favour appears to be due, not to any difference in the underlying theoretical approach, but to their use of the *ex post* actual experience, using, for example, Treasury Bills as a risk-free asset, whereas we believe that, over the relevant period, the *ex ante* expected returns on Treasury Bills or on other fixed-interest investments appeared sufficient to investors at the time any investment was made, but in the event turned out to make inadequate allowance for the *ex post* actual inflation experienced (see also ¶4.9.2).

4.3.13 Dimson & Marsh (1994) stated that their contacts with companies suggested that many had not responded sufficiently to falling inflation and lower real interest rates; as a result managements were demanding an excessive return from new investment projects and running the risk of failing to share fully in economic recovery. BZW (June 1994) confirmed this conclusion. They reported that Hanson, which had required paybacks from its industrial investments in three to four years for most of its life, had recently announced that it had extended its payback period to five or six years. They also reported an informal Bank of England survey of 250 large and medium-sized companies, which found that:

— 72% had not changed their required returns;
— most firms set target rates of between 6% and 20% in real terms, and between 10% and 25% in nominal terms; and
— 40% of firms used payback criteria typically requiring payback in just two or three years.

4.3.14 BZW also reported that early results from their own research indicated that, while some companies had reduced their hurdle rates of return, others had moved in the opposite direction (see also ¶3.7.3).

4.3.15 The use of a discount rate which is too high could distort the relative weights placed on the short term and on the longer term, thereby leading to mistaken decisions. If it is assumed, mistakenly, that the deliberate use of a very high discount rate provides a contingency margin which reduces the need for rigorous risk analysis, there is the danger of the incorrect acceptance of a risky

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

project with a high apparent NPV or the incorrect rejection of a low-risk project which would have a positive NPV if it were based on the returns expected by those who invest in the company's shares.

4.4 *The Portfolio Selection Model*

4.4.1 The theoretical methodology on which the portfolio selection model is based is reviewed briefly in Appendix C. The model was originally put forward by Markowitz (1952, 1959), and is most recently presented in full in Markowitz (1987). It is described also in a paper to the Institute by Moore (1972), and in many books on investment, including those referred to in ¶1.3.

4.4.2 Markowitz started by considering the investor with a given sum to invest, who has the choice of investing in any or all of a set of assets of any kind, which are denoted in general as 'securities' and might be bonds or shares or a capital project of any kind, in any chosen quantities, subject perhaps to upper and lower limits on the amount that may be invested in each security (the lower limit might well be zero, implying that negative holdings of any security are not allowed); he assumed that the investor has a time horizon of a single period; that he (or she) is willing to treat the returns on each of the available securities over this time period as random variables with known (at least known to him) means, variances and covariances; and that he is willing to base his decisions on a consideration of the first two moments, the mean and variance of the available portfolios.

4.4.3 It is assumed that investors like a high expected return, but dislike uncertainty about the amount of that return, as expressed by the variance. Investors, therefore, like a high expected return and a low variance.

4.4.4 This mean-variance framework is a simplification. It can be shown, however, to be valid, sometimes with small modifications, for a wide range of cases where the distribution of returns can be described by only two parameters. The normal and lognormal distributions fall into this category. Some other distributions are not so amenable to this framework. Examples include the payoffs from option contracts, which may concentrate one or other tail of a distribution into a mass at some point; or distributions of returns on capital projects which have discrete probabilities of success or failure. These may be more analogous to insurance contracts, whether life or non-life, which are better dealt with as single cases in a large portfolio, applying the principles of probabilistic risk discussed in Section 4.2.

4.4.5 The analysis in Appendix C shows that the 'efficient frontier' in the mean-variance plane is formed by a series of sections of parabolas, which join at points representing 'corner portfolios', points at which one of the securities reaches its upper or lower limit.

4.4.6 In Appendix C we also demonstrate one of the widely quoted results of the portfolio selection model, namely the linear relationship, subject to certain conditions, between the expected returns and the beta-factors of securities. This is usually derived as a consequence of the CAPM, in relation to the 'market

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

portfolio' of the CAPM; but it can also be derived, with suitable assumptions and definitions, directly from the portfolio selection model. This result is of some significance in the assessment of capital projects, and it deserves careful consideration.

4.4.7 The result applies to any stretch of the efficient frontier between two corners, for the set of securities included in a chosen efficient portfolio which are not at their upper or lower limits, and applies to the expected returns on those securities and the coefficients of their regression equation for the return on that security on the return on that portfolio, the beta-factor of the security relative to the portfolio.

4.4.8 The result can be expressed as:

$$E_i - r = \beta_{i,P}.(E_P - r)$$

where:

— R_i is the return on security S_i;
— R_P is the return on portfolio P;
— $E_i = \mathrm{E}[R_i]$ is the expected value of R_i;
— $E_P = \mathrm{E}[R_P]$ is the expected value of R_P;
— $\beta_{i,P} = \mathrm{Cov}[R_i,R_P]/\mathrm{Var}[R_P]$ is the beta-factor of R_i on R_P; and
— r is the return on a hypothetical security for which the beta-factor is zero (which might be a security with zero variance in the numeraire used, or a 'risk-free' security).

4.4.9 Thus the expected return on such a security in excess of the return on the zero-beta security is the product of the expected return on a chosen portfolio in excess of the return on the zero-beta security and the beta-factor of that security on that portfolio. If the adjusted discount rate, i.e. $r + \beta_{i,P}.(E_P - r)$, is used to discount the expected cash flows, then the NPV is zero.

4.4.10 This applies to any security which enters freely into portfolio P and is not at its upper or lower constraint, i.e. $L_i < x_i < U_i$, where x_i is the proportion invested in security S_i. However, for any security at its upper limit, U_i, then:

$$E_i - r > \beta_{i,P}.(E_P - r)$$

so its expected return lies above the beta-line, and, if its expected cash flows are discounted at the risk-adjusted rate, its NPV is positive. For any security at its lower limit, L_i, (which might be zero):

$$E_i - r < \beta_{i,P}.(E_P - r)$$

so its expected return lies below the beta-line, and, if its expected cash flows are discounted at the risk-adjusted interest rate, its NPV is negative.

4.4.11 In order to carry out a portfolio selection exercise, we need to decide on the values of all the means, variances and covariances required. We can do this by making our own subjective estimates of these values, or by investigating past data, and then we need to assume some sort of stochastic model that describes how we believe the distributions of returns on the various securities we are considering are determined.

4.4.12 The random walk model is popular with American financial economists, and this has many advantages in the fairly short term, but for longer terms it is better to use a specific stochastic model that takes account of the realities of the market. The model developed by Wilkie (1986a, 1986b, 1987, 1992) is one such which has been widely used and discussed, but any model of this type that takes account of the correlations over time would be in the same category.

4.5 *Using the Portfolio Selection Model to Assess Projects*

4.5.1 The portfolio selection model we have described gives a framework within which a new project for an existing company can be considered. One could, perhaps, start from the assumption that the portfolio of existing projects undertaken by the company is optimal for the company, and the portfolio can be assumed to lie on the company's efficient frontier in a mean-variance plane.

4.5.2 It would then be necessary to estimate the expected returns on the individual projects making up the company's present portfolio, and also their variances and covariances with the total portfolio, from which the beta-factors, relative to the total portfolio, could be derived. This could be done by a study of returns on these projects over the past, provided these can be identified; there could be a problem with this in practice. From this the company's beta-line can be estimated.

4.5.3 We can then consider a proposed new project. What is its expected return, and what is the covariance, and hence beta-factor, of that return with the total portfolio? Where does this project lie relative to the beta-line for the company? If it lies above the beta-line then the project is attractive; if it lies below the beta-line it is unattractive.

4.5.4 The position of future prospects for existing projects relative to the beta-line can also be considered. They will not necessarily all lie on the beta-line if the portfolio is not on the efficient frontier that would be derived using this methodology, and assuming that projects could be 'purchased' in any quantities. However, projects whose expected return lies on or above the beta-line remain attractive, whereas projects whose expected returns lie below the beta-line are possible candidates for closure or disposal, depending on whether the scrap value or the sale value is greater.

4.5.5 The argument so far takes an enormous number of assumptions for granted. The portfolio selection model assumes that 'securities' can be purchased in any desired quantity. In reality projects are discrete 'lumpy' investments, which often cannot be divided. The space of feasible portfolios is then not a

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

continuous region, but a number of points, each one representing a particular selection of projects. The efficient frontier is not a continuous line, but a series of points that lie to the furthest south-east (in our diagrams; in other diagrams they may be to the furthest north-west).

4.5.6 When projects are discrete, selection of optimum portfolios becomes a problem of 'integer programming', rather than 'quadratic programming'. When the number of possibilities is small, simple enumeration of all the combinations is practicable, but when there are larger numbers of possibilities, many of which may be mutually exclusive, the mathematical methods become more complex.

4.5.7 We have simplified reality by assuming that it is possible to assess all the projects undertaken by the company over the same single time horizon. To do otherwise complicates the problem considerably, since it requires companies to make an assessment of their preferences between profits at different dates. We discuss this further in Section 4.6.

4.5.8 It is not simply a matter of looking at the returns over one year. A suitable common time horizon should be used, which accords with the typical or average lifetime of projects undertaken by the company. This may be relatively short or rather long, depending on the type of business the company is in.

4.5.9 We have also assumed that it is possible for the company to estimate, not only the expected (i.e. mean) total return, over the prescribed time horizon, for each project, but also the covariance of that return with the return on the portfolio of projects. How this is done would have to depend on the circumstances of each particular case. A very diversified company may consider that the returns on its separate enterprises are connected no more than through the general well-being of the economy. A different company with a single purpose, expressed through a variety of similar products manufactured at different plants, or different branches selling a range of similar products, may view them as being more closely associated.

4.5.10 The point, however, is not whether the projects are widely diversified or closely connected, but how they rank relative to the company's total portfolio. The weighted average beta-factor of all the projects in the portfolio must, by definition, be unity. Our approach takes the variability of the existing portfolio as being roughly what the company wishes, and considers whether a new project increases or diminishes that variability, and whether the expected return justifies the change.

4.5.11 The method outlined so far does not tell us what to do about a wholly new project, which may require the establishment of a new company and the raising of capital for it. This may require consideration of whether the expected return for prospective shareholders, taking into account the costs of fixed-interest capital, and the consequent gearing of the return to shareholders, is sufficient to be attractive to possible shareholders who may use their own portfolio selection models, or indeed the CAPM, to assess the attractiveness of particular shares.

4.5.12 Note that, except where a new company is to be floated or where a significant new issue of shares is proposed, the company's share price and the

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

observed or expected return on its shares, and the observed or estimated beta-factor of its share relative to other shares over any time period, does not enter into consideration. Note also that no assumption is made about the existence of a risk-free asset, or what it would be if it were to exist; nor does the methodology rely on the CAPM.

4.5.13 The method does, however, require that the units of measurement, or 'numeraire', be defined. Returns could be defined in money terms, in sterling or in some other currency, or they could be defined 'in real terms', i.e. after allowing for inflation. Calculating in real terms as opposed to nominal terms does not alter the ranking of projects in respect of their expected returns, since converting from nominal to real involves dividing all returns by the same divisor. However, changing from nominal to real returns may have a substantial influence on the variances and covariances, and hence on the beta-factors of projects relative to a given portfolio, depending on whether the return on the project is itself more or less correlated with inflation.

4.6 *Multiple Time Periods*

4.6.1 The portfolio selection model is essentially a one-period model. In reality time is continuous; but it is practical to consider it as a series of discrete periods, and for many purposes one uses years, because companies typically do their accounting in yearly periods. Many projects can be expected to last more than one year, and many projects provide returns irregularly over their lifetime. It is rather easier if the returns can actually be assumed to be uniform year by year over the lifetime of the project, because various conventional methods like redemption yield or rate of return on equity can be used; but it is inappropriate to force projects into this mould if the returns are not uniform. Further, unless the assumed future rate of inflation is zero, a project that gives level returns in nominal terms cannot give level returns also in real terms, and vice versa.

4.6.2 Ways of extending the portfolio selection model to multiple time periods have been discussed by Fama (1977), Sherris (1987) and Mehta (1992). They justify the method of using adjusted discount rates to assess projects, over multiple periods; but if the basic portfolio selection model is to be used, it is necessary to choose a specific time horizon over which to assess the project, and to use the same horizon for all other relevant projects.

4.6.3 A convenient way of doing this is to use the method developed by Wise in relation to pension funds. See Wise (1984a, 1984b, 1987) and Wilkie (1985). In this method it is assumed that all the initial capital is invested at the commencement of the project. If it is not needed for paying for equipment at the time, the excess can be assumed to be invested in 'cash', or some other liquid asset until it is needed. The usual pattern of investments is that significant expenditure is incurred in the early years, and profits come in later. The net income resulting from the project is assumed to be reinvested in a suitable asset, which may not need to be particularly liquid, until the final horizon.

4.6.4 For some projects terminal capital expenditure has to be incurred. If the

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

project overall is profitable, then this outgo can be paid out of the accumulated proceeds of earlier profits. Calculations can generally be effected by assuming that the sums set aside to meet these later expenditures earn the same rate of interest as the accrued profits have been assumed to be earning. The 'interest' earned by accumulated profits might well be a stochastic variable, and does not need to be related to interest rates on bank deposits. A company could assume that the available funds were invested in the same sort of way as the funds of the company are usually invested, with the same sort of returns; but it is important that the assumption should be realistic, and that an unrealistic target is not used in this part of the calculation.

4.6.5 It is necessary also to take tax into account on a realistic timeframe. This has been a feature of properly conducted investigations into the financing of capital projects since the seminal work of Merrett & Sykes (1962).

4.7 *Assessing Means and Variances*

4.7.1 In order to assess all the means and variances that are required, it is also necessary to assume as stochastic all those variables in which there is uncertainty, and it is therefore necessary to define the final return in probabilistic terms. For general investment variables, price inflation, salaries, market interest rates and so on, it may well be useful to use a stochastic investment model such as the 'Wilkie model', which is familiar to actuaries, or some other suitable model. Stochastic investment models are discussed further in Sections 4.10 and 4.11.

4.7.2 While a general stochastic investment model is useful for those general factors that affect all projects, there will be many other factors specific to each particular project, and it will be necessary to specify a probability model for these factors. The numerical parameters for such a model may be assessed subjectively, on the basis of the experience and good sense of those familiar with such projects; or they may be assessed on the basis of empirical investigation, which may be carried out specially for the assessment of the project, or may be part of the common knowledge available about this class of project.

4.7.3 A convenient methodology for deriving estimates of the means, variances and covariances of the total returns on various projects is stochastic simulation. This has the advantage that the methodology is well established and widely applicable, and that all the interactions and correlations between the different parts of the model can be taken into account. Simulation is a way of estimating the theoretical distribution of the final results, given the theoretical distributions of the relevant inputs.

4.7.4 An alternative to stochastic simulation, which is sometimes practicable, is direct analytical manipulation in order to derive the desired means, variances, etc. This method should not be overlooked if it is conveniently practicable; but it is not a good idea to simplify a problem excessively just in order to make an analytical methodology possible.

4.7.5 Stochastic simulation allows the overall pattern of the distribution of total return, at the final time horizon, to be estimated. One needs the mean, variance and relevant covariances to be calculated as summary statistics from the total distribution, but it may also be useful to calculate the higher moments (and hence the skewness and kurtosis coefficients) and also to investigate the overall shape of the distribution. Sometimes it will be approximately normal, though more often approximately lognormal; this will generally be true if the distributions of the input variables are treated as normal or lognormal. If some of the input variables have very skew distributions, or are binomial or multinomial discrete distributions, or mixed discrete and continuous distributions, then the distribution of final total return may retain some of these characteristics.

4.7.6 Some examples of such non-normal distributions that are familiar in other actuarial contexts include the following situations:

— The life assured under an endowment assurance policy may die at any time before the maturity date (a continuous distribution) or survive to the maturity date (a discrete mass of probability).
— A member of a pension scheme may die before normal retirement age, or survive to normal retirement age and die beyond it; the distribution of age at death is continuous, but the financial consequences of whether this occurs before or after retirement are discontinuous.
— A substantial liability claim that goes to court may have a discrete probability of zero amount, if the defender wins the case, or the complementary probability of a significant award, if the pursuer wins; further, the probability distribution for the amount of this award, if it is made, may be very skew.

4.7.7 These give some analogies for other circumstances: many projects may be either successful, with very varying degrees of success, or may be failures; theatrical productions are often of this kind. Products may gain effective monopoly of a market, until a competing technology intervenes. Several competing technologies may be launched at once, and it is uncertain which will win out (e.g. the format of video tapes). A process may be successful if particular large contracts are obtained, a failure otherwise. Government regulation may act as a binomial random variable; an activity may be permitted or prohibited. The examples could be continued indefinitely.

4.7.8 Note, however, that what is important to shareholders is often not the variance of the project, but the covariance with the existing portfolio of projects, expressed through the beta-factor. The distribution of the total portfolio may be relevant, and any individual project may contribute to that distribution; but if the number of projects is sufficiently large, then the distribution of the return on any portfolio will be reasonably close to normal. This depends on the extent of non-normality of the distribution of returns on individual projects, and on their relative weights, and it may well be worth constructing a portfolio simulation methodology in order to investigate the shape of the distribution of portfolio returns. However, it is this distribution that is important to the owners of the business, not the distribution for individual projects.

4.8 *The Capital Asset Pricing Model (CAPM)*

4.8.1 In ¶4.5.12 we observed that the methodology of using the portfolio selection model to assess capital projects did not rely on the CAPM. The portfolio selection model is for the use of an individual investor, using his or her own assumptions about the expected returns and other statistics of the possible projects in which he is interested.

4.8.2 The CAPM, on the other hand, is an economic equilibrium model, describing what, theoretically, would happen if certain conditions were fulfilled. The most important conditions are that all investors have the same choice of available investments, and make the same estimates of the means, variances and covariances of the returns on those investments. It also relies on all investors using the same numeraire, whether this is nominal or real, and having the same single time horizon.

4.8.3 In effect, it says that if all investors agreed about the prospects for all possible investments, then the market would settle down in a certain equilibrium position, in which all investors would hold different quantities of the same portfolio of risky investments, together with greater or lesser quantities of whatever asset they all agree is risk-free. In this model investors are assumed to have different degrees of risk-aversion, and so choose different proportions of risky and risk-free assets.

4.8.4 Such a model is instructive for considering certain aspects of stock exchange investment, and also as a theoretical ideal, but we do not feel that it is a wholly satisfactory model of the real world; our reasons for this view are elaborated in Section C.10.

4.9 *The Risk Premium on Shares*

4.9.1 In this section we consider the risk premium on ordinary shares as compared with fixed-interest stocks, a measure which is used explicitly in the CAPM, as the risk premium on all risky assets. Many recent articles, for example Grubb (1993) and Dimson (1993), have put forward the proposition that the risk premium on shares can be derived from an analysis of the actual returns of shares as compared with fixed-interest over some past time period, and that over a suitable time period in the U.K. it has been observed to be about 7% p.a.

4.9.2 Applying this to the CAPM, it is suggested, in such articles, that the expected return on shares in aggregate is 7% p.a. or so higher than the expected yield on a risk-free asset. Sometimes this risk-free asset is assumed to be short-term cash or Treasury Bills, sometimes long-term fixed-interest stock.

4.9.3 We consider that, unless the investor's time horizon is really only very short, it is more appropriate to compare returns on shares with returns on long-term fixed-interest stock. Thus, when redemption yields on long-term U.K. Government Stock are 8½%-9% p.a. (as they were at 30 June 1994), a risk premium of 7% would imply an expected gross return on shares of 15½%-16% p.a.

4.9.4 These assumptions are wholly at variance with the typical assumptions

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

made by actuaries in the valuation of pension schemes. See, for example, Thornton & Wilson (1992). One is much more likely to find actuaries assuming that shares might yield 1% to 2% p.a. more than fixed-interest stock; but there may be an argument that for actuarial reserving against liabilities it is proper to use a discount rate that is lower than one would use for the assessment of a potential asset.

4.9.5 Note that all the returns discussed in this section are gross returns to an investor such as a pension fund.

4.9.6 The risk premium on risky assets appears in the CAPM as the term $(E_P - r)$, where E_P is the expected return on the 'market portfolio', strictly the portfolio of all possible risky assets, but in practice taken as the portfolio of all shares, and r is the fixed return on the risk-free security.

4.9.7 In our development of the portfolio selection model, E_P is the expected return on the portfolio of projects undertaken by a particular company, and r is the return on a hypothetical risk-free security, defined simply by where the beta-line meets the E axis. The 'risk premium', defined in our way, depends on the company's assessment of the prospects for the projects it undertakes or is considering undertaking, and has nothing to do with the experience of its own or any other shares in any market over any past period.

4.9.8 However, it is also important to explain why we do not think that the generality of investors on the stock market have expected, or should be expecting, a risk premium on shares as high as 7% p.a. In the accompanying paper 'The Risk Premium on Ordinary Shares' A. D. Wilkie reviews the historical evidence and explains why he considers that a risk premium on shares of 0% p.a. to 3% p.a. over fixed-interest stock is appropriate for the future. He also explains why, for the assessment of capital projects, it may be appropriate to use a discount rate up to 2% higher than this, i.e. between 2% p.a. and 5% p.a. higher than the real yields on long-term index-linked U.K. government stock, all gross of tax. Some of us would prefer to use 6% as the upper end of the latter range.

4.10 *Stochastic Investment Models*

4.10.1 A number of actuaries have found it useful to simulate possible 'futures' by using a stochastic investment model such as that put forward by the Maturity Guarantees Working Party (1980), or the 'Wilkie model', described by Wilkie in a series of papers (1986a, 1986b, 1987, 1992), which was discussed by Geoghegan *et al*. (1992). Such models contrast with the typical 'random walk' model for total returns on investments put forward by many financial economists, in that the actuarial models recognise that certain features, like the dividend yields on ordinary shares and interest rates in real terms, are statistically stationary, and also recognise how price inflation influences share dividends after a suitable time lag, and how the inflation experienced affects what one might take as 'the market's' view of expected future inflation. Such models have the

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

advantage of taking into account the whole structure of the variables included, and the relevant correlations between them, both simultaneously and lagged.

4.10.2 These actuarial models are similar to many econometric models, but differ from most of these in the time horizon used, which is for many years at annual intervals, rather than for only a few years, possibly at quarterly intervals; and also differ because the actuarial models are wholly self-contained, and do not include any exogenous variables, such as current government policy. Such exogenous variables are of great use in short-term forecasting, but would themselves require to be forecast if they were to be used over a long period. The actuarial models are also designed to represent the probability distributions of possible futures, whereas the usual econometric models emphasise single point forecasts (although distributions must be readily available internally).

4.11 *Using a Stochastic Investment Model in the Assessment of a Capital Project*

4.11.1 Stochastic simulation or 'Monte Carlo' methods are familiar to the actuarial profession. Amongst other references we can cite Benjamin (1966), Maturity Guarantees Working Party (1980) and Daykin, Pentikäinen & Pesonen (1994). They are described in many textbooks, such as Rubinstein (1981) and Johnson (1987), and there are now software packages available that facilitate both the construction and the presentation of stochastic simulations.

4.11.2 This is not the place for a lengthy explanation of stochastic methods. Points to be considered are:

— For each discrete time step a series of independent random variables must be generated, using appropriate distributions (and a great variety of these is available).
— These independent variables must be combined to reflect the desired correlations in the model in order to simulate the desired variables.
— Consideration must be given to the information available at any time, and 'foresight' must not be used.
— Careful consideration needs to be given to what results are required, because otherwise the volume of output can be overwhelming.

Possible outputs include:

— the distribution of final returns or of NPVs;
— the distribution of profits (revenues, costs,...) in each future year, or selected years;
— the distribution of events, such as the first year a profit (loss, ...) occurs; and
— the distribution of the time until such an event.

4.11.3 We believe that the actuarial stochastic investment models referred to above may be found to be of considerable practical use in carrying out the simulation of the financial returns on capital projects, because of the particular features referred to in Section 4.10. Part of the model-building process for project simulation will then be to develop specific links between the economic variables simulated by the actuarial stochastic model (e.g. price inflation, wages, interest rates and investment returns) and the cash flows for the project under consideration.

4.11.4 Although simulation methods are powerful and readily accessible, it should not be forgotten that, in some circumstances, analytical methods are possible. Both analytical and simulation methods have their advantages.

5. Some Practical Considerations

5.1 *Working as a Team Member*

5.1.1 The actuary will normally work on projects as a member of a multi-disciplinary team. The extent of the work needing to be done by the actuary will vary from case to case, but the principal areas in which his assistance is likely to be needed are:

— identification, quantification and minimisation of risks;
— financial model building, choice of numerical assumptions, application of discounted cash flow techniques, simulations, etc.; and
— designing funding structures which enable the risks to be assumed where they can best be handled.

5.1.2. Other team members are likely to be more concerned with such aspects as:

— liaison between all interested parties;
— estimation of costs of construction, maintenance and disposal;
— estimation of the market for the project's product and the likely revenues obtainable;
— method and costs of operation;
— schedule of cash flows in each period;
— tax implications;
— grants available; and
— effect on company's future financial results.

5.1.3. The actuary will need to work closely with the other members of the team in order to obtain the inputs he needs and to understand the implications. The results of the actuary's work will need to be considered by the team, to ensure that the actuary has correctly interpreted their inputs and to enable all concerned to gain a greater insight into the nature of the project.

5.2 *General Approach to Appraisal Work*

5.2.1 It is suggested that the actuary's work on appraisals should normally proceed in the following way:

(a) There needs to be a good understanding of the basis on which the estimated cash flows have been prepared. In particular, it must be clear whether each figure represents the mode, median or mean of the underlying probability

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

distribution, and whether (and how) allowance has been made for tax and inflation. All non-cash items (e.g. depreciation) must be rigorously excluded from the cash flow analysis. Any effects on the company's existing business must be taken into account. In practice, the determination of the appropriate incremental cash flows attributable to a project often gives rise to difficulties and requires a clarity of thought in which actuaries would be well placed to assist if required.

(b) The actuary will question every team member about the risks involved in the project and attempt to get a 'feel' for the underlying probability distributions of the various principal elements comprising the cash flows. He may supplement this with his own research on the industry and include over-riding risks which are not specific to the project.

(c) A series of discounted cash flow calculations will be carried out to assess the NPV of the project at various discount rates, allowing for the cash flows to take alternative values. Computer simulations may be carried out, where justified.

(d) The IRR of the project for each set of cash flows will also usually be calculated, and the payback period, but it will be pointed out that these figures are shown for information only, and the main criterion for decision should be the NPV.

(e) The results will be presented in the form of a draft report which sets out clearly the main inputs and assumptions and the numerical results obtained. The report will identify the key risks and suggest ways in which their potential financial impact can be minimised and managed. If required, the report will also suggest criteria for monitoring the implementation of the project. The report will be expressed as simply as possible, so that its conclusions can be understood by non-specialists.

(f) After discussion of the draft report with the rest of the team, the actuary will complete the final version of his report — this should preferably be submitted in full to the company's senior management rather than its contents being summarised. If the report has to be summarised, the actuary should insist on the right to approve the wording.

5.2.2 Given the controversies referred to in Sections 3 and 4 about the 'correct' discount rate to use, it may be useful to the decision-maker to see how sensitive the overall outcome is to changes in the discount rate. This is why it would normally be appropriate to show results for at least two discount rates, e.g. 6% p.a. and 10% p.a. if applied to cash flows expressed at constant price levels, with a commentary on which result is likely to be more appropriate in the circumstances. If the company already has a 'standard' discount rate, the calculations should also be carried out on that basis. If, however, the actuary considers, after discussion with the company's finance director, that the circumstances are such that the company's standard discount rate is inappropriate for the project in question, he should not hesitate to point this out (with reasons) in his report.

5.2.3 In the remainder of this section we indicate how we think the actuary might proceed with his work, starting with simple appraisals, where a good grasp of the principal risks and net present values is required, but detailed risk analysis is not justified. This could apply either for a straightforward project or for the early stages of analysis of a complicated project. Some of the techniques we suggest include some approximations in the risk analysis which are theoretically inexact, but easy to apply in practice. We then consider what additional work might be carried out in the case of a more complex appraisal, where the initial indications for a large project look promising and detailed analysis is now required. We look at practical considerations regarding the possibility of insuring some of the risks in a project. We also examine the special practical points which can arise in the case of Government projects. This section concludes with a discussion of the Government's Private Finance Initiative, and the ways in which both the public and private sector parties can find acceptable deals if risk is properly managed.

5.3 *Simple Appraisals*

5.3.1 Let us assume that the actuary has been engaged as a member of a team which is appraising a relatively straightforward construction project for a commercial firm. The size of the project is not large enough to justify the expense and effort involved in extensive investigations for elaborate computer-based simulations, though simple computerised methods might well be employed to manage the arithmetic. What is required is a straightforward method of appraisal which can easily be applied in practice and which is easy to understand.

5.3.2 The simplest method of all is to start with a schedule of the most likely cash flows (expressed at a constant price level) for each year of the project's construction and operation, and use standard compound interest techniques to calculate:

(a) the IRR; and
(b) the NPV at two specimen rates of interest, e.g. 6% p.a. and 10% p.a.

5.3.3 However, the use, in this way, of the mode rather than the mean (or 'expected value') of the distribution of each year's cash flows is very likely to give rise to misleading results (see Appendix D for an illustration of this point). The method can also lead to competing projects with differing underlying probability distributions being ranked in the wrong order. At least some risk analysis is, therefore, essential.

5.3.4 The simplest form of risk analysis is to consider, in conjunction with other team members, the extent to which the net overall cash flows in each year are likely to vary from the most likely value, and repeat the exercise of ¶5.3.2 for an 'optimistic' and a 'pessimistic' set of cash flows. An element of subjective judgement will enter into the determination of just how 'optimistic' or 'pessimistic' the net cash flows used are likely to be. In principle, if cash flows in each year have an underlying probability distribution which can be estimated with some degree of confidence, the aim might be to set the optimistic cash flows

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

at the level at which only 5% of the cash flows are more favourable. Similarly, the pessimistic cash flows would be set at the level at which only 5% of the cash flows are less favourable. There would thus be only a 10% chance that the cash flow in any year falls outside the range which has the chosen optimistic and pessimistic values as its limits. In practice, the uncertainties about the underlying probability distribution are likely to be so great that precision is unattainable. Moreover, the figures for successive years are unlikely to be independent of each other, and it would complicate the method considerably to take proper account of this.

5.3.5 Despite the theoretical limitations of this technique, it is easy to apply and understand, and may sometimes help in focusing attention on the variability of the possible financial outcomes. The actuary will also investigate and advise on such questions as:

(a) Which underlying assumptions are the least predictable? If their impact on the financial results of the project would be significant, could further investigations reduce the degree of uncertainty?
(b) How sensitive are the results to alterations in the basic assumptions on which the appraisal is based? Examples of basic assumptions, which might otherwise go unchallenged, are: stability of existing law and tax regimes, availability of labour and raw materials at constant real price levels, absence of extreme economic conditions, etc.
(c) What are the key downside risk areas for the project, i.e. which events, if they occurred, would have a major adverse impact on the financial results obtained? What is the worst that can happen?
(d) How would the project, if undertaken, affect the risk profile of the company's portfolio of projects? Would the effect be to reduce or increase the overall risk to the company, and by how much? What would be the likely effect on investors' perceptions of the company?
(e) Can the project be financed from cash resources already available to the company or is external finance necessary? How should the financial package be structured?
(f) Can the downside risks of the project for the actuary's client be minimised by insurance or by otherwise passing some of the risks to another party? Does it make sense to share the project with another company, on the basis that they will share one of their own projects, so that both companies achieve a reduction in overall risk?

5.3.6 Another, and perhaps better, way of approaching the risks involved is illustrated by the example in Appendix D. A specific estimate is made by experts of the underlying probability distributions for ranges of financial results in the three key areas of:

(a) capital costs;
(b) operating costs; and
(c) receipts.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

5.3.7 The mid-points of these ranges are then combined into a series of 'outcomes' for the project, and the probability of each outcome is calculated by multiplying the underlying independent probabilities.

5.3.8 The example given is simplified for ease of presentation. In practice it might be desirable to have, say, three ranges of capital costs, four ranges of operating costs and five ranges of receipts, leading to 60 possible outcomes for the project. Whereas the technique suggested in ¶¶5.3.2 and 5.3.4 focuses mainly on the net cash flow in each year, i.e. receipts less costs, and is somewhat imprecise in the degree of optimism or pessimism assumed in the upper and lower outcomes presented, the technique illustrated in Appendix D focuses on receipts separately from costs, and starts with the best estimates available of the probability distribution for each item. As Appendix D illustrates, assumptions may also be made about specific future events and the likelihood to be attached to them — in the case illustrated, the possibility of a competitive product appearing on the market after three years.

5.4 *Complex Appraisals*

5.4.1 In addition to the techniques described for simple appraisals, there are some extra steps which can be taken in a large and complex project where the expense can be justified.

5.4.2 One such step might be the preparation of a 'risk matrix', which will identify each of the downside risks likely to be incurred at every stage of the project. An outline of a risk matrix is shown at Appendix E, and this framework could be used as a checklist in the identification of the risks involved in almost any project.

5.4.3 Once the risks have been identified, each one needs to be quantified. One way of doing this is to ask an expert to attach degrees of likelihood to a few of the possible financial outcomes for the item concerned, and to estimate the mean outcome, so that a smooth line can be drawn between these points to estimate the underlying probability distribution. Alternatively, the expert can be shown graphs of some possible probability distributions (e.g. uniform, normal, lognormal, etc.), and be asked to state which distribution most nearly coincides with his view of the underlying probability distribution for the item in question. This might be done for hundreds of independent items, for many years into the future. The resulting distributions could be used in a simulation program from which the expected value and distribution of the overall financial outcome for the project can be estimated. In effect, this method can be regarded as a more sophisticated version of the technique described in ¶5.3.6. Possible dangers are that an unwarranted degree of precision may be attached to the results of the simulations, and that the key assumptions may be lost sight of in a welter of detail. Moreover the employment of many experts to assist in the analysis described may weaken the coherence and internal consistency of the overall framework.

5.4.4 In the appraisal of a complex project, it may be worthwhile to devote more attention to the underlying conceptual model. Thus, for example, the

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

assumption that items are independent of each other may not be true, and some specific linkages might need to be built in. For instance, suppose that the distribution of the cost of a particular item is normal and that the distribution of a particular item of receipts is normal; both items may take unfavourable values together if extreme economic conditions occur, so a correlated bivariate normal distribution may be appropriate. Moreover, the value taken by a variable in year t may be correlated with the value it had in year $(t - 1)$ or even in previous years, and this correlation, too, may need to be taken into account through an appropriate time-series model. Unless such adjustments are made, the extent of the variability of the overall financial outcome of the project may be over- or under-estimated, if the simulation technique of ¶5.4.3 is used.

5.4.5 In some cases it may be appropriate to allow in the simulations for possible variations in the rate of future inflation, and stochastic models such as those developed by Wilkie (1986a, 1986b, 1987, 1992) or by others might be useful here (see Sections 4.10 and 4.11). Each simulation would allow for a rate of inflation in each future year determined from the stochastic model, allowing for the fact that the rate of inflation in any one year is not independent of the rate of inflation in the previous year. The financial results for certain other items in the simulations might depend on the general economic conditions prevailing each year. It might therefore be appropriate for actuaries to extend their existing stochastic models to reflect such factors as the rate of GDP growth, the rate of unemployment, etc.

5.4.6 A useful guide to the detailed methods and probabilistic models which can be used for the risk analysis of large projects is given by Cooper & Chapman (1987). Drawing on their practical experience and using a variety of case studies, they demonstrate techniques which enable the views of specialists to be modelled mathematically. In particular, they discuss how to identify risks, the two basic ways being to use a panel group for a brainstorming session or to question individual specialists. Some practical methods are described whereby the views of specialists who are unfamiliar with probability concepts may be quantified. The authors say that they have found, in practice, that it is very useful to carry out initial analyses using pocket calculators alone, rather than computers, even for a very complex project. They conclude that they no longer see a numerical 'solution' to a 'problem' as the most important part of risk analysis. Instead, a major purpose is to provide a structure which will allow a variety of people with diverse disciplinary backgrounds to communicate and gain insight. Another most useful paper is H.M. Treasury (1993b); although drafted for the benefit of Government Departments, much of the guidance given would equally be applicable to large projects in the private sector. The paper summarises in just a few pages the whole process of risk identification and management, and gives a number of valuable practical hints.

5.5 *Insurance of Project Risks*

5.5.1 In Appendix E we consider a number of ways of dealing with downside risk, one of which may be to take out insurance. The example in Appendix D illustrates how insurance (if available) may be used to minimise the risks involved in the project, as far as the sponsor is concerned. Such insurances are likely, in the long run, to provide good profits for insurers, despite the uncertainties. In effect, part of the project sponsor's upside potential in some outcomes is sacrificed in order to reduce the downside potential of other outcomes. Part of the sponsor's 'expected gain' (in the mathematical sense) is also sacrificed, on account of the insurer's generally more cautious approach to risk and his expenses and profit margin.

5.5.2 However, the availability of insurance does not alter a project's aggregate expected NPV, and whether the project is attractive or not (from the viewpoint of shareholders as a whole) depends only on whether the expected NPV, discounted at the appropriate risk-adjusted discount rate, is positive. In essence, the risks of all projects can be spread among all shareholders in the market, and insurance simply transfers profits or losses from one pocket to another of the same shareholders. Hence, from the viewpoint of shareholders as a whole, all companies should undertake the projects with the highest expected NPV, even if there is a significant downside risk in such projects. Even if some companies become insolvent as a result, the expected overall return to shareholders holding a widely spread portfolio of shares will, thereby, be maximised. The problem with adopting a 'no insurance approach', however, from the viewpoint of shareholders as a whole, is that it may lead, in practice, to some projects with very high expected NPVs being rejected by cautious managers. It may, therefore, be in the interests of shareholders to encourage insurance, to the extent that it enables such projects to become acceptable to the company managements concerned.

5.5.3 From the viewpoint of a company's management, the question of whether they would wish to insure the downside risks in a particular project is likely to depend on whether the project could have a significant impact on the company's finances if it went wrong. For a project which would not have a significant impact, the company might well decide not to insure, on the basis that this is the course of action which provides the maximum expected rate of return (because part of the profit is not being diverted to the insurer), with poor returns on one project being more than counterbalanced by good returns on another. For a project which could have a significant adverse impact, however, the company's board might well feel that insurance (if available) would be an attractive option, in order to safeguard the future of the company, its managers and its employees. If such insurance is not available, and the downside risks cannot be passed to third parties in other ways, the company might decide not to proceed with the project.

5.5.4 We have made enquiries about the extent to which insurance is likely to be available, in practice, to cover project risks. An insurance market already exists for insuring against physical catastrophes — e.g. fire, storm or adverse ground conditions. We are unsure about the extent to which companies could, in practice, insure against adverse business risks arising from other causes. Much will perhaps depend, in a particular case, on the scale of the cover envisaged, whether the insurer can be protected against 'moral hazard' by suitable excesses and other policy conditions, and whether the insurer envisages a long-term relationship with the company extending over a number of future projects.

5.6 *Government Projects*

5.6.1 By 'government projects' we mean those projects which are sponsored and funded by a country's central or local government. In view of the world-wide tendency towards privatisation of commercial activities hitherto undertaken by the state, government projects will increasingly relate to activities which have little commercial potential, but do offer the promise of benefit to the community as a whole. One of the difficulties is how to measure that benefit.

5.6.2 A pioneering study in this field was that by Foster & Beesley (1963) in relation to the proposal to build London Underground's Victoria Line. They found that much of the 'social benefit' took the form of time savings: it was necessary to value these time savings, and a somewhat arbitrary figure of five shillings per hour was selected. It was also necessary to place a monetary value on passengers' comfort and convenience. Calculations had shown that, if social benefit was ignored, the Victoria Line was almost certain not to earn enough at then current fare levels to meet its interest charge after covering operating costs and providing for depreciation. Taking the estimated social benefits into account, however, there was a net present value (at 6% p.a. real) of £31m. The ratio of discounted benefits to total discounted capital and running costs was 1.57. The internal rate of return was 10.5% p.a. real. These calculations were sufficient to justify construction of the Victoria Line, despite the expected financial loss it would cause for London Transport. In the event, after more than 25 years of operation, many people would probably say that the Victoria Line has proved a great asset to London, and that the decision to go ahead was justified.

5.6.3 Analysis of this kind, generally described as cost benefit analysis, has been used for many subsequent government projects (notably in transport), and it seems clear that it is here to stay. The quantified social benefits can be treated in the appraisal calculations just as though they were cash flows. It is important to ensure that all the social disbenefits of a project are measured, as well as its positive benefits. From the viewpoint of risk and sensitivity analysis, the uncertainties necessarily involved in quantifying social benefits pose wider challenges than if financial considerations alone were involved.

5.6.4 After carrying out the appraisal calculations, including cost benefit analysis if appropriate, together with risk and sensitivity analysis, a decision must be made on whether the project offers a sufficiently high rate of return to justify a decision to proceed. Since 1989 the cost of capital for the U.K. central government has been taken to be 6% p.a. real and this has also normally been used as a discount rate for public service projects. For public enterprises a required rate of return (or RRR) of 8% p.a. real has been seen as an average

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

return on their investment programme, and is generally used in those areas as a discount rate. The procedures are set out in H.M. Treasury *Green Book* (1991), and the justification for choosing these particular rates is set out in an interesting paper by Spackman (1991).

5.6.5 We suggest that the use of a fixed discount rate at all stages of the economic cycle may be inappropriate, and that consideration should be given by the Government to using lower rates in periods of recession if the real cost of capital reduces at that time. This would lead to more government projects being undertaken when there are unutilised resources in the economy, and fewer at times when the economy is booming and there is pressure on scarce resources: in our opinion, this could have advantages from social, economic and political viewpoints. Possibly the Government's discount rate could be adjusted annually, with a linkage in some appropriate way to the yield on index-linked gilts. It is possible that, if the Government adjusted its discount rate in this way, this could also act as a signal to the private sector to adjust its own corresponding rates. The arguments which have been advanced against this suggestion include the following points:

(a) a small change in the discount rate would not make much difference to the acceptance or non-acceptance of most projects;
(b) the amount of capital spending is controlled more by the level of public expenditure allocations than by the discount rate;
(c) the weightings of future years relative to each other should not vary with short-term cycles; and
(d) there could be practical difficulties in promulgating a new rate to all the public sector bodies involved and avoiding confusion regarding projects where negotiations are already in progress.

We are not convinced, however, that these objections, powerful as they are, outweigh the logic underlying our suggestion.

5.6.6 At times of high unemployment, it is sometimes argued that one of the benefits arising from new government projects which use people who would otherwise be unemployed is the saving to the public purse in unemployment benefit, etc., and the gain from the tax such people would start to pay. This argument raises complex issues, and at present we understand that it is not the practice to take such factors into account. We feel, however, that the subject deserves further study and discussion.

5.6.7 Some government decisions involve time preference, but not capital investment, for example decisions on whether to use finite resources now or in some years' time. One case in point is the question of whether to permit natural gas to be 'flared' (i.e. wasted) in order to get early production from combined oil and gas fields in the North Sea. We think that the conventional public sector discount rate should not be used automatically for such calculations, and that a lower rate, based on considerations relating to the time preference of the community as a whole, should be used instead. This would help to ensure that long-term considerations in such questions are given due weight.

5.6.8 In Appendix F we suggest a very simple technique for cost benefit analysis. The technique as described does not involve discounting methods, and so is suitable only for projects which are expected to have a life of up to about five years. Its advantage is that it is capable of being applied by those who have had little prior training. It might have applications both in the U.K. and overseas, even in developing nations.

5.7 *The Private Finance Initiative*

5.7.1 The U.K. Government announced its Private Finance Initiative in the November 1992 Budget, since when the rules governing it have gradually been more clearly defined (see Appendix G).

5.7.2 As the Initiative gathers momentum, it is likely that increased prominence will be given to the possibility of long-term projects being financed by life assurance companies and pension funds. This seems entirely practicable, provided that the prospective returns are deemed adequate and provided that an appropriate structure is developed. The contracts sometimes used for town centre redevelopment schemes in the 1970s may provide a useful precedent. For example, a local authority made the site available and the construction work was carried out by a developer, with the aid of bank finance. The developer accepted the risk of cost over-run and the risk that the shops could not be let. The contract signed at the outset specified that, once the building was substantially complete and pre-let, the funding institution would pay the developer for the costs incurred and interest thereon, up to a specified overall limit, thus enabling him to repay the bank and take his profit. From that point onwards the rental income from the property would be shared between the ground landlord (the local authority) and the institution in a predetermined ratio. Thus, the institution was not exposed to the construction or initial letting risks and, from its point of view, the transaction was similar to that of an ordinary purchase of a pre-let shopping centre, with the exception that a commitment was being entered into for an investment in two or three years' time rather than immediately.

5.7.3 It seems probable that insurance companies and pension funds will often be more interested in very long concessions (e.g. 99 years) or freeholds (subject to legitimate safeguards for the public interest) than they will in concessions of around 25 years, where there may be little opportunity for recovery if things go wrong in the early stages.

5.7.4 To proceed as in ¶5.7.2 would involve institutions in a considerable amount of appraisal work, project by project. This might be difficult for medium-sized institutions having only limited staff resources. Increased support might, therefore, be attracted from the institutions if a suitable funding vehicle were developed, which would employ its own specialist staff and negotiate directly with developers and the Government. If such a vehicle were an ordinary commercial company, with debt and equity finance, its debentures and shares could be regarded as a part of ordinary institutional investment portfolios. Such

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

a company could not only invest in projects where third parties are developers, but could, where appropriate, act as the developer itself to a limited extent.

5.7.5 The Government is understandably insisting that, where a public project is financed in whole or in part by the private sector under the Initiative, not only should the private sector benefit from much of the upside potential of the project, but they should also bear most of the risks involved. There is no difficulty in principle with this, provided that the deal is struck on a basis which gives the private sector upside potential commensurate with the risks they run. This assumes that the Government is prepared to retain some risks which are outside the control of the private sector, particularly those risks which relate to Parliamentary processes, any risks of Government control over the revenues from the project being introduced at a later stage in the public interest, and the risks of a future government-backed competing project.

5.7.6 In order to achieve an acceptable deal, it will sometimes be necessary for the Government to make a contribution to the costs of the project. One key question is how the size of such a contribution should be determined. A very simple example to illustrate the principles involved is set out in Appendix H.

6. Company Procedures

6.1 *Scope for Actuarial Involvement*

This section addresses the procedures that companies use to identify, appraise, control and monitor projects, and indicates possible areas for actuarial involvement. Although labelled company procedures, they could apply equally to a variety of different types of sponsors.

6.2 *The Search for Investment Opportunities*

6.2.1 Most companies are engaged in a search to identify ventures that will deploy existing capital more effectively or that will attract additional capital, consistent with their overall strategies. It is important, therefore, that companies should manage the search process in such a way that the optimum quality and variety of proposals is generated, and that worthwhile ideas are not stifled at birth.

6.2.2 The process may be organised with varying degrees of formality and structure. Often the arrangements are relatively informal, with proposals having their origins in a wide variety of sources. Typically these might include employees' suggestions, competitor information or general surveys of literature. More formally, a company may be large enough to support a full-time specialist unit with specific accountability for identifying and evaluating new ventures. Such units are typically attached to the finance function of a company and are part of the finance director's responsibilities. In this context, it is natural for the search process to be extended to address candidates for disposal and to determine the optimal strategy for replacement of assets.

6.2.3 While the search process should engage with as wide a variety of sources as possible, for reasons of efficiency it should be carried out within a comprehensive framework that communicates the company's objectives and policies.

6.3 *Differing Roles of Divisions and Head Office*

6.3.1 The circumstances under which proposals are prepared and championed within organisations have received much attention. Of particular interest is the evidence that has been acquired in respect of highly divisionalised organisations (Willey & Marsh, 1992).

6.3.2 Typically the operating divisions will propose ventures for approval by head office, and head office will have the perceived role as custodian of the organisation's capital. There is evidence to suggest that, while head office will see allocation of capital to divisions as consistent with the organisation's strategic plan, divisional managers often perceive the allocation as a function of divisional profitability.

6.3.3 The tendency of this mismatch of perceptions is to encourage divisional managers to concentrate on short-term results at the expense of potentially profitable longer-term investments.

6.3.4 True long-term strategic development needs to be supported by head office behaviour designed to discourage divisional short-term attitudes. This can be achieved explicitly by head office through its systems, criteria and decisions, and implicitly by indications, priorities and dialogues with managers.

6.4 *Avoidance of Rejection of Worthwhile Proposals*

6.4.1 If the search for investment opportunities is to be successful, the circumstances under which proposals are rejected are worthy of special attention. If proposals are to be encouraged, rejection without adequate rationale, in the absence of a communicable framework, is unlikely to serve the organisation well.

6.4.2 Most companies will have some, perhaps unverbalised, view of their attitude towards risk. However, most will not be large enough to engage in a sufficiently large number of projects to make this attitude both tangible and plausible.

6.4.3 The appropriate investment in the feasibility study stage of project appraisal will enable the company's attitude to become a demonstrable and explicit element in its decision making process.

6.5 *The System for Approving Projects*

6.5.1 Systems for project approval usually consist of procedures grouped in phases. The extent and depth of the procedures and the phase groupings will depend upon the needs of the company and industry concerned. The procedures should be such as to enable due consideration to be given to all aspects of the project by the relevant managers of different functions, with a view to ensuring that the company undertakes only those projects that are likely to benefit it.

6.5.2 However, as it is important to maintain the momentum of proposals

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

during the approval process, a lengthy and recursive programme is undesirable. In all cases the feedback from the approval process to the search process should be designed to strengthen and maintain interest in the search activity and focus it on the company's strategic objectives.

6.5.3 The system should be designed in such a way as to minimise any bias arising from an undue concentration on short-term issues or from undue optimism in its various forms.

6.6 *Control and Monitoring of Projects*

6.6.1 Techniques for project management are well established, (for example, Williams, 1982), and have been considerably facilitated in recent years by the wider availability of suitably powerful computing platforms.

6.6.2 The extent to which such techniques are applied in practice depends upon a number of factors. In general, complex and large projects will require some form of computerised project management system. Cheap computing power now means that even small projects can make effective use of these tools. The use of these tools and techniques represents a project overhead. In established industries with many years of project management experience, the value of this overhead and its role in the project management process is well understood. Typically 30% of project management effort may be expended on the application of specific technique, the balance being aimed at securing an environment for the project that is conducive to success.

6.6.3 The lessons of project management have been much studied. A model of environmental preconditions for project success is set out in Appendix I, adapted from Morris & Hough (1987).

6.6.4 In practice, choosing the tools and techniques applicable in a given set of circumstances is often a matter of preference and experience. In particular, the appropriateness of the consequent overhead expense is a matter for careful judgement. It is tempting to acquire and apply all the tools at one's disposal. However, without considerable practical experience in their application this is probably an error, and practice suggests that the simplest techniques have the highest payoff in terms of their contribution to project success when balanced with the percentage of project effort they consume. There is evidence to suggest that most projects that run into difficulties do so, not for lack of control and monitoring technique, but for lack of regard to the project environment.

6.6.5 This suggests that, from a risk management viewpoint, any project control and monitoring system should provide fast and systematic reporting on environmental aspects of the project as well as on physical progress. Any changes in the risk outlook should also be monitored.

6.7 *Reappraisals in Changing Circumstances*

6.7.1 Not all project risks last for the duration of a project, and risks may subsequently become apparent that were not evident at the outset. A desirable project design is not only one which has designed out as many risks as possible

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

at the outset, but is also one which has specified and ordered activities in such a way as to reduce the inherent riskiness of a project as it progresses.

6.7.2 The appropriate appraisal framework for updating the risk outlook at various key points in the progress of a project can, therefore, provide a deeper insight into the value of a facility being developed, as well as highlight departures from the project plan that might otherwise be hidden from traditional progress and earned value reporting systems. The latter tend only to highlight difficulties arising due to optimism contained in estimates built into the project plan.

6.7.3 It is important that financial reappraisals conducted during the progress of a project are based on future benefits and future costs, with 'sunk costs' being ignored. Among other things, such reappraisals should take account of any improvements which may have occurred in the net revenues expected to be earned by the project.

6.8 *Audit*

6.8.1 There is evidence to suggest that relatively few organisations carry out post-project audits of strategic investments as a matter of course (see Willey & Marsh, 1992). This is in sharp contrast to the extensive analysis which is widely conducted into the effectiveness of a fund manager's decisions concerning portfolio performance.

6.8.2 The value of a systematic approach to a post-project audit process lies in the identification of lessons learned. H.M. Treasury have produced a valuable paper (1993c), which sets out the steps which can be taken in order to monitor projects at various stages, particularly after completion. Although intended for use by Government Departments, this paper would be equally useful for private sector projects.

6.9 *Documentation*

6.9.1 The plan for project documentation should be an integral part of project design. It is central to an effective reporting process, and the difficulty of integrating it with procedures should not be underestimated.

6.9.2 Project documentation should reside in a reference or 'filing' system designed specifically to support the project. Examination of this system from time to time can be a useful project 'health-check', and can often give an early indication of project difficulties that have yet to emerge.

6.9.3 Project documentation should be recognisable, identifiable and therefore bound by rules. There is a danger, however, that paper can be produced in a ritual way and that project credibility can suffer as a result.

6.10 *The Actuary's Contribution*

6.10.1 The actuarial approach, taking specific account of each risk, constructs a coherent framework for appraisal and provides a similar process management tool to evaluate and communicate departures from a plan effectively.

6.10.2 The approach we are recommending has a parallel in process control in production and operations management. The technique known as Failure Mode Effect and Criticality Analysis (see Muhleman, Oakland & Lockyer, 1993) is generally used to analyse engineering component manufacture with particular reference to reliability issues.

ACKNOWLEDGEMENTS

We should like to acknowledge gratefully the willing help we have received from many quarters in formulating the ideas underlying this paper. Particular thanks are due to: Roy Brimblecombe, Chris Waites, Geoff Lindey, Gregor Campbell, John Bratley, Paul Remington, Ian Duncan, Tony Ridley, Owen Simon, Sheila Farrell, Robert Clarkson, Michael Spackman and Robert Weeden. We should also like to express our gratitude to Sally Grover and Roy Park for their assistance on aspects of the bibliography and to Rachel Wood and Mary McCamley for valiantly typing the paper and its numerous drafts.

REFERENCES

The following list includes, not only the works referred to in the paper, but also the details of some other publications, derived from a variety of sources, which might prove helpful as a starting point for further research. The literature on this subject is quite extensive and we are aware that there are many other books and papers, not mentioned here, that may also contain relevant material.

ADELSON, R.M. (1965). Criteria for capital investment — an approach through decision theory. *Operational Research Quarterly, March 1965.*

ARMITT, J. (1994). The tendering process. *Papers of Bow Group Conference on Public-Private Sector Partnerships, 2 March 1994.*

BAKER, A.J. (1986). *Valuing information about risks.* University of Leicester, Discussion Paper No. 51.

BEEDLES, W.L. (1984). Some notes on the cost of new equity. *Journal of Business Finance and Accounting,* **11**, 245-251.

BEEDLES, W.L. (1986). Some notes on the cost of new equity: a reply. *Journal of Business Finance and Accounting,* **13**, 153-154.

BENJAMIN, S. (1966). Putting computers on to actuarial work. *J.I.A.* **92**, 134-192.

BENNETT, J. & ORMEROD, R.N. (1984). Simulation applied to construction projects. *Construction Management and Economics,* **2**, 225-263.

BIERMAN, H. JNR. & SMIDT, S. (1960). *The capital budgeting decision.* Macmillan, New York.

BREALEY, R.A. & MYERS, S.C. (1991). *Principles of corporate finance,* Fourth Edition. McGraw-Hill, New York.

BZW (1994). *BZW investment management briefing,* June 1994. Barclays de Zoete Wedd Investment Management, London.

CHAMBERS, SIR PAUL (1966). Problems in forecasting in the chemical industry. *J.I.A.* **92**, 117-133.

CHARETTE, R. (1989). *Risk management.* McGraw-Hill, New York.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

COMMISSION ON THE THIRD LONDON AIRPORT (1971). *Report by the Chairman, The Hon. Mr Justice Roskill.* H.M.S.O., London.

CONFEDERATION OF BRITISH INDUSTRY (1994). *Realistic returns; how do manufacturers assess new investment?* C.B.I., London.

COOPER, D. & CHAPMAN, C. (1987). *Risk analysis for large projects.* John Wiley, New York.

COOPER, D.F., MACDONALD, D.H. & CHAPMAN, C.B. (1985). Risk analysis of a construction cost estimate. *Project Management*, **3**, 141-149.

COX, M.B. (1991). *Risk management for the department head — an integrated approach.* Cox Publications, Billings, MT.

CULPIN, M.F. (1989). *The management of capital projects.* BSP Professional, Newark, NJ.

DARNELL, H. & DALE, M.W. (1991). *Total project management, an integrated approach to the management of capital investment projects in industry.* British Institute of Management, Corby.

DARVISH, T. & ECKSTEIN, S. (1988). A model for simultaneous sensitivity analysis of projects. *Applied Economics*, **20**, 113-123.

DAYKIN, C.D., PENTIKÄINEN, T. & PESONEN, M. (1994). *Practical risk theory for actuaries.* Chapman & Hall, London.

DEPARTMENT OF THE ENVIRONMENT (1993a). *Working together: private finance and public money.* H.M.S.O., London.

DEPARTMENT OF THE ENVIRONMENT (1993b). *Partnership opportunities, A summary of new schemes.* H.M.S.O., London.

DEPARTMENT OF THE ENVIRONMENT (1993c). *Guidelines for the use of rental guarantees by urban development corporations.* H.M.S.O., London.

DIMSON, E. (1993). Appraisal techniques. *Proceedings of Capital Projects Conference, Investing for Tomorrow, 25 May 1993.* Institute of Actuaries.

DIMSON, E. & MARSH, P. (1994). Unhappy returns. *Financial Times*, 9 February 1994.

FAMA, E.F. (1977). Risk-adjusted discount rates and capital budgeting under uncertainty. *Journal of Financial Economics,* **5**, 3-24.

FISHER, C. (EDITOR) (1991). *Risk avoidance in construction contracts.* HLK Global Communications Inc.

FLANAGAN, R. & NORMAN, G. (1993). *Risk management and construction.* Blackwell Scientific Publications, Oxford.

FOSTER, C.D. & BEESLEY, M.E. (1963). Estimating the social benefit of constructing an underground railway in London. *Journal of the Royal Statistical Society, Series A (General)*, **126**, 46-93.

FRENCH, S. (1988). *Decision theory: an introduction to the mathematics of rationality.* Ellis Horwood, Chichester.

GADELLA, J.W. (1992). Post project appraisal — a chance to invest in better capital projects. *Management Accounting*, **70**, 52-53.

GALE, G. (1991). Managing capital projects successfully. *Management Accounting*, **69**, 53-54.

GEOGHEGAN, T.J., CLARKSON, R.S., FELDMAN, K.S., GREEN, S.J., KITTS, A., LAVECKY, J.P., ROSS, F.J.M., SMITH, W.J. & TOUTOUNCHI, A. (1992). Report on the Wilkie stochastic investment model. *J.I.A.* **119**, 173-228.

GRUBB, M. (1993). A second generation of low inflation. *Professional Investor. Dec. 1993/Jan. 1994*, 53-57.

GULLIVER, F.R. (1988). Post project appraisals pay. *Harvard Business Review*, **65**, 128-131.

HACKNEY, J.W. (1991). *Control and management of capital projects.* McGraw-Hill, New York.

HAYES, R., PERRY, J., THOMPSON, P. & WILLMER, G. (1987). *Risk management in engineering construction: an SERC project report.* Thomas Telford, London.

HEAD, G.L. (1984). *Risk management process.* Risk Management Society Publishing Inc.

HERTZ, D.B. & THOMAS, H. (1983). *Risk analysis and its applications.* John Wiley, London.

HIRST, I.R.C. (1988). *Business investment decisions.* Philip Allen, Hemel Hempstead.

H.M. TREASURY (1991). *Economic appraisal in Central Government: a technical guide for Government Departments* (*'The Green Book'*). H.M.S.O., London. [Appendix G contains a note on discount rates and required rates of return.]

H.M. TREASURY (1993a). *The private finance initiative — breaking new ground towards a new partnership between the public and private sectors.* H.M.S.O., London.

H.M. TREASURY (1993b). *Managing risk and contingency for works projects.* Central Unit on Procurement Guidance Paper No. 4. H.M.S.O., London.

H.M. TREASURY (1993c). *Project evaluation.* Central Unit on Procurement Guidance Paper No. 43. H.M.S.O., London.

H.M. TREASURY (1994). *Competition and the private finance initiative — guidance for departments.* H.M.S.O., London.

HUTCHINSON, R. (1991). *Economic appraisal and the prioritisation of capital projects.* Department of Finance and Personnel for Northern Ireland, Policy Planning and Research Unit.

INSTITUTE OF CHARTERED ACCOUNTANTS (1986). *Investment appraisal.* ICAEW, London.

JOHNSON, M.E. (1987). *Multivariate statistical simulation.* John Wiley, New York.

KNIGHT, F. (1921). *Risk, uncertainty and profit.* Houghton Mifflin Company, Boston.

LAWTON, T. (1986). How Britoil evaluates its project work. *Accountancy,* **102**, 54-87.

LENZ, M. (JNR.) & RALSTON, H. (1988). *Risk management manual.* Meritt Co., Santa Monica, CA.

LEVY, H. (1963). Stochastic dominance among log-normal prospects. *International Economic Review,* **14**, 601-614.

LEVY, H. (1977). Multiperiod stochastic dominance with one-period parameters, liquidity preference, and equilibrium in the log-normal case. In *Natural Resources, Uncertainty and General Equilibrium Systems,* edited by A.S. Blinder and P. Friedman. Academic Press, New York.

LEVY, H. & SARNAT, M. (1990). *Capital investment & financial decisions,* Fourth Edition. Prentice Hall, New York.

LEWIN, C.G. (1967). The appraisal of capital projects. *J.I.A.S.S.* **18**, 150-177.

LEWIN, C.G. (1981). The opportunities for capital investment. *J.I.A.* **108**, 19-74.

LEWIN, C.G. (1993). Capital projects — investing for tomorrow. Report of Conference 25 May 1993. *J.I.A.* **120**, 475-479. [A more detailed report is held in the Institute's Library.]

LUMBY, S. (1991). *Investment appraisal and financing decisions,* Fourth Edition. Chapman & Hall, London.

MCCARTHY, J. (1994). An overview of the private finance initiative. *Papers of the AK Conference, 18 April 1994.*

MAJOR PROJECTS ASSOCIATION (1994). *Beyond 2000: a source book for major projects.* Templeton College, Oxford.

MARCET, J. (1816). *Conversations on political economy.*

MARKOWITZ, H.M. (1952). Portfolio selection. *The Journal of Finance,* **7**, 77-91.

MARKOWITZ, H.M. (1959). *Portfolio selection: efficient diversification of investments.* Yale University Press, New Haven.

MARKOWITZ, H.M. (1987). *Mean-variance analysis in portfolio choice and capital markets.* Basil Blackwell, Oxford.

MATURITY GUARANTEES WORKING PARTY (1980). Report of the Maturity Guarantees Working Party. *J.I.A.* **107**, 101-212.

MEHTA, S.J.B. (1992). Allowing for asset, liability and business risk in the valuation of a life office. *J.I.A.* **119**, 385-455.

MERRETT, A.J. & SYKES, A. (1962). *The finance and analysis of capital projects.* Longmans, London.

MILLS, R.W. (1990). *Post-completion audit of capital expenditure projects.* Chartered Institute of Management Accountants, London.

MOORE, P.G. (1972). Mathematical models in portfolio selection. *J.I.A.* **98**, 103-148.

MOORE, P.G. (1983). *The business of risk.* Cambridge University Press.

MOORE, P.G. & HODGES, S.D. (1968). The consideration of risk in project selection. J.I.A. **94**, 355-378.

MOORE, P.G. & THOMAS, H. (1976). *The anatomy of decisions.* Penguin.

MORRIS, P.W.G. & HOUGH, G.H. (1987). *The anatomy of major projects — a study of the reality of project management.* Major Projects Association & John Wiley, Chichester.

MUHLEMAN, A., OAKLAND, J.S. & LOCKYER, K.G. (1993). *Production and operations management,* 6th Edition. Pitman, London.

NEALE, W. (1991). A revolution in post completion audit adoption. *Management Accounting,* **69**, 44-46.

NEALE, C.W. & HOLMES, D.E.A. (1990). Post auditing capital projects. *Long Range Planning*, **23**, 88-96.

PDFM LIMITED (1994). *Pension fund indicators.* PDFM Limited, London.

PATTERSON, C.S. (1986). Some notes on the cost of new equity : a comment. *Journal of Business Finance and Accounting*, **13**, 149-152.

PIKE, R. (1982). *Capital budgeting in the 1980's. A major survey of the investment practices in large companies.* Institute of Cost and Management Accountants.

RUBINSTEIN, R.Y. (1981). *Simulation and the Monte Carlo method.* John Wiley, New York.

SAMUELS, J.M., WILKES, F.M. & BRAYSHAW, R.E. (1990). *Management of company finance*, Fifth Edition. Chapman & Hall, New York.

SHARPE, W.F. (1970). *Portfolio theory and capital markets.* McGraw-Hill, New York.

SHERRIS, M. (1987). On the risk adjusted discount rate for determining life office appraisal. *J.I.A.* **114**, 581-589.

SMYTH, D. (1990). How to structure an effective post completion audit. *Accountancy*, **106**, 90-93.

SMYTH, D. (1990). Keeping control with post completion audits. *Accountancy*, **106**, 163-164.

SPACKMAN, M. (1991). *Discount rates and rates of return in the public sector: economic issues.* H.M. Treasury, Government Economic Service Working Paper No. 113 (Treasury Working Paper No. 58). H.M.S.O., London.

STRAW, B.M. & ROSS, J. (1988). Knowing when to pull the plug. *Harvard Business Review*, **65**, 68-74.

TV CHOICE LTD. (1990). *Financial management and planning.* (Training video). TV Choice Ltd., London.

TAYLOR, A. (1994). Pounds, pioneers and more pounds. *Financial Times*, 3 May 1994.

THOMPSON, P.A. & PERRY, J.G. (1983). *Engineering construction risks — a guide to project risk analysis and risk management.* Thomas Telford, London.

THORNTON, P.N. & WILSON, A.F. (1992). A realistic approach to pension funding. *J.I.A.* **119**, 229-312.

WESTON, J.F. & COPELAND, T.E. (1988). *Management finance*, Second U.K. Edition, adapted by A.F. Fox & R.J. Limmack. Cassell.

WILKES, F.M. (1980). On multiple rates of return. *Journal of Business Finance and Accounting*, **7**, 569-583.

WILKIE, A.D. (1985). Portfolio selection in the presence of fixed liabilities; a comment on 'The matching of assets to liabilities'. *J.I.A.* **112**, 229-277.

WILKIE, A.D. (1986a). A stochastic investment model for actuarial use. *T.F.A.* **39**, 341-403.

WILKIE, A.D. (1986b). Some applications of stochastic investment models. *J.I.A.S.S.* **29**, 25-51.

WILKIE, A.D. (1987). Stochastic invesment models — theory and applications. *Insurance: Mathematics and Economics*, **6**, 65-83.

WILKIE, A.D. (1992). Stochastic investment models for XXIst Century actuaries. *Transactions of the 24th International Congress of Actuaries*, Montreal, **5**, 119-137.

WILLEY, K. & MARSH, P. (1992). Does top management add value to investment decisions? *Long Range Planning Journal*, **25**, 43-58.

WILLIAMS, A.J. (1982). Techniques of project management. *J.I.A.* **109**, 39-82.

WILLIAMS, C.A. & HEINZ, R.M. (1989). *Risk management and insurance.* Witherby & Co. Ltd., London.

WISE, A.J. (1984a). A theoretical analysis of the matching of assets to liabilities. *J.I.A.* **111**, 375-402.

WISE, A.J. (1984b). The matching of assets to liabilities. *J.I.A.* **111**, 445-485.

WISE, A.J. (1987). Matching and portfolio selection, Parts 1 and 2. *J.I.A.* **114**, 113-133 and 551-568.

APPENDIX A

ESTIMATED ASIAN INFRASTRUCTURE EXPENDITURE REQUIREMENT, 1991-2000
(U.S. $ Billion)

	ASEAN	China	Hong Kong	Korea	Taiwan	Totals
Transportation	74.0	968.4	23.2	132.3	124.3	1,322.2
Power	64.1	54.0	12.8	46.2	28.5	205.6
Telecom	21.0	25.2	1.2	32.3	9.6	89.3
Environment	35.5	N/A	3.5	3.6	19.2	61.8
Public Housing	14.3	N/A	15.5	86.9	39.2	155.9
Other	13.5	N/A	10.6	55.2	25.7	105.0
Total	222.4	1,047.6	66.8	356.5	246.5	1,939.8

Notes: ASEAN includes Brunei, Indonesia, Malaysia, Philippines, Singapore, Thailand.

The table does not include Central Asia (India, Pakistan, Sri Lanka, Bangladesh).
See also ¶1.12.

Source: Asian Development Bank, World Bank, Government Five Year Plans (information supplied in conjunction with the Asian Infrastructure Fund, co-sponsored by Peregrine Investments Holdings Limited and the Frank Russell Company, 1994).

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

APPENDIX B

FILMCO AND BREADCO

B.1 *Introduction*

B.1.1 In this appendix we discuss an example of what would happen if managers adopted the suggestion that they should use the same discount rate for all projects, as referred to in ¶4.2.7.2.

B.1.2 Consider two companies, FilmCo and BreadCo. FilmCo is a company which makes films. The market regards this activity as high risk and, accordingly, expects a rate of return from FilmCo (and from its competitors) of 15% p.a.. BreadCo, on the other hand, makes loaves of bread and is regarded altogether as less risky. The market expects a return from BreadCo (and from its competitors) of just 5% p.a..

B.1.3 Suppose now that the shareholders of FilmCo and BreadCo are asked to consider a merger of the two companies into a films-to-bread conglomerate known as MergedCo, and they agree to it. We completely ignore questions here as to why the shareholders of the companies should agree to such a merger.

B.2 *What should be the Discount Rate for MergedCo?*

B.2.1 Under the Project Discount Rate Hypothesis, MergedCo will continue to apply target discount rates of 5% p.a. to new bread projects and 15% p.a. to new film projects, but if the Single Discount Rate Hypothesis were adopted, MergedCo would apply a single company-wide discount rate to all projects. This rate would be somewhere between 5% p.a. and 15% p.a.. Let us suppose, for simplicity, that the two businesses were of identical size and that, therefore, the cost of capital for MergedCo is the straight average of the two predecessor companies' cost of capital, i.e. 10% p.a.

B.2.2 It will immediately be apparent that the result of adopting 10% p.a. as the target rate following the merger is for MergedCo to accept all of the new film projects that FilmCo would have accepted and some more projects that FilmCo would have rejected, but it will now reject some bread projects that BreadCo would have accepted (i.e. all those bread projects with an expected return between 5% p.a. and 10% p.a.). At the very least, the balance of the business in MergedCo will tilt towards films and away from bread. So MergedCo is now more heavily into films than bread, whereas BreadCo and FilmCo had been of equal size.

B.2.3 Furthermore, it is possible that, on these figures, *all* new bread projects will be rejected, because none of them exceeds 10% p.a.. This follows from a basic rule of economics that, if there are plenty of new projects offering a return in excess of, say, 10% p.a. in an industry which requires only, say, 5% p.a., new companies will enter the market until competition forces prices back down to the expected level for the industry, i.e. 5% p.a. It would take time for competitors to exploit the excess returns and compete them away, but we can safely assume that the bread industry has been around long enough for that to have happened already.

B.3 *Subsequent Developments*

B.3.1 As soon as the market learns of MergedCo's new-found proclivity for films, it will adjust its risk assessment of the company. MergedCo is no longer split 50:50 between films (rated at a 15% p.a. expected return) and bread (5% p.a.). There is a greater film content. The share price of MergedCo will, therefore, fall, so that the cost of capital (i.e. the expected rate of return) rises. Let us suppose, for the sake of simplicity, that the new split between films and bread is 60:40 (Note: this is the split across all projects — new and old — not just the new ones.) So the price of MergedCo shares will fall such that the cost of capital increases from 10% p.a. to 11% p.a..

B.3.2 Once management realises that the fall in price is a systematic response to increased risk, not an arbitrary blip, it will have to adjust its discount rate to 11% p.a.. This immediately squeezes out even more new bread projects (if there were any at the 10% level). It will also squeeze out some film projects (those with an expected return between 10% p.a. and 11% p.a.), but there will clearly be a greater volume of new film projects that can generate returns of 11% p.a. than there are bread projects.

B.3.3 So the Single Discount Rate Hypothesis is not only squeezing out bread projects, it is pushing up the discount rate to be used for all new projects. Over time, as the old projects terminate, they will be replaced by a disproportionate volume of film projects. This pushes the discount rate even higher. If the merger has not already choked off all bread projects by introducing a 10% p.a. discount rate, the increasing discount rate eventually will. Once that happens the company will be taking on only film projects and will eventually become just a film company.

B.3.4 In fact one does not have to wait that long. The process of conversion into a film company will be hastened by the process described in the next paragraphs.

B.4 *What about the Existing Projects?*

B.4.1 We now turn our attention to the company's pre-merger projects. A bread project which generates £5 p.a. (net of tax) was valued by the company at £100 before the merger, but was implicitly de-valued to £50 immediately after the merger when the discount rate doubled (and to £45.45 when the rate rose to 11% p.a.). By the same token, a film project which generates £20 p.a. would appear to increase in value to the company from its pre-merger level of £133.33 to £200 immediately post-merger (and then fall back slightly to £181.82 when the discount rate increased to 11% p.a.).

B.4.2 Faced with these figures, the head of the film division of MergedCo puts the following business plan up to the Board immediately following the merger. He proposes that one of the bread projects generating £5 p.a. should be

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

sold off to a company which makes only bread. The buyer will value the project at £100 compared to MergedCo's valuation at only £50. With the £100 from the sale, the money can be invested in a film which generates £12.50 p.a.. This was a project which was rejected just prior to the merger because it failed the 15% discount rate test (i.e. NPV < 0), but now comfortably passes the 10% test. Finally, the head of the film division points out, the project will have a value of £125 in MergedCo, which is £75 more than the value to MergedCo of the bread project it sold and £25 more than MergedCo got for it from the sale!

B.4.3 This analysis is, of course, nonsense. It argues that a perfectly viable low-risk bread project should be sold and replaced with a film that a specialist film company would not touch; and the analysis appears to suggest that the value of the new risky project (£125) is two-and-a-half times the value of the old safe project (£50).

B.4.4 Nevertheless, any Board (or any actuary) which has been seduced by the Single Discount Rate Hypothesis would accept this proposal. The enterprising head of the film division now turns his attention to all the other bread projects, making out the case for their sale also. Eventually, all of the bread projects will be sold off.

B.5 *The Result*

B.5.1 As a result of either, or both, of the two processes described in Sections B.3 and B.4, MergedCo will (sooner or later) become a film-only company.

B.5.2 More generally, any multi-product company which adopts the Single Discount Rate Hypothesis will tend towards those of its products with the highest risk until that product (or product(s), if it has several of equally high risk) is the sole product in the company.

B.6 *Is there an Alternative?*

There is, of course, an alternative to the above scenario. It is to adopt different discount rates for projects which have different risk levels, i.e. 15% p.a. for MergedCo's films and 5% p.a. for its bread projects. The merged company will take on the same projects as the separate companies would, and will not be falsely persuaded to sell off good projects in order to finance bad ones.

APPENDIX C

THE PORTFOLIO SELECTION MODEL

C.1 *Introduction*

C.1.1 In this appendix we review the portfolio selection model, originally put forward by Markowitz (1952, 1959), and most recently presented in Markowitz (1987) (see also Sections 4.4-4.7).

C.1.2 Markowitz started by considering the returns on securities, i.e. any sort of investments, whether bonds or shares or a capital project, over some given time period, as random variables with known (and finite) means and variances. He assumed that investors liked a high expected return, but disliked uncertainty about the amount of that return, as expressed by the variance. Investors therefore liked a high expected return and a low variance.

C.1.3 This mean-variance framework is a simplification. It can be shown, however, to be valid, sometimes with small modifications, for a wide range of cases where the distribution of returns can be described by only two parameters. The normal and lognormal distributions fall into this category. Some other distributions are not so amenable to this framework. Examples include the payoffs from option contracts, which may concentrate one or other tail of a distribution into a mass at some point; or distributions of returns on capital projects which have discrete probabilities of success or failure. These may be more analogous to insurance contracts, whether life or non-life, which are better dealt with as single cases in a large portfolio, applying the principles of probabilistic risk discussed in ¶4.2.2.

C.1.4 We consider first the investor who is limited to only two securities, in order to set the scene, and then go on to multiple securities. At this stage we assume that the investor can buy any amount he chooses of either security, and we assume that he has no specific liabilities.

C.2 *Two Securities*

C.2.1 Let us consider first an investor with one unit of cash (say £10,000,000) to invest. He can choose in general from any of N securities available to him: S_i, $i = 1, 2, ..., N$, though in the first place he will be limited to $i = 1, 2$. The return on security S_i over one period is a random variable R_i with expected value (or mean) $\mathrm{E}[R_i] = E_i$ and variance $\mathrm{Var}[R_i] = V_i = \sigma_i^2$. The covariance of R_i and R_j, $\mathrm{Covar}[R_i, R_j] = C_{ij} = \rho_{ij}.\sigma_i.\sigma_j$ where the correlation coefficient between R_i and R_j is ρ_{ij}.

C.2.2 Returns can be measured either as the total return per unit invested, e.g. 1 invested becomes R_i, or as percentage returns, e.g. 1 invested becomes $1 + 100R_i$, with corresponding changes in the means, variances and covariances. Either measure can be used and the algebra is the same.

C.2.3 The investor first considers only the two securities, S_1 and S_2. He creates his portfolio by putting x_1 in S_1 and x_2 in S_2. We have $(x_1 + x_2) = 1$, whence $x_2 = (1 - x_1)$. The return on his portfolio is:

$$R_p = x_1.R_1 + x_2.R_2.$$

Then:

$$E = \mathrm{E}[R_p] = x_1.E_1 + x_2.E_2$$

and

$$V = \mathrm{Var}[R_p] = x_1^2.V_1 + 2x_1.x_2.C_{12} + x_2^2.V_2.$$

Now:

$$E = E_2 + x_1.(E_1 - E_2)$$

so

$$x_1 = (E - E_2)/(E_1 - E_2)$$

and by substitution we get:

$$V = A.E^2 + B.E + C$$

where:

$$A = \frac{(V_1 - 2C_{12} + V_2)}{(E_1 - E_2)^2}$$

$$B = \frac{-2\,\{E_1(V_2 - C_{12}) + E_2(V_1 - C_{12})\}}{(E_1 - E_2)^2}$$

$$C = \frac{(E_1^2.V_2 - 2E_1E_2C_{12} + E_2^2.V_1)}{(E_1 - E_2)^2}.$$

We can find the minimum value of V when:

$$\frac{\mathrm{d}V}{\mathrm{d}E} = 0.$$

Now: $\dfrac{\mathrm{d}V}{\mathrm{d}E} = 2A.E + B$, which $= 0$ when $E = -B/2A$

or

$$E = \frac{E_1(V_2 - C_{12}) + E_2(V_1 - C_{12})}{V_1 - 2C_{12} + V_2}$$

at which point: $x_1 = \dfrac{V_2 - C_{12}}{V_1 - 2C_{12} + V_2}$ and $x_2 = \dfrac{V_1 - C_{12}}{V_1 - 2C_{12} + V_2}$.

C.2.4 Now let us choose S_1 and S_2 so that:

$$E_1 = 4\% \qquad V_1 = 4\%\% \qquad \sigma_1 = 2\%$$

$$E_2 = 8\% \qquad V_2 = 36\%\% \qquad \sigma_2 = 6\%$$

and choose $\rho_{12} = +1.0, +0.75, +0.25, -0.25, -1.0$ in turn. Note that we define returns in the percentage form.

C.2.5 Then we get what is shown in Figure C.1, where the horizontal axis represents expected values, E, and the vertical axis represents variances, V. The securities S_1 and S_2 are represented by the points shown, and the possible portfolios are represented by lines, which in this diagram are parabolas, one for each value of ρ_{12}.

C.2.6 In many books the E and V axes are reversed, but we follow Markowitz in putting E on the horizontal axis and V on the vertical.

C.3 *Efficient Portfolios*

C.3.1 'Efficient portfolios' in the mean-variance framework are those to the right of the minimum value of V in each case. A portfolio is inefficient if the investor can find another portfolio with the same E and lower V, or the same V and higher E and a portfolio is efficient if he cannot find a better one. How the investor chooses between different efficient portfolios is another problem. At this

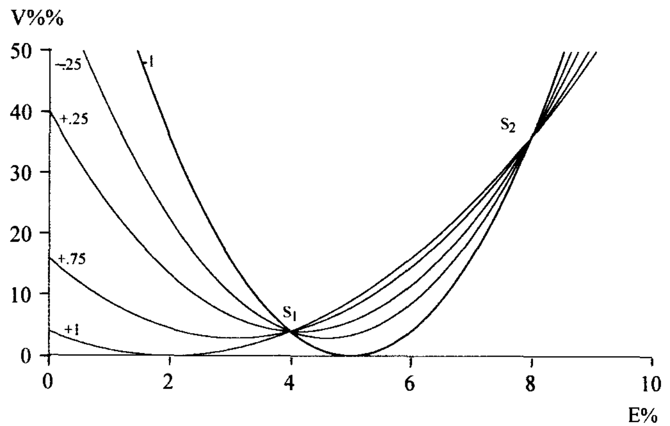


Figure C.1. Portfolios formed from two securities in the E-V plane

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

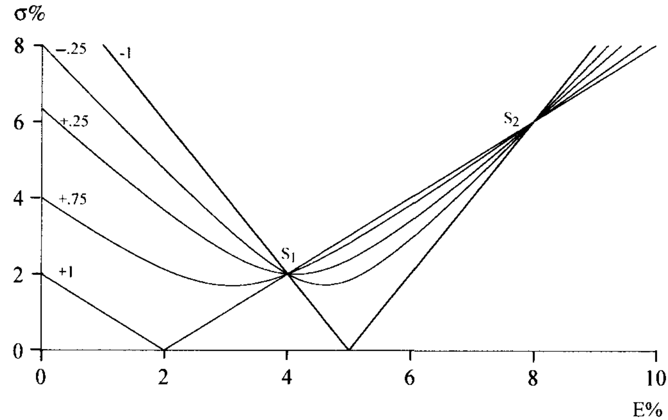


Figure C.2. Portfolios formed from two securities in the E-σ plane

stage we are just trying to identify efficient portfolios. See also Section C.7 for the situation when returns are lognormally distributed.

C.3.2 When $\rho_{12} = +1$ or -1 the parabola touches the E-axis, so that there is some portfolio with $V = 0$. Such a portfolio can be described as risk-free. When $\rho_{12} = -1$ this occurs between points S_1 and S_2, representing a portfolio with positive values of x_1 and x_2, whereas when $\rho_{12} = 1$ it occurs with a negative holding of S_2 and a large holding of S_1.

C.3.3 If we plot these lines as in Figure C.2 with the vertical axis showing the standard deviation, σ, then the parabolas change in general to hyperbolas, except for those that touch the E-axis, which change to pairs of straight lines. Strictly the whole diagram can be reflected in the E-axis, but it is conventional to describe standard deviations as positive, rather than negative.

C.4 N *Securities*

C.4.1 Now we allow the investor to invest in any of the N securities, S_i, $i = 1, \ldots, N$. He invests x_i in security S_i. The return on his portfolio is:

$$R_P = \Sigma_i \, x_i.R_i.$$

C.4.2 The expected return on his portfolio is:

$$E = \mathrm{E}[R_P] = \Sigma_i \, x_i.E_i$$

and the variance is:

$$V = \mathrm{Var}[R_P] = \Sigma_i \, \Sigma_j \, x_i.x_j.C_{ij} \text{ where we write } V_i = C_{ii}.$$

C.4.3 We wish to choose x_i to minimise V subject to the constraints:

$$\Sigma_i \, x_i = 1$$

and

$$E = E_P, \text{ say.}$$

C.4.4 We can do this by creating the 'Lagrangian' function W, where:

$$W = V - 2\lambda(E - E_P) - 2\mu(\Sigma_i \, x_i - 1).$$

C.4.5 Then we set the partial derivatives of W with respect to all the x_i and also with respect to λ and μ equal to 0, and solve the resulting simultaneous equations by matrix inversion.

C.4.6 If we wish to put in constraints so that all the x_i are non-negative, i.e. $x_i \geq 0$ for all i then we have a 'quadratic programming' problem, which is more difficult to solve, but methods are known, and computer programs are available.

C.4.7 We may wish to put in other constraints such as:

$$0 \leq L_i \leq x_i \leq U_i \leq 1.$$

C.4.8 The full solution to this problem, allowing for all special cases, is discussed in Markowitz's most recent book (1987). Sharpe (1970) also gives a very clear explanation.

C.5 *Example with Three Securities*

C.5.1 For another simple example we choose S_1, S_2 and S_3 so that:

$$E_1 = 4\% \quad V_1 = 4\%\% \quad \sigma_1 = 2\%$$

$$E_2 = 6\% \quad V_2 = 18\%\% \quad \sigma_2 = 4.2426\%$$

$$E_3 = 8\% \quad V_3 = 36\%\% \quad \sigma_3 = 6\%$$

and

$$\rho_{12} = \rho_{13} = \rho_{23} = 0.25.$$

C.5.2 We shall restrict all the $x_i \geq 0$. We consider first those portfolios with only two securities, which are shown as the lines joining the points in pairs in Figure C.3. Then we consider minimum variance portfolios with three securities and no constraints on the x_i. These are also shown in Figure C.3 by the line tangential to and below the other lines.

C.5.3 Then we consider minimum variance portfolios with three securities and all $x_i \geq 0$. This is only one part of the former curve, that lying between the two tangential points, A and B in Figure C.3.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

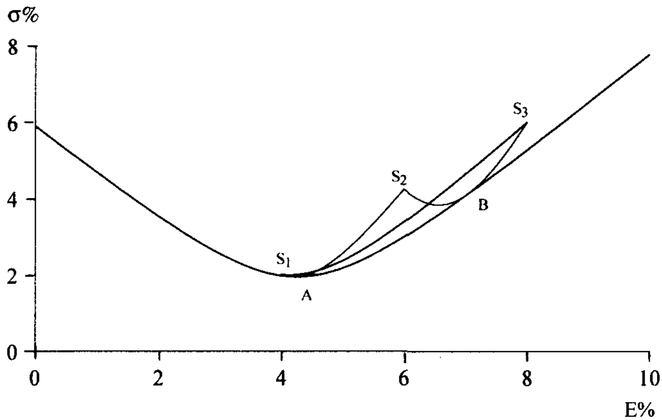


Figure C.3. Portfolios formed from pairs of three securities and from all three

C.6 *Corner Portfolios*

C.6.1 What are called 'corner portfolios' occur where one $x_i = 0$ so that a particular S_i comes in or drops out of consideration as we move along what is called the 'efficient frontier'.

C.6.2 In this case corner portfolios occur at:

$$E = 4.1822\% \quad \text{where } x_3 = 0$$

and

$$E = 7.0231\% \quad \text{where } x_1 = 0.$$

C.6.3 All the x_i are linear in E between corners, as can be seen from Figure C.4, in which the values of the x_i for the optimum portfolio (i.e. minimum variance portfolio) for each value of E are shown. One can see how the lowest feasible mean return is obtained with a portfolio invested 100% in S_1, giving a return of 4% in this case, and the highest value of E is 8% obtained with a portfolio invested 100% in S_3. All intermediate portfolios have some mixture of securities.

C.7 *Normal and Lognormal Distributions*

C.7.1 The distribution of returns is of some importance. Normal and lognormal distributions appear very close over short periods, but diverge over longer periods. Figure C.5 shows this. First we have a normal and a lognormal distribution with the same means and the same standard deviation of 0.05. The

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

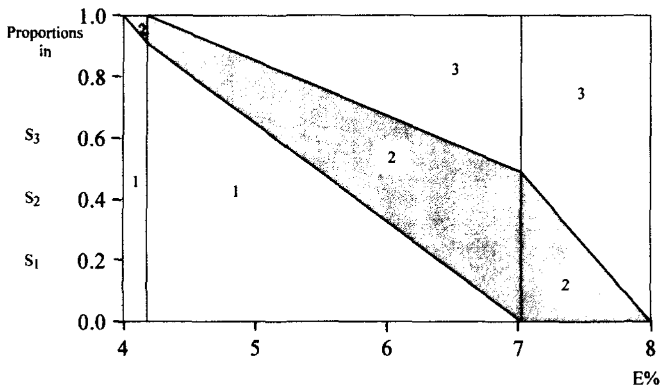


Figure C.4. Proportions of each security in portfolios with different values of *E*

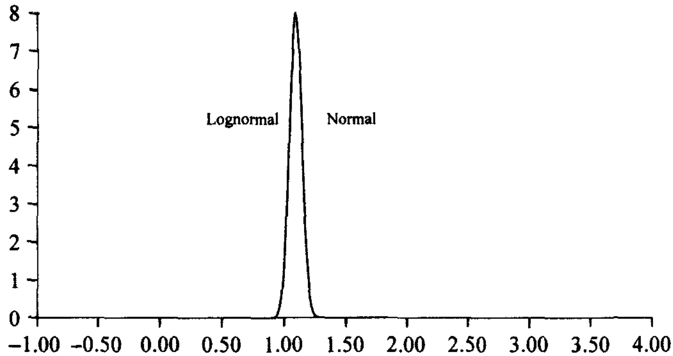


Figure C.5. Normal and lognormal distributions each with mean 1.1 and standard deviation 0.05

graphs are indistinguishable. This is typical of share price movements over one month. In Figure C.6 we show a standard deviation of 0.25, appropriate over a period of a year or so. The distributions begin to diverge. In Figure C.7 we have a standard deviation of 0.5, and the distributions are quite a bit different.

C.7.2 If the distribution of returns is normal and the investor is risk-averse, even to a small extent, then efficient portfolios are as described in the mean-

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

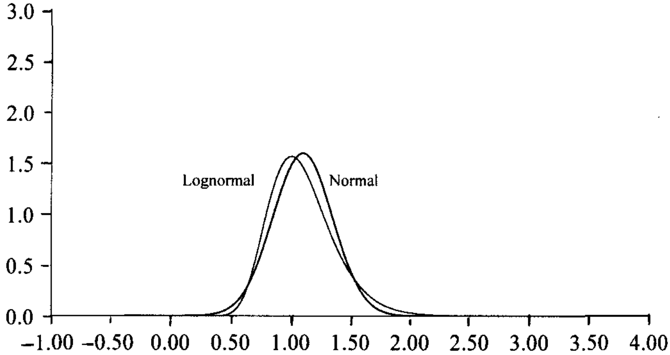


Figure C.6. Normal and lognormal distributions each with mean 1.1 and standard deviation 0.25

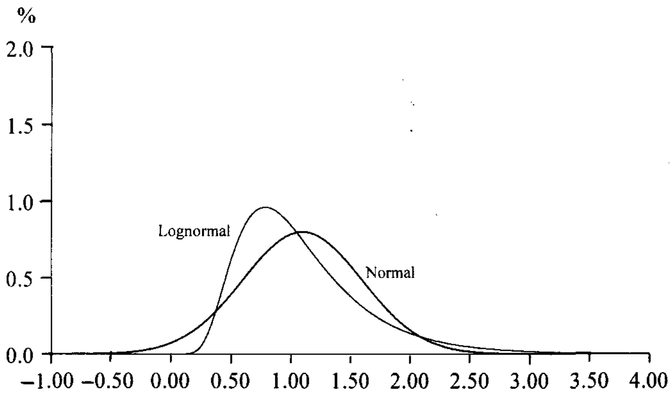


Figure C.7. Normal and lognormal distributions each with mean 1.1 and standard deviation 0.50

variance framework. The investor may choose whereabouts on the efficient frontier he wishes to be, according to his degree of risk-aversion.

C.7.3 However, if the distribution of returns on individual securities is lognormal, and the portfolio is purchased and held for a fixed period, then the distribution of returns on the portfolio is neither normal nor lognormal, and tidy results are not known; but if the distributions of the returns on individual

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

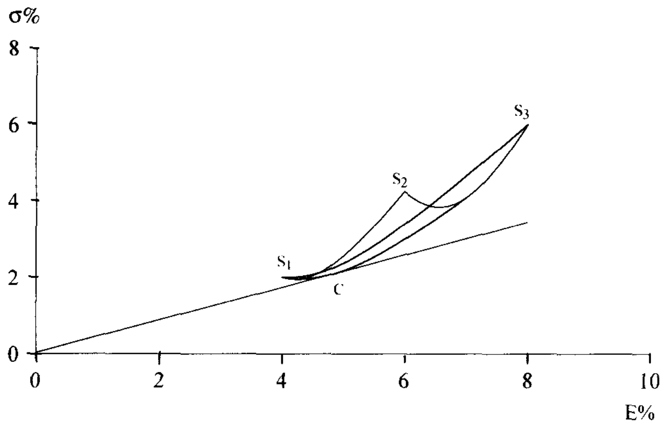


Figure C.8. Efficient portfolios for lognormal distributions are to the right of point C

securities are lognormal, and the portfolio is continuously rebalanced in accordance with the initially chosen proportions, then it has been shown (Levy, 1977) that the distribution of the final return is also lognormal. For a lognormal portfolio it has also been shown (Levy, 1963) that efficient portfolios lie to the right of the point where the tangent from the origin meets the curve of minimum σ, as shown in Figure C.8. There is not very much difference in this case between this point and the point with overall minimum σ.

C.8 *Estimation of Parameters and Stochastic Models*

C.8.1 In order to carry out such a portfolio selection exercise we need to decide on the values of all the E_i, V_i and C_{ij} required. We can do this by making our own subjective estimates of these values, or by investigating past data, and then we need to assume some sort of stochastic model that describes how we believe the distributions of returns on the various securities we are considering are determined.

C.8.2 The random walk model is popular with American financial economists, and this has many advantages in the fairly short term, but for longer terms it is better to use a specific stochastic model that takes account of the realities of the market. The model developed by Wilkie (1986a, 1986b, 1987, 1992) is one such which has been widely used and discussed, but any model of this type that takes account of the correlations over time would be in the same category.

C.9 *Beta-Factors in the Portfolio Selection Model*

C.9.1 One of the widely quoted results of the portfolio selection model is the

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

linear relationship, subject to certain conditions, between the expected returns and the 'beta-factors' of securities. This is often derived as a consequence of the CAPM, but it can also be derived, with suitable assumptions and definitions, directly from the portfolio selection model. This result is of some significance in the assessment of capital projects, and it deserves careful consideration.

C.9.2 We consider further the problem of finding the efficient frontier, with no constraints on the x_i for the time being, except that the fractions invested in the various securities sum to unity. We start by minimising the variance for a given value of E, E_P. We use the same Lagrangian function W:

$$W = V - 2\lambda(E - E_P) - 2\mu(\Sigma_i x_i - 1).$$

C.9.3 In order to find the minimum value of this function we calculate the partial derivatives of W with respect to each of the x_i and also with respect to λ and μ.

$$\frac{\partial W}{\partial x_i} = 2\Sigma_j x_j.C_{ij} - 2\lambda.E_i - 2\mu$$

$$\frac{\partial W}{\partial \lambda} = -2(E - E_P)$$

$$\frac{\partial W}{\partial \mu} = -2(\Sigma_i x_i - 1).$$

C.9.4 We then set all these partial derivatives to zero. The latter two constraints simply say that the expected return on the portfolio is equal to some chosen value, and the sum of the fractions invested sum to unity.

C.9.5 We denote the solutions by a subscript P; thus x_{Pi} is the fraction invested in security S_i, i=1, 2, ... N, at the solution and λ_P and μ_P are the values of λ and μ at the solution.

C.9.6 At the solution each partial derivative is zero. Therefore:

at $x_i = x_{Pi}$ $$\frac{\partial W}{\partial x_i} = 0$$

so:

$$\Sigma_j x_{Pj}.C_{ij} - \lambda_P.E_i - \mu_P = 0.$$

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

C.9.7 Now:

$$\text{Covar}[R_i,R_P] = \Sigma_j\, x_{Pj}.C_{ij}$$

and the usual beta-factor or regression coefficient between R_i and R_P, $\beta[R_i,R_P] = \beta_{i,P}$, is given by:

$$\beta_{i,P} = \text{Covar}[R_i,R_P]/V_P$$

so we get:

$$\beta_{i,P}.V_P = \text{Covar}[R_i,R_P] = \Sigma_j\, x_{Pj}.C_{ij} = \lambda_P.E_i + \mu_P$$

which is a linear relationship between E_i and $\beta_{i,P}$, which we rearrange to give:

$$E_i = \beta_{i,P}.V_P/\lambda_P - \mu_P/\lambda_P. \qquad \text{(C.1)}$$

C.9.8 Consider now any portfolio, Q, formed from some selection of the securities S_i with fractions x_{Qi}.

$$E_Q = \text{E}[R_Q] = \Sigma_i\, x_{Qi}.E_i$$

so:

$$E_Q = \Sigma_i\, x_{Qi}.\{\beta_{i,P}.V_P/\lambda_P - \mu_P/\lambda_P\}$$

$$= \Sigma_i\, x_{Qi}.\beta_{i,P}.V_P/\lambda_P - \mu_P/\lambda_P \qquad (\text{since } \Sigma_i\, x_{Qi} = 1)$$

$$= \beta_{Q,P}.V_P/\lambda_P - \mu_P/\lambda_P \qquad (\text{since } \Sigma_i\, x_{Qi}.\beta_{i,P} = \beta_{Q,P}).$$

C.9.9 So the linear relationship applies to any portfolio formed from the S_i. In particular it applies to portfolio P, so:

$$E_P = V_P/\lambda_P - \mu_P/\lambda_P \qquad (\text{since } \beta_{P,P} = 1).$$

C.9.10 We can consider the intercept with the E-axis: consider a hypothetical security or portfolio R such that $\beta_{R,P} = 0$. Such a security or portfolio does not need to exist, but if it does we call it the risk-free security or risk-free portfolio. Then:

$$E_R = -\mu_P/\lambda_P \qquad (\text{since } \beta_{R,P} = 0).$$

Put:

$$E_R = r$$

then:

$$E_P = V_P/\lambda_P + r$$

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

or

$$V_P/\lambda_P = E_P - r.$$

C.9.11 So we can rewrite the relationship (C.1) as:

$$E_i - r = \beta_{i,P}.(E_P - r).$$

C.9.12 It follows also that:

$$E_Q - r = \beta_{Q,P}.(E_P - r), \text{ for any portfolio } Q \text{ formed from the } S_i.$$

C.9.13 These are the usual beta-factor relationships, if P is taken as the 'market portfolio' of the CAPM. In E-β space the relationship can be represented by a line, sometimes called the 'Security Market Line' (SML), but which we prefer to call the 'beta-line'. The difference is that the assumption of the CAPM is that there is only one SML, the same for all investors, whereas we are postulating separate beta-lines for each investor, for each section of his efficient frontier between corners, as we now consider.

C.9.14 The argument above was applied to a portfolio with no constraints. Now consider the situation when there are constraints, say:

$$L_i \leq x_i \leq U_i.$$

C.9.15 We consider any portfolio on the efficient frontier which is not a corner portfolio, and consider only those securities that enter freely into the portfolio, not at their upper or lower limits. Then the same relationship applies. But for any security, S_i, at its upper limit then:

$$E_i - r > \beta_{i,P}.(E_P - r)$$

so its expected return lies above the beta-line, and for any security, S_i, at its lower limit (which might be zero):

$$E_i - r < \beta_{i,P}.(E_P - r)$$

so its expected return lies below the beta-line.

C.9.16 The proof of this is more complicated, and can be done by use of what are called the Kuhn-Tucker conditions for the maximising or minimising of a function subject to constraints. One can adduce an intuitive argument: if any security is at its upper limit, we could create a better portfolio with more of it, so it is attractive, so its expected return is above the beta-line; if any security is at its lower limit, we could create a better portfolio with less of it, so it is not attractive, so its expected return is below the beta-line.

C.9.17 We note also that $\sigma_P^2 = V_P = \Sigma_{ij} x_i x_j C_{ij}$, so:

$$2\sigma_P \frac{\partial \sigma_P}{\partial x_i} = 2\Sigma_j x_j C_{ij} = 2\,\mathrm{Covar}[R_i, R_P] = 2\sigma_P^2 \beta_{i,P}.$$

So:

$$\beta_{i,P} = \frac{1}{\sigma_P} \frac{\partial \sigma_P}{\partial x_i}.$$

C.9.18 The beta-factor of S_i relative to portfolio P is therefore equal to the proportionate change in the standard deviation of R_P for a small change in x_i.

C.9.19 The 'systematic risk' for a security can be represented by $\frac{\partial \sigma_P}{\partial x_i}$, the marginal change in the portfolio standard deviation for a small change in the proportion invested in security S_i. It is equal to $\sigma_P \beta_{i,P}$. The independent risk of the security can be thought of as the standard deviation of the security itself or $\sigma_i = \sqrt{V_i}$. Since $\sigma_i \geq \sigma_P \beta_{i,P}$ the systematic risk cannot be greater than the independent risk.

C.10 *The Capital Asset Pricing Model (CAPM)*

C.10.1 In the CAPM it is assumed that all investors agree about the values of all E_i, all V_i and all C_{ij}, and that all securities enter into some portfolios. If there is a risk-free security then the efficient frontier collapses to a straight line, and any efficient portfolio can be created by a mixture of the risk-free security and the 'market portfolio', which is the portfolio on the efficient frontier containing a zero fraction of the risk-free security. All risky securities must be held by someone, so this market portfolio contains all risky securities in proportion to their market value. There is now only one beta-line, the same for all investors, called the Security Market Line, and the market values of securities must adjust so that the expected returns E_i lie on the required beta-line.

C.10.2 In our view the CAPM makes too many assumptions to be realistic. It is a single period model, and in reality investment is done over multiple periods. Different investors measure using different numeraires, have different time horizons, different liabilities, and different views about the expected returns, variances and covariances of different possible assets. They may have different restrictions on the assets they may hold. Some may not use mean-variance optimisation.

C.10.3 The CAPM is no more a description of the real world than are certain models of perfect competition, in which, for example, all purchasers are assumed to have perfect knowledge of the prices offered by all entrepreneurs, whereas they actually only know the prices quoted by a few suppliers, and all entrepreneurs are aware of the demand curves for their products, whereas in reality they know only how much they have sold at a particular price.

C.10.4 In reality source investors are individuals. These may be the beneficiaries or potential beneficiaries under pension schemes, or they may have entrusted their savings to life assurance companies or unit trust managers, or they may have invested a large proportion of their assets in a house financed by a mortgage loan. They do not have the opportunity of investing in the theoretical range of all possible assets in any desired quantity. Large investment institutions are able to spread a share portfolio in such a way, but even they cannot subdivide individual real properties in any way they choose.

C.10.5 All investment intermediaries have some sort of liabilities, and these differ from one intermediary to another. The liabilities of a pension fund may be quite different from those of an insurance company, not just in the degree of risk-aversion of the beneficiaries, but in the guarantees offered, the timing of payments, and whether payments are nominal or real. A building society cannot adopt the same investment policy as the managers of an investment trust. The trustees of a closed pension fund which has only pensioners, of a young pension fund which offers final salary defined benefits, or of a defined contribution or money purchase pension scheme will also have quite different investment requirements.

C.10.6 Where international investment is involved, again all investors are not on an equal footing: there are still many restrictions which prevent domestic and foreign investors from taking the same attitudes to shares; some companies and some countries have restrictions on the proportion of foreign shareholders; most countries have, in effect, different taxation for domestic and foreign shareholders; and, above all, investors in different countries measure in different currencies. Some American writers present the CAPM as if the 'risk free asset' for all possible investors in all countries of the world was U.S. Treasury Bills!

C.10.7 Finally, and probably most important, the CAPM is a static equilibrium model, whereas reality is dynamic. Different investors have different attitudes to particular shares or particular projects. Some are optimistic and wish to purchase the shares or undertake the projects. Others are pessimistic and prefer to sell the shares or avoid the projects. Further, participants in the market change their minds about the prospect for companies. It is not the case that a new piece of information leads all investors to agree that the value of a share should change from X to Y. Rather, it leads some investors to consider that the share is now worth more than X and worth buying, and others to consider that it is now worth less than X and worth selling. The net effect of these forces in the market causes the share price to settle down at Y, but only temporarily, since more information will become available and more investors will change their views.

C.10.8 These limitations of the CAPM do not mean that the linear relationship between expected returns and beta-factors falls away, but it needs to be interpreted as above, for a single investor, for his time horizon, and his set of

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

constraints, for one of the particular portfolios on his efficient frontier, and for the beta-factors for the securities that enter freely into that specific portfolio measured against that portfolio. Each investor has multiple beta-lines, one for each sector of his efficient frontier between corner portfolios, and each investor may have a different set of beta-lines from any other investor.

C.10.9 This diversion about the CAPM in no way affects our argument that the portfolio selection methodology is a useful tool for the individual investor, particularly an institutional investor, or for the individual company assessing capital projects. However, we consider that it is important that results that purport to be derived from the CAPM do not distort the application of the portfolio selection model.

C.11 *The Risks Taken on by Different Providers of Capital*

C.11.1 A portfolio of securities or of projects might be financed by different investors, who agree to provide the initial capital in pre-determined amounts and to divide the total return on the portfolio in any pre-arranged way (which might not be proportionate), so that the sum of their interests equals the total return. If the assumptions of the CAPM are met, so that they all agree about the means, variances and covariances, and all use the same numeraires and have the same time horizon, then the weighted beta-factors of their interests relative to the portfolio must sum to the beta-factor of the portfolio, and if the portfolio lies on the Security Market Line, then so must the values of their respective shares.

C.11.2 In these circumstances it can be seen that the way in which a project is financed does not affect the value of the project, or its relative attractiveness. If the project lies above the SML, so that it has a positive NPV, then the surplus may be divided among the providers of capital in any way they choose to negotiate.

C.11.3 However, this neat result is affected by imperfections such as taxation. The present U.K. system of Corporation Tax favours debt capital, so it is sensible for any project to be financed by as much debt capital as the shareholders can afford. An insurer can also be treated as contributing to the capital of the project, in exchange for a rather special payoff, i.e. a premium if the insured event does not arise, and a claim payment if it does. There are, however, many other considerations to take into account when deciding whether or not insurance is either necessary or desirable for a project (see Section 5.5).

APPENDIX D

APPRAISAL AND RISK MANAGEMENT — AN EXAMPLE

D.1 This is a very simplified hypothetical example (referred to in ¶¶5.3.3, 5.3.6 and 5.5.1) designed to demonstrate principles. In practice the analysis would normally be more extensive, even for a straightforward project.

D.2 It has been suggested that a factory should be constructed, at a cost of about £40m which will be spent in a single lump sum at the outset. The factory will then be in operation for 20 years. It is most likely that the income derived from the sale of the factory's products will total about £15m p.a. (at the factory gate) and that the costs of running and maintaining the factory (including the costs of raw materials) will be about £5m p.a. (For simplicity in this example, all future cash flows are expressed in terms of constant price levels with no allowance for future inflation and assume receipt at the end of each year.)

D.3 At the simplest level we can obtain the return from the equation:

$$40 = 10\, a_{\overline{20|}}$$

which leads to an IRR of 24.7% p.a. The NPV is calculated by the formula:

$$NPV = 10a_{\overline{20|}} - 40.$$

The result is £45.1m at 10% p.a. or £74.7m at 6% p.a. The project, therefore, looks worthwhile. Those who use payback periods would not all approve it, though, with a payback period of four years.

D.4 Let us suppose, however, that the company seeks actuarial advice. The consulting actuary familiarises himself with the project and its background, and questions the technical experts on the production and marketing sides of the company. He identifies the possible risks for the project as being:

(a) *Construction risks.* The site may have unexpected difficulties, e.g. archaeological remains, pollution or drainage problems, leading to extra cost. Alternatively, there is the possibility of a small cost saving if all goes according to plan. The distribution of possibilities is assumed to be as follows:

Scenario	Cost range	Midpoint	Probability
	£m	£m	
C_1	39 - 41	40	0.7
C_2	41 - 45	43	0.3

The probability of the capital cost being outside the range £39-£45m is so small that it can be ignored. Using the midpoints of the ranges, the mean value of C can be calculated as £40.9m.

(b) *Running Costs.* It is rather uncertain what the annual running costs will be, because the factory uses new processes and the industrial relations implications in terms of remuneration and manning levels are as yet unclear. However, the distribution is assumed to be:

Scenario	Cost range £m	Midpoint £m	Probability
R_1	4 - 6	5	0.8
R_2	6 - 8	7	0.2

These figures include the costs of labour, raw materials and property maintenance, but exclude interest charges and depreciation. Using the midpoints of the ranges, the mean value of R can be calculated as £5.4m p.a.

(c) *Income.* A considerable amount of market research has been done, and it is possible to predict the likely value of the products (at the factory gate) with a reasonable degree of confidence, as shown in scenarios 1 to 4 below. However, there is also a small possibility that recent developments by another firm will emerge in the form of a competing product in about three years' time; if this happened, the income from the factory's products would decline by about 40%, as shown in scenario 5 below. The annual income distribution is assumed to be thus:

Scenario	Income range £m	Midpoint £m	Probability
I_1	10 - 12	11	0.1
I_2	12 - 14	13	0.1
I_3	14 - 16	15	0.6
I_4	16 - 20	18	0.1
I_5	13 - 17 initially, but decreasing by 40% after three years	15 reducing to 9 after three years	0.1

Using the midpoints of the ranges, the mean values of I can be calculated as £14.7m p.a. for the first three years and £14.1m p.a. thereafter.

D.5 There are 20 (=2×2×5) scenarios, as listed in columns (1) and (2) of Table D.1. The amounts of capital, running costs and income are assumed to be independent of each other, and the probability of each outcome on this assumption is shown in column (3) of Table D.1. The NPV of each outcome is calculated by the formula:

$$\text{NPV} = I(1)\, a_{\overline{3}|} + I(2)\, v^3\, a_{\overline{17}|} - R\, a_{\overline{20}|} - C$$

where $I(1)$ is the income for the first three years and $I(2)$ is the income for the next seventeen years. The IRR is calculated as the interest rate such that this expression for NPV equals zero.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

D.6 The mean NPV is calculated from the expression:

$$E[NPV] = 14.7\, a_{\overline{3}|} + 14.1\, v^3\, a_{\overline{17}|} - 5.4\, a_{\overline{20}|} - 40.9$$

and this is £34.7m at 10% p.a. and £60.5m at 6% p.a. The NPVs, not unnaturally, differ substantially from those quoted in Section D.3, being based on the mean rather than the mode of each outcome. If a 'mean IRR' is calculated by taking IRRs for each scenario and weighting by the probability of occurrence, one gets 21.2% p.a. A more meaningful concept is to calculate a mean IRR such that the mean NPV equals zero; on this basis the result is 21.5% p.a.

D.7 The example could be made more complicated by assuming a distribution for the duration of the project, like an annuity on a life. Note that the expected present value of an annuity for uncertain duration is not, in general, equal to the present value of an annuity certain for the expected duration, as is well known for a life annuity.

D.8 The actuary presents columns (1) to (6) of Table D.1 to the company's senior management, together with the following comments:

(a) The IRRs and NPVs are not very sensitive to the differences assumed in the capital invested.

(b) The two differing assumptions on running costs make a big difference to the IRR and the NPVs.

(c) If the lower income scenarios coincide with the higher running cost scenarios, the IRRs and NPVs do not look attractive. However, the combined probability of such outcomes (6, 10, 16 and 20) is only 4%.

(d) The analysis does not take account of unlikely, but foreseeable risks, such as:

- terrorism, major strikes and other man-made disruption;
- earthquake, fire, flood, etc.;
- extreme economic conditions;
- social or environmental developments which render the factory's products unsaleable;
- competition or technological developments in later years which cannot at present be foreseen;
- legislation which leads to higher running costs; and
- safety hazards at present unforeseen.

D.9 The company's management focuses its attention on outcomes 6, 10, 16 and 20 and, after due consideration and discussion, they ask the actuary what steps can be taken to manage the risks inherent in these outcomes.

D.10 The actuary considers the possibility of reducing the perceived risks by the various methods outlined in Section E.4 and he comes to the conclusion that the most practicable possibility in this case is to explore the question of whether the risks can be insured. We shall assume, for the purpose of illustration (though much will depend in practice on all the circumstances), that insurances can be effected to cover the risks of excessive capital costs and unexpectedly low sales, though both insurances will need to contain suitable 'excesses' so as to give the company incentives to manage the project to its best abilities. Although an insurance against high running costs would be useful, we shall assume that the actuary fails to find one.

D.11 We shall assume, therefore, that the actuary identifies the following two insurances which could be taken out in this case:

(a) *Capital cost insurance.* Insurer (A) is prepared to pay a sum equal to 90% of the excess capital cost of construction over £41m, in return for a single premium of £0.8m.

(b) *Income insurance.* Insurer (B) is prepared to offer a ten-year insurance policy under which he will pay out 80% of such sum as is necessary to build the income up to £12m p.a., subject to a maximum payout of £3m in any one year. The single premium he would charge is £4.4m.

D.12 The effect of taking out both of these insurances is to change the capital cost from £40m to £45.2m in outcomes 1-10 and from £43m to £46.4m in outcomes 11-20. There is additional income from insurance policy (b) in outcomes 1, 5, 6, 10, 11, 15, 16 and 20, ceasing at the end of 10 years from the outset.

D.13 The table of outcomes, if both of these insurances are effected, becomes as set out in columns (7), (8) and (9) of Table D.1. The figures take the insurance premiums as an addition to the capital cost and any claim under the capital cost insurance as a reduction in the capital cost. Any claim under the income insurance is added to the income in the year concerned. (It would be possible to refine the figures to allow for any delay thought likely in settling the insurance claims.) The mean NPV becomes £31.4m at 10% p.a. and £57.5m at 6% p.a. (which may be contrasted with the corresponding pre-insurance figures in ¶D.6).

D.14 Let us now see how these contracts might look from the point of view of the insurers.

D.15 Insurer (A) assesses the probability of the capital cost exceeding £41m as being 35%; the probability it exceeds £43m as 10%; the probability it exceeds £45m as 2%; and the probability it exceeds £47m as negligible. These probabilities differ from those assumed by the company, because the insurer is here assumed to be taking a somewhat cautious view. He therefore calculates the pure premium (using mid-points) as:

$$0.9\ [0.25 \times £1\text{m} + 0.08 \times £3\text{m} + 0.02 \times £5\text{m}] = £0.531\text{m}.$$

Allowing for a margin of about 50% for expenses, uncertainty and profit, he quotes a single premium of £0.8m.

D.16 Insurer (B) assesses the probability of the initial income falling below

£12m as 12% and his assessment of the sum he will have to pay out in this event is 80% of £1.5m p.a. (as opposed to the company's estimate of £1m p.a.) for ten years, since he assumes that the probability of significant recovery of the income level is small if it commences below £12m p.a. He assesses the probability of the competitor marketing a successful new product after three years as being 15%, and he believes that he will, in the event of this occurring, have to pay out 80% of £3m p.a. for seven years, commencing in three years' time. He disregards as being negligible the probability of other scenarios which could lead to a policy claim. He therefore calculates the pure premium (using 5% p.a. interest, which for this purpose is conservative) as:

$$0.8[0.12 \times £1.5\text{m} \times a_{\overline{10|}} + 0.15 \times £3\text{m} \times v^3\, a_{\overline{7|}}] = £2.91\text{m}.$$

Allowing for a 50% margin, he quotes a single premium of £4.4m.

D.17 We may draw the following conclusions applicable to this example:

(a) Without insurance, there is a 4% chance of an unsatisfactory financial outcome if NPVs are calculated at 10% p.a., or a 2% chance if calculated at 6% p.a.

(b) If capital and income insurance can be obtained, on the basis discussed, the chance of an unsatisfactory outcome is still 4% if NPVs are calculated at 10% p.a., but is reduced to nil if calculated at 6% p.a.

(c) Taking out insurance reduces the upside financial potential to some extent, and it may also worsen the financial results from some outcomes which are already unsatisfactory.

(d) Taking out insurance also reduces the expected rates of return and NPVs to the sponsor, because of the insurer's normally more cautious risk assessment and his expenses and profit margins.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table D.1. Possible outcomes

Outcome	Combination	Probability	With no insurance			With separate capital and income insurance		
			IRR % p.a.	NPV at 10% p.a. £m	NPV at 6% p.a. £m	IRR % p.a.	NPV at 10% p.a. £m	NPV at 6% p.a. £m
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	$C_1R_1I_1$	0.056	13.9	11.1	28.8	13.5	10.8	29.5
2	$C_1R_1I_2$	0.056	19.4	28.1	51.8	16.9	22.9	46.6
3	$C_1R_1I_3$	0.336	24.7	45.1	74.7	21.7	39.9	69.5
4	$C_1R_1I_4$	0.056	32.4	70.7	109.1	28.6	65.5	103.9
5	$C_1R_1I_5$	0.056	14.2	9.0	21.9	15.1	12.6	28.0
6	$C_1R_2I_1$	0.014	7.8	−5.9	5.9	7.8	−6.2	6.6
7	$C_1R_2I_2$	0.014	13.9	11.1	28.8	11.9	5.9	23.6
8	$C_1R_2I_3$	0.084	19.4	28.1	51.8	16.9	22.9	46.6
9	$C_1R_2I_4$	0.014	27.3	53.6	86.2	24.0	48.4	81.0
10	$C_1R_2I_5$	0.014	5.5	−8.1	−1.0	7.9	−4.5	5.0
11	$C_2R_1I_1$	0.024	12.7	8.1	25.8	13.0	9.6	28.3
12	$C_2R_1I_2$	0.024	17.9	25.1	48.8	16.4	21.7	45.4
13	$C_2R_1I_3$	0.144	22.9	42.1	71.7	21.1	38.7	68.3
14	$C_2R_1I_4$	0.024	30.1	67.7	106.1	27.8	64.3	102.7
15	$C_2R_1I_5$	0.024	12.6	6.0	18.9	14.5	11.4	26.8
16	$C_2R_2I_1$	0.006	6.8	−8.9	2.9	7.5	−7.4	5.4
17	$C_2R_2I_2$	0.006	12.7	8.1	25.8	11.5	4.7	22.4
18	$C_2R_2I_3$	0.036	17.9	25.1	48.8	16.4	21.7	45.4
19	$C_2R_2I_4$	0.006	25.3	50.6	83.2	23.4	47.2	79.8
20	$C_2R_2I_5$	0.006	4.3	−11.1	−4.0	7.4	−5.7	3.8
	Expected values:		21.2	34.7	60.5	19.2	31.4	57.5

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

APPENDIX E

RISK MATRIX

E.1 *Introduction*

E.1.1 As mentioned in ¶5.4.2, we have developed a risk matrix in order to assist in the identification, quantification and management of a project's risks. The matrix is still at an experimental stage. 'Risk' has been defined as 'an undesirable implication of uncertainty' (Cooper & Chapman, 1987), and it is in that sense that we use the word here.

E.2 *Risk Identification*

E.2.1 The matrix in Table E.1 shows down the left-hand column the stages into which a typical project may be split, with subheadings for some of the principal things which can go wrong at each stage:

— promotion of concept;
— design;
— contract negotiations;
— project approval;
— raising of capital;
— construction;
— operation and maintenance; } (these two activities occur simultaneously,
— revenues; and } but it is convenient to consider them separately)
— decommissioning.

E.2.2 Each of these events may be categorised according to the underlying cause of risk, as set out across the top of the matrix:

Generic Cause	Sub-Categories
A. Political	(a) Government (b) Public opinion (c) Environmental change (d) Legislation (e) Wars, terrorism, riots (f) Public relations
B. Business	(a) Demand failure (b) Competition (c) Premature obsolescence (d) Safety standards

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

C. Economic	(a) Cost inflation and interest rates (b) Currency fluctuations (c) Extreme economic conditions
D. Project	(a) Definition (b) Technical innovation (c) Leadership (d) Technical competence (e) Commitment (f) Planning and control (g) Resourcing (h) Legal framework (i) Progress (j) Labour relations (k) Human error or incompetence
E. Natural	(a) Weather (b) Earthquake (c) Fire or explosion (d) Ground conditions
F. Financial	(a) Inadequate margins (b) Unbalanced sharing of risk

E.2.3 An '×' in a cell of the matrix indicates that the project under review is subject to some degree of risk from the cause in question at the stage indicated. The precise pattern will vary from one project to another.

E.3 *Risk Quantification*

E.3.1 Having identified each risk, it needs to be quantified. Risks may vary from the very likely to the extremely improbable. If the event which is the subject of the risk occurs, the impact on the project may be large or small. Risks may be controllable or non-controllable. They may be dependent on other risks or there may be a degree of independence. Work can proceed separately for each cell in the matrix where an '×' is shown, but it must be borne in mind that a particular underlying cause of risk may affect other cells in the same column, so that there may be a lack of independence when working down a particular column.

E.3.2 Let us now concentrate on a particular cell. The cell is first given a classification according to the following scheme:

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Risk level (L)	—	Very likely (1)
	—	Likely (2)
	—	Even chance (3)
	—	Unlikely (4)
	—	Very unlikely (5)
Dependence (D)	—	Very high degree (1)
	—	Quite a high degree (2)
	—	Fair (3)
	—	Quite a low degree (4)
	—	A very low degree (5)
Controllability (C)	—	Very controllable (1)
	—	Quite controllable (2)
	—	Controllable to some extent (3)
	—	Rather uncontrollable (4)
	—	Very uncontrollable (5)
Impact (I)	—	Project cancelled (1)
	—	Considerably increased cost or delay (2)
	—	Much reduced revenues (3)
	—	Slightly reduced revenues (4)
	—	Slightly increased cost or delay (5)

E.3.3 Thus a cell which is classed as L4, D5, C4, I3 would represent an unlikely, independent, rather uncontrollable risk, which would, if the event in question occurred, give rise to much reduced revenues and therefore have a potentially serious impact on the project. Any cell in categories I1, I2 or I3 should be the subject of careful thought and analysis.

E.4 *Risk Management*

E.4.1 Once a cell has been classified, consideration will be given to how the particular risk should be managed. There are a number of possible responses:

(a) *Absorb*. An appropriate contingency margin would be added to the costings.

(b) *Reduce*. Improve the design, carry out further research or site investigation, strengthen the management, optimise timing, avoid new technology, avoid risky construction techniques, change the implementation plan.

(c) *Pool*. Share the risk with other parties.

(d) *Transfer*. Transfer the risk to another of the parties, e.g. to a contractor, who is better able to control it.

(e) *Insure*. Pay a premium to pass the whole or part of the risk to an insurance company.

(f) *Avoid*. Do not proceed with project in a form which involves this risk.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

At the end of this process, a typical cell might contain the information: L4, D5, C4, I3, Insure.

E.5 *Other Processes*

In order to complete the matrix, a considerable amount of background work may have to be carried out. For example, in order to assess the risk level, it may be necessary to question specialists and translate their views into probability distributions. Deciding on the management of each risk may involve complex processes of negotiation between the interested parties and exploration of the insurance markets. A single cell may sometimes be backed by a comprehensive file of working papers specific to the cell concerned.

E.6 *Conclusion*

The risk matrix does not reduce the amount of labour involved in identifying, quantifying and managing the risks of a project, but it is hoped that it will provide a convenient check-list and a disciplined framework within which some complex work can be carried out and controlled, without losing sight of the wood for the trees. Above all, it may help to prevent particular kinds of risk being overlooked altogether.

Table E.1. Risk matrix: framework for risk identification (overview)

Principal Project Risks by stage of project	Cause of risk: A. Political	B. Business	C. Economic	D. Project	E. Natural	F. Financial
Promotion of concept **Design** **Contract negotiations** **Project approval**	Table E.1a			Table E.1d		
Raising of capital **Construction**	Table E.1b			Table E.1e		
Operation and maintenance **Revenues** **Decommissioning**	Table E.1c			Table E.1f		

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table E.1a. Risk matrix: framework for risk identification (part 1)

Cause of risk	A. Political						B. Business				C. Economic		
	a	b	c	d	e	f	a	b	c	d	a	b	c
Principal project risks by stage of project	Government	Public opinion	Environmental change	Legislation	Wars, terrorism, riots	Public relations	Demand failure	Competition	Premature obsolescence	Safety standards	Cost inflation/interest rates	Currency fluctuations	Extreme economic conditions
Promotion of concept													
loss of intellectual property rights													
claims for infringement of intellectual property rights													
Design													
non-compliant design (failure to meet specified standards)													
design based on inadequate site investigation data													
professional negligence													
Contract negotiations													
failure to agree development framework with client													
failure to resolve conflicts of interest within promoting consortium													
contractual terms and conditions worse than expected													
Project approval													
failure to obtain approval/consents	×	×											
long delay before approval granted	×	×				×							
unforeseen modifications to project		×											
cost of obtaining approval higher than expected	×	×				×							
inclusion of contingent liabilities (e.g. environmental clean up)	×	×								×			
introduction of regulatory controls (fares, competition policy)	×		×	×				×					

Table E.1b. Risk matrix: framework for risk identification (part 2)

Cause of risk

Principal project risks by stage of project	A. Political						B. Business				C. Economic		
	a	b	c	d	e	f	a	b	c	d	a	b	c
	Government	Public opinion	Environmental change	Legislation	Wars, terrorism, riots	Public relations	Demand failure	Competition	Premature obsolescence	Safety standards	Cost inflation/interest rates	Currency fluctuations	Extreme economic conditions
Raising of capital													
capital requirements increased by inflation											×	×	×
interest costs higher than expected											×	×	×
capital not available due to market conditions													×
capital not available due to poor market perceptions of project						×							
capital not available due to withdrawal of support by key organisations						×		×					
refinancing not available or terms worse than expected						×		×					×
default due to insufficient project revenue							×		×				
default due to external factors			×		×			×					×
Construction													
inability to obtain land, access right, wayleaves		×											
compensation costs higher than expected	×	×				×							
delays due to force majeure					×								
delays due to other causes													
cost over-runs										×	×		
insolvency of promoter													
insolvency of contractor													
third party damages													
failure of project to gain technical acceptance			×										
structural failure post completion													

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table E.1c. Risk matrix: framework for risk identification (part 3)

Principal project risks by stage of project	Cause of risk: A. Political						B. Business				C. Economic		
	a	b	c	d	e	f	a	b	c	d	a	b	c
	Government	Public opinion	Environmental change	Legislation	Wars, terrorism, riots	Public relations	Demand failure	Competition	Premature obsolescence	Safety standards	Cost inflation/interest rates	Currency fluctuations	Extreme economic conditions
Operation and maintenance													
unforeseen operating costs										×			
major repairs										×			
third party claims													
accident damage													
Revenues													
operating volume less than expected			×			×	×	×					
unit revenues lower than expected				×				×					
unforeseen competition			×	×				×					
revenue collection costs higher than expected													
revenues negotiable (influence of large customers)								×					
loss of revenue due to late completion or temporary closure													
revenue insufficient to cover debt servicing													
revenue loss due to fraud			×										
Decommissioning													
lifetime below expectations			×	×			×	×	×	×			
residual value less than expected			×	×					×	×			
dismantling costs higher than expected										×			

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table E.1d. Risk matrix: framework for risk identification (part 4)

Principal project risks by stage of project	Cause of risk: D. Project											E. Natural				F. Financial	
	a	b	c	d	e	f	g	h	i	j	k	a	b	c	d	a	b
	Definition	Technical innovation	Leadership	Technical competence	Commitment	Planning and control	Resourcing	Legal framework	Progress	Labour relations	Human error/incompetence	Weather	Earthquake	Fire or explosion	Ground conditions	Inadequate margins	Unbalanced sharing of risk
Promotion of concept																	
loss of intellectual property rights								×									
claims for infringement of intellectual property rights						×		×			×						
Design																	
non-compliant design (failure to meet specified standards)	×			×		×	×										
design based on inadequate site investigation data	×			×		×	×										
professional negligence	×			×		×	×	×			×						
Contract negotiations																	
failure to agree development framework with client	×		×		×			×								×	×
failure to resolve conflicts of interest within promoting consortium	×		×		×			×								×	×
contractual terms and conditions worse than expected																×	×
Project approval																	
failure to obtain approval/consents			×		×												
long delay before approval granted																	
unforeseen modifications to project	×	×				×											
cost of obtaining approval higher than expected						×											
inclusion of contingent liabilities (e.g. environmental clean up)																	
introduction of regulatory controls (fares, competition policy)																	

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table E.1e. Risk matrix: framework for risk identification (part 5)

Principal project risks by stage of project	Cause of risk: D. Project a Definition	D. b Technical innovation	D. c Leadership	D. d Technical competence	D. e Commitment	D. f Planning and control	D. g Resourcing	D. h Legal framework	D. i Progress	D. j Labour relations	D. k Human error/incompetence	E. Natural a Weather	E. b Earthquake	E. c Fire or explosion	E. d Ground conditions	F. Financial a Inadequate margins	F. b Unbalanced sharing of risk
Raising of capital																	
capital requirements increased by inflation																	
interest costs higher than expected																	
capital not available due to market conditions																	
capital not available due to poor market perceptions of project																	
capital not available due to withdrawal of support by key organisations			×		×				×								
refinancing not available or terms worse than expected			×		×				×								
default due to insufficient project revenue																	
default due to external factors																	
Construction																	
inability to obtain land, access right, wayleaves					×												
compensation costs higher than expected																	
delays due to force majeure												×	×	×	×		
delays due to other causes		×	×		×	×	×			×	×						
cost over-runs		×	×		×	×	×			×	×	×	×	×	×		
insolvency of promoter																×	×
insolvency of contractor																×	×
third party damages						×					×						
failure of project to gain technical acceptance	×	×		×		×	×										
structural failure post completion		×		×													

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

Table E.1f. Risk matrix: framework for risk identification (part 6)

Principal project risks by stage of project	Cause of risk: D. Project											E. Natural				F. Financial	
	a	b	c	d	e	f	g	h	i	j	k	a	b	c	d	a	b
	Definition	Technical innovation	Leadership	Technical competence	Commitment	Planning and control	Resourcing	Legal framework	Progress	Labour relations	Human error/incompetence	Weather	Earthquake	Fire or explosion	Ground conditions	Inadequate margins	Unbalanced sharing of risk
Operation and maintenance																	
unforeseen operating costs		×				×											
major repairs		×				×											
third party claims						×					×						
accident damage						×					×						
Revenues																	
operating volume less than expected						×											
unit revenues lower than expected						×											
unforeseen competition						×											
revenue collection costs higher than expected						×											
revenues negotiable (influence of large customers)																	
loss of revenue due to late completion or temporary closure									×								
revenue insufficient to cover debt servicing																×	
revenue loss due to fraud						×											
Decommissioning																	
lifetime below expectations												×	×	×			
residual value less than expected																	
dismantling costs higher than expected		×															

APPENDIX F

EXAMPLE OF APPLICATION OF SIMPLIFIED COST BENEFIT ANALYSIS

F.1 As mentioned in ¶5.6.8, this appendix describes a simplified form of cost benefit analysis which can be used for public sector projects with lifetimes of up to about five years.

F.2 One possible field of application is the Department of the Environment's new Single Regeneration Budget. This Budget will be worth some £1.3 billion in 1995/96, and brings together a variety of existing programmes, many of which still have a number of years of unexpired life. The overall objective of the Budget programme is not specified, and it is understood that a variety of social and political judgements will be applied, at local and central level, in deciding which new proposals should be funded. The Budget is described in the official Bidding Guidance note as: "a fund of public money which will complement or attract other resources — public, private or voluntary. It will help to improve local areas and enhance the quality of life of local people by tackling need, stimulating wealth creation and improving competitiveness". We think that, as an aid to judging proposals for new schemes, consideration might be given to introducing a very rudimentary form of cost benefit analysis, which is capable of being applied by junior civil servants and those who have little prior training. Each proposal would have to be accompanied by a statement showing:

(a) *Cost*. i.e. the total of capital and running costs from public sector sources over the project's lifetime (up to five years).

(b) *Benefit*. i.e. the total social benefits over the project's lifetime (up to five years), on the basis of guidelines issued centrally after appropriate research, e.g.

- every new job created is worth £*a*;
- every nursery school place created is worth £*b*;
- every homeless person housed is worth £*c*;
- every green area created is worth £*d* per acre; and
- every new business start up is worth £*e*, etc.

Any cash receipts generated by the project would also be taken into consideration.

(c) *Benefit/cost ratio*. i.e. (b) divided by (a).

F.3 The use of discounting techniques may not be necessary in this instance because of the comparatively short life of most projects (up to five years) and the lack of financial sophistication on the part of the users. Apart from this lack of discounting, the technique is analogous to the receipts/costs ratio method described in ¶3.2.4.

F.4 The practical difficulties of estimating the parameters *a*, *b*, *c*, etc.,

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

meaningfully should not be under-estimated. It would probably be necessary to adopt a methodology which achieved at least a degree of consensus from interested parties. Moreover, it would be necessary to allow a degree of flexibility to cope with local variations — for example because the demand for a project's output in a particular location may be genuinely different from the promulgated national average figure. It would also be legitimate for the numerical values chosen to reflect, to some extent, the Government's own social priorities.

F.5 We believe that the introduction of such analysis on a routine basis would be of material assistance in the appraisal and control of projects, and that it would help to ensure that good value for money is obtained.

F.6 Suppose that it is proposed to establish a new local enterprise agency, which will cost £50,000 spread over the first year. The staff and running costs will be £60,000 p.a. for the next four years. It is estimated that the value of each job the agency helps to create is £500 and that it will help to create 200 new jobs per annum. Then:

(a) Cost:	£50,000 + £60,000 × 4	=	£290,000
(b) Benefit:	200 × 4 × £500	=	£400,000
(c) Benefit/Cost Ratio:	£400,000/£290,000	=	1.38

F.7 Sensitivity analysis indicates that if the capital costs could be reduced to £40,000 and running costs to £50,000 p.a., the benefit/cost ratio would become 1.67 — quite a large difference.

F.8 Suppose that the project is authorised on the basis of the cheaper costs and the benefit/cost ratio of 1.67. However, at the end of two years the capital costs have totalled £100,000 instead of £40,000 and the running costs in the first full year of operation have totalled £80,000 instead of £50,000; the running costs will continue at this higher level. Should the project be discontinued, bearing in mind that by the end of the five-year period the total costs will be £420,000 and the benefits achieved only £400,000? The analysis for the next three years proceeds thus:

(a) Cost:	£80,000 × 3	=	£240,000
(b) Benefit:	200 × 3 × £500	=	£300,000
(c) Benefit/Cost Ratio:	£300,000/£240,000	=	1.25

F.9 The project is, therefore, worth continuing, providing the benefits each year are likely to be as estimated, unless there are competing projects offering higher benefit/cost ratios for the scarce resources available.

APPENDIX G

THE PRIVATE FINANCE INITIATIVE

G.1 The U.K. Government's Private Finance Initiative (see Section 5.7) was discussed in detail at the public conference held by the actuarial profession on 25 May 1993 (see Lewin, 1993). Since then the Government has issued (HM Treasury, 1994) an important paper on competition in this field, which was one of the aspects discussed at the conference.

G.2 This paper makes it clear that the advantages in terms of stimulating innovation may, in exceptional cases, justify direct negotiation with a single promoter as an alternative to competitive tendering. If a competition is held, the Government is prepared to consider compensation if the project cannot proceed for reasons outside the control of the bidders. The number of bidders selected to tender will normally be limited to three or four and, in some cases, the Government may announce advance arrangements for contributing to bidders' costs. A useful paper on the tendering process is that by Armitt (1994).

G.3 Such steps may well help to stimulate bids from companies in respect of specific schemes and, as a further incentive to progress, the Government has appointed a Private Finance Panel to promote the flow of transactions.

G.4 An informative overview of the Initiative, as seen by one Government Department (Environment), appears in a paper by McCarthy (1994). Some of the key points he mentions are:

— For financially freestanding projects, undertaken by the private sector on the basis that costs will be recovered entirely from the final customer, Government bodies may contribute in terms of giving consents, meeting minor start-up costs or carrying out feasibility studies.

— Value for money must be demonstrated for any expenditure by the public sector: it must be shown that the preferred approach is the most cost effective of the options available.

— There are no predetermined rules for risk transfer and risk sharing — but the private sector must genuinely bear risk. The public and private sectors must each take on those risks they are best placed to manage. The Government is happy for the private sector to make big profits where they are prepared to take real risks. The advantage for the public sector is that risk transfer delivers the benefits of private sector management disciplines and efficiency gains, thus ensuring that public resource inputs are used with maximum effect.

— One of the risks most likely to be borne by the public sector would be the procedural risk (changes of public policy, or failure to obtain Parliamentary powers, planning permission, public consents or licences). The public sector might also sometimes be prepared to take part of the operating risk.

Even part of the revenue risks might be assumed by the public sector in some circumstances.

G.5 As one example of the flexibility of the Government's approach, Urban Development Corporations may give rental guarantees to private developers for up to five years.

APPENDIX H

THE PRIVATE FINANCE INITIATIVE: AN EXAMPLE OF SHARING OF RISKS AND REWARDS

(see ¶5.7.6)

H.1 Suppose we have a project which is estimated to cost £100m at the outset and which will bring in a net revenue of £8m p.a. for 20 years (annually in arrears), with an additional net social benefit of £10m p.a. for 20 years (all figures being expressed at a constant price level). The overall real internal return from the project is therefore expected to be 17.3% p.a., taking account of both revenues and social benefits. The overall NPV is £53.2m at 10% p.a. or £106.5m at 6% p.a. If the Government were to carry out the project, the public sector (including the recipients of social benefits) would obtain all this return. However, the private investor can only capture the net revenues, not the social benefit. If he is expected to invest the whole £100m he will probably conclude that the project is not viable, because (ignoring the social benefits) it has an internal return of only 5.0% p.a. The internal return and NPVs the investor and the Government can expect to receive from different levels of capital investment are as follows:

Case	Capital required from: Private Investor £m	Government £m	Return for private investor IRR % p.a.	NPV £m 10% p.a.	6% p.a.	Return for Government IRR % p.a.	NPV £m 10% p.a.	6% p.a.
1	100	0	5.0	−31.9	−8.2	∞	85.1	114.7
2	90	10	6.2	−21.9	1.8	100.0	75.1	104.7
3	80	20	7.8	−11.9	11.8	50.0	65.1	94.7
4	70	30	9.6	−1.9	21.8	33.2	55.1	84.7
5	60	40	11.9	8.1	31.8	24.7	45.1	74.7
6	50	50	15.0	18.1	41.8	19.4	35.1	64.7
7	40	60	19.4	28.1	51.8	15.8	25.1	54.7
8	30	70	26.4	38.1	61.8	13.1	15.1	44.7
9	20	80	40.0	48.1	71.8	10.9	5.1	34.7
10	10	90	80.0	58.1	81.8	9.2	−4.9	24.7
11	0	100	∞	68.1	91.8	7.8	−14.9	14.7

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press

H.2 The band of private sector capital investment between £60m and £40m (i.e. cases 5, 6, and 7) gives the private sector a real expected yield of between 11.9% p.a. and 19.4% p.a. A figure somewhere in this range would probably be regarded as a reasonably good rate of return by most investors, though much would depend on the risks involved. The real return to the Government (including the value of social benefits) would then be between 24.7% p.a. and 15.8% p.a., which would easily meet the target rate of return for Government projects at the current 6% p.a. or 8% p.a. level, though the lower end is below the 17.3% p.a. available to the Government if it were to undertake the whole project. The split at which each party gets 17.3% p.a. is when costs are split in proportion to benefits, i.e 8:10 or £44.4m from the private sector and £55.6m from the Government. The Government might, however, be willing to put in more than this if there were other advantages in the private sector carrying out and managing the investment.

H.3 These figures provide a range within which a negotiated solution could be found which would be likely to satisfy both sides. Exactly where the deal should be struck may depend on each party's perceptions of the risks and the upside potential, both for themselves and the other party. Each side may wish to take professional advice during the course of the negotiations, to help them to understand these risks and rewards better.

H.4 Clearly the private investor will be prepared to accept more of a project's capital cost if his own risk can be limited, for example by taking out appropriate insurance. In the case of the more unusual risks, if commercial insurance is impossible to obtain, it may even be sensible for the Government itself to act as insurer of last resort in return for an appropriate premium from the private investor (though it is thought that the Government would prefer any such arrangement to be submerged in the contract terms agreed for the project rather than being specifically identified as an insurance arrangement). This might enable the private investor to accept a much higher share of the capital cost, whilst still leaving him with much of the project risk.

APPENDIX I

RESEARCH MODEL: PRECONDITIONS OF PROJECT SUCCESS

adapted, by permission of the Major Projects Association, from:
"The Anatomy of Major Projects" by P.W.G. Morris & G.H. Hough (1987) (See ¶6.6.3)

EXTERNAL FACTORS

- Effects of external factors on definitions of project success properly recognised (e.g. prices, regulation, technical developments, government/corporate changes).
- Political support obtained:
  - requisite sponsorship available;
  - political support for necessary management actions;
  - nationalist aspirations constrained.
- Community factors properly considered and controlled.

ATTITUDES

- Good positive client, parent company and senior management attitudes, interrelationships and commitment.

PROJECT DEFINITION

- Comprehensive and clearly communicated project definition:
  - prefeasibility and feasibility and design study phases carried out in an orderly fashion;
  - objectives related to participants;
  - clarity not forced prematurely;
  - premature over-commitment to project avoided;
  - magnitude of task properly recognised;
  - the project organised appropriately.
- Good design/technology management, especially where there is technical uncertainty or complexity:
  - the extent to which R & D is completed recognised as affecting the accuracy of the estimate;
  - design tested adequately before final project commitment is made;
  - interface management recognised as important where there are significant interdependencies;
  - replication wherever possible;
  - design frozen once agreed.

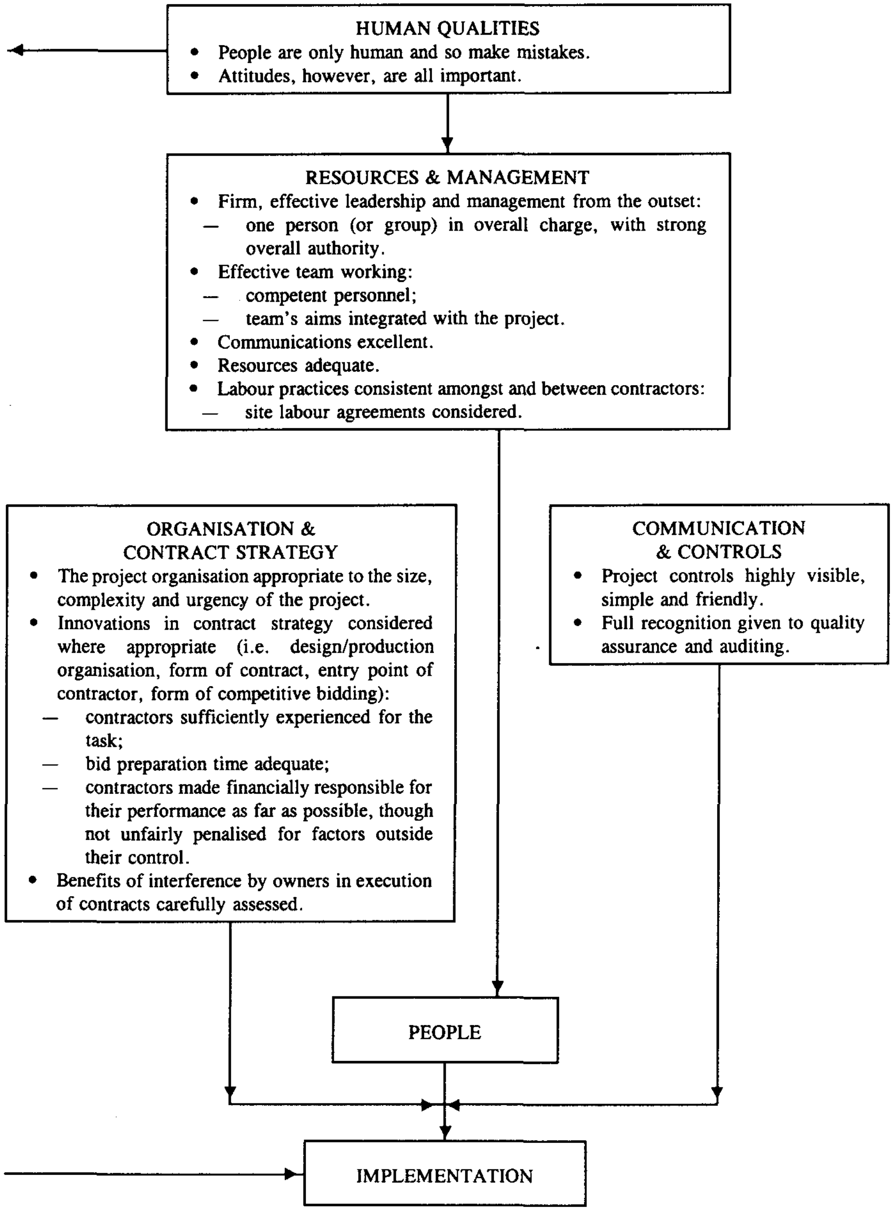
SCHEDULE

- Good planning, clear schedules and adequate back-up strategies:
  - the broad 'systems' aspects of the project recognised;
  - the project definition phased and developed as appropriate;
  - sub-objectives identified, assessed and developed clearly;
  - full account taken of phasing, logistics, geophysical uncertainties, environmental problems and the relationship between design and production;
  - back-up strategies prepared for high risk areas;
  - switching design authority during different phases of project avoided;
  - attention paid to detail.
- Full cognisance given to the potentially harmful effects of urgency.
- Concurrency avoided where possible.

FINANCE

- Full financial analysis of all project risks undertaken:
  - sponsors interested in success of project *per se*;
  - availability of funding appraised in relation to perceived success of project at key review points.

https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press



https://doi.org/10.1017/S1357321700001082 Published online by Cambridge University Press