Laser and Particle Beams (2009), *27*, 27–32. Printed in the USA.

doi:10.1017/S0263034609000044

Electron acceleration in the wakefield of asymmetric laser pulses

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(Received 2 October 2008; Accepted 10 October 2008)

Abstract

Electron acceleration in the plasma wakefield driven by asymmetric laser pulses is investigated analytically. It is found that the asymmetric laser pulse can significantly modify the phase portrait of the electron dynamics and enhance the maximum energy of the accelerated electrons. There exists an optimum ratio of the lengths of the rising and falling segments of the asymmetric laser-pulse. A linear scaling law relating the accelerated electrons' energy and the plasma density is obtained. This result differs from the power-law dependence often associated with symmetric laser pulses.

Keywords: Asymmetrical laser pulse; Wakefield electron acceleration; Optimal energy gain

INTRODUCTION

Laser wakefield acceleration (LWFA) of electrons was proposed by Tajima and Dawson (1979) almost 30 years ago. Since ultraintense laser pulses obtained from the chirped pulse amplification technique became available, LWFA have been intensively investigated theoretically and experimentally, and compared with other acceleration methods (Nakamura, 2000; Lotov, 2001; Balakirev, *et al.*, 2001, 2004; Geddes *et al.*, 2004; Faure *et al.*, 2004; Mangles *et al.*, 2004; Li *et al.*, 2004; Reitsma *et al.*, 2005; Reitsma & Jaroszynski, 2004; Hoffmann *et al.*, 2005; Mourou *et al.*, 2006; Joshi, 2007; Hora, 2007; Shi, 2007; Nickles *et al.*, 2007; Chen *et al.*, 2008; Kulagin *et al.*, 2008; Limpouch *et al.*, 2008).

In LWFA, the ponderomotive force of the leading front of the propagating laser pulse drives out the local electrons. The resulting space charge field then pulls the background electrons into the cavity left behind, so that a wake plasma wave is excited. Many theoretical models have qualitatively and quantitatively described the basic properties of LWFA (Tajima & Dawson, 1979; Joshi, 2007; Mourou *et al.*, 2006). It is found that the laser-driven plasma wakefield can be excited most effectively when (Tajima & Dawson, 1979) $L_L = c\tau_L \sim \lambda_p/2$, where L_L and τ_L are the spatial and temporal laser pulse lengths, c is the light speed in vacuum, and λ_p is the plasma wavelength, which is inversely proportional to the square root of the background plasma density n_e. For plasma densities in the range of $n_e \sim 10^{17}$–10^{19} cm^{-3}, we have $\tau_L \sim 150-15$ fs (Joshi, 2007).

Since an intense laser pulse can ponderomotively expel most of the electrons in it, to accelerate a sufficiently large number of electrons, it is necessary to introduce external electrons into the wakefield by direct injection or by creating a suitable beat wave using another laser (Joshi, 2007). Since the first cycles of the wakefield [are] the strongest, the electrons there can gain the most energy. In their recent analytical investigation of electron LWFA by a short ultraintense Gaussian laser pulse, Esirkepov *et al.* (2006) found that in the first cycle of the wakefield, the acceleration there is more effective than by direct laser ponderomotive-force acceleration, and there are at least three types of separatrices in the electron phase space. Nonlinear charged-particle dynamics also appears in the stochastic heating and acceleration of electrons in colliding laser fields (Sheng *et al.*, 2002), the three-dimensional bubble-regime acceleration for

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producing monoenergetic electrons (Pukhov *et al.*, 2004; Xie *et al.*, 2007), as well as other processes (Niu *et al.*, 2008).

Often the laser-plasma interaction is such that an initially symmetric laser pulse can become asymmetric because of steepening of the pulse front arising from externally induced (say due to strong plasma inhomogeneity), backscattering of the laser light, or nonlinear self-modulation and compression of the pulse front (Esarey *et al.*, 2000). In general, any large variation of the plasma permittivity can lead to asymmetry of the laser pulse propagating in it (Reitsma *et al.*, 2005; Reitsma & Jaroszynski, 2004). Asymmetric pulse modulation can also occur in plasma channels, except that here a steepening of the backside of the pulse occurs (Gordon *et al.*, 2003). On the other hand, smoothing and stochastic pulsation can also occur in intense laser-plasma interaction (Hora, 2006, 2007). Up to now, LWFA by asymmetric laser pulses has still not been investigated in much detail.

In this paper, we consider electron LWFA by intense asymmetric laser pulses. The plasma density is assumed to be low, so that the response of the laser pulse can be neglected. It is found that an asymmetric laser pulse can significantly modify the phase portrait of the electrons in the first wakefield cycle as well as the maximum electron energy gain. Furthermore, depending on the laser and plasma parameters, there can exist several local minimums in the wakefield potential and the deepest minimum leads to maximum electron acceleration. There also exists an optimum asymmetry ratio of the rising to falling segments of the laser pulse profile. Scaling laws for the maximum energy of the accelerated electrons are also found. In contrast to symmetric laser profiles that usually lead to a power law between the maximum energy and the density of the electrons, for asymmetric laser pulses the density-dependence of the electron energy always remains linear.

THE MODEL AND NUMERICAL RESULTS

The relativistic electron dynamics in intense electromagnetic fields have been studied by many authors and many solutions have been obtained (Yu *et al.*, 1978; Esirkepov *et al.*, 2006). The Hamiltonian for the relativistic motion of an electron in an electromagnetic field is

$$H = \sqrt{m^2c^4 + c^2p_\parallel^2 + (cp_\perp + eA_\perp)^2} - e\phi, \qquad (1)$$

where m and $-e$ are the mass and charge of the electron, $A_\perp$ is the vector potential of the laser light, ϕ is the electrostatic potential of the wakefield, $p_\parallel$ and $p_\perp$ are the longitudinal and transverse momentum of the electron, respectively. In the fast LWFA process, the motion of the much heavier ions is negligible. Since for LWFA, the background plasma density is low, dissipation of the laser during the interaction is also negligible.

In the moving frame $x - v_g t$ of the laser-pulse group velocity v_g (or the wakefield phase velocity), the Hamiltonian can be rewritten as

$$h = \sqrt{1 + p_x^2 + a^2} - \varphi - \beta_{\rm ph} p_x, \qquad (2)$$

where $p_x = p_\parallel/mc$, $a = eA_\perp/mc^2$, $\varphi = e\phi/mc^2$, and $h = H/mc^2$ are the normalized electron longitudinal momentum, laser-field vector potential, wakefield scalar potential, and Hamiltonian, respectively. In the last term of Eq. (2) on the right-hand-side, the quantity $\beta_{\rm ph} = v_g/c$ is the normalized wake-wave phase speed. The corresponding relativistic factor is $\gamma_{\rm ph} = (1 - \beta_{\rm ph}^2)^{-1/2}$. We shall assume that the plasma is cold and ion motion is negligible.

The electrostatic wakefield potential in the electron plasma is given by the Poisson's equation (Yu *et al.*, 1978; Esirkepov *et al.*, 2006; Xie & Wang, 2002)

$$\frac{\partial^2 \varphi}{\partial \xi^2} = k_p^2 \gamma_{\rm ph}^2 \beta_{\rm ph} \left[\frac{\gamma_{\rm ph}(1+\varphi)}{\sqrt{\gamma_{\rm ph}^2(1+\varphi)^2 - (1+a^2)}} - 1 \right], \qquad (3)$$

where $\xi = (x - v_g t)/\lambda_L$ is the moving coordinate normalized by the laser wavelength. The Hamiltonian system Eq. (2) leads to the following canonical equations of motion

$$\begin{aligned} \xi' &= \partial h/\partial p_x = p_x/\sqrt{1 + p_x^2 + a^2} - \beta_{\rm ph}, \\ p_x' &= -\partial h/\partial \xi = -aa'/\sqrt{1 + p_x^2 + a^2} + \partial\varphi/\partial\xi, \end{aligned} \qquad (4)$$

where the superscript *prime* denotes derivative with respect to t.

For convenience, we shall mainly consider asymmetric Gaussian laser pulses of the form $a = a_0\{\exp[-4\ln(2)\xi^2/l_p^2] - 1/16\}\theta(\xi + l_p)$ for $\xi < 0$ and $a_0\{\exp[-4\ln(2)\xi^2/r_p^2] - 1/16\}\theta(r_p - \xi)$ for $\xi \geq 0$, where $\theta(\xi) = 1$ for $\xi \geq 0$ and $\theta(\xi) = 0$ for $\xi < 0$. The plasma density is $n_e = 0.01 n_c$, where $n_c = m\omega^2/4\pi e^2$ is the critical density corresponding to the laser frequency ω. Figure 1 shows the wakefield of a laser pulse with $a_0 = 2$, $l_p = 30\lambda$, and $r_p = 15\lambda$, where λ is the laser wavelength. We see that within the laser pulse there are already three plasma-wave cycles, with their local minimums at $\xi \sim -10$, -20, and -30. The wake plasma oscillations started inside the laser pulse because before the trailing segment of the pulse has passed, the space-charge field already pulls plasma electrons into the electron deficient region created by the ponderomotive force of the short, and steeply rising front segment of the laser pulse. That is, the wavelength of excited wake plasma waves is shorter than the length of the trailing segment of the laser pulse.

It is of interest to investigate the fixed points of Eq. (4) and see how they are related to electron acceleration. We found that near the local minimums $\xi \sim -10$ and ~ -20 of the potential there are two saddle-type fixed points at $(\xi, p_x) \approx$

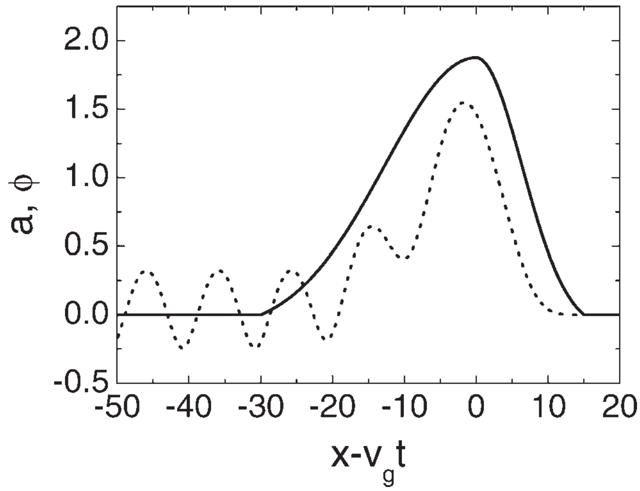

Fig. 1. The wakefield potential (dotted line) φ excited by an asymmetric Gaussian laser pulse (solid line) with $a_0 = 2$, $l_p = 30$, and $r_p = 15$.

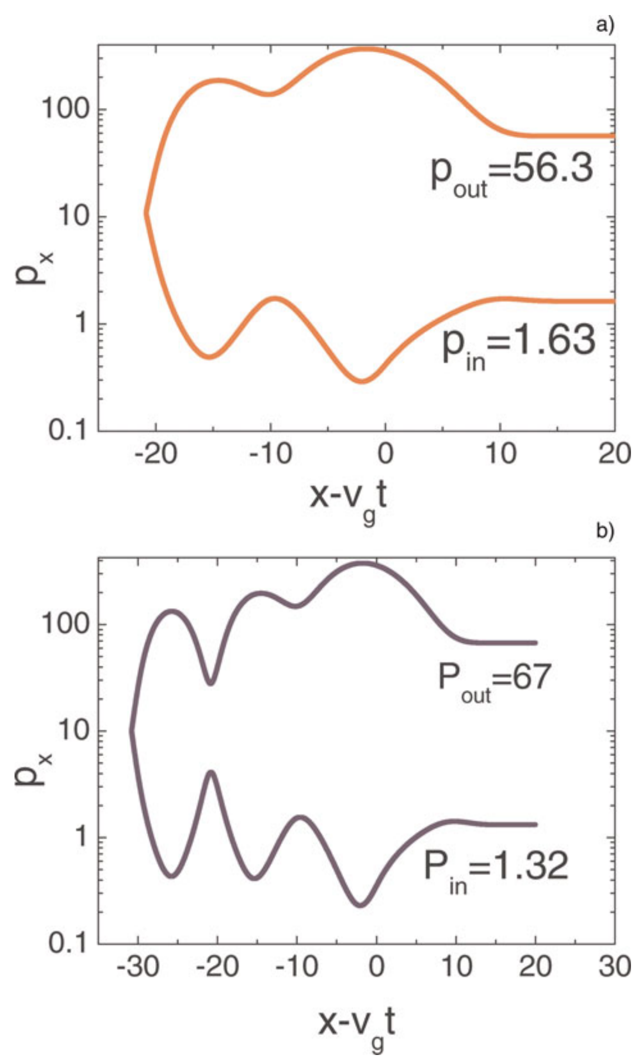

Fig. 2. (Color online.) The electron phase space. The two separatrices correspond to the two saddle points $(\chi, p_x) \approx (-20, 10)$ and $(-30, 10)$. The other parameters are the same as in Figure 1.

$(-20, 10)$ and $(-30, 10)$. In Figure 2, the electron phase space (ξ, p_x) is shown. There are two separatrices of the electron trajectories. Clearly, electrons passing the neighborhood of $(-30, 10)$ would achieve more net energy gain than those passing the neighborhood of $(-20, 10)$. Since the acceleration efficiency and net energy gain is associated with the minimum of the wakefield potential, the problem reduces to finding out how $|\varphi_{\min}|$ depends on the system parameters.

We first studied the effects of the laser-pulse rising length r_p on the electron acceleration efficiency for a given pulse falling length l_p. For comparison, we also consider the asymmetric sine pulse $a = a_0 \sin[(1 + \xi/l_p)\pi/2]\theta(\xi + l_p)$ for $\xi < 0$ and $a_0 \sin[(1 - \xi/r_p)\pi/2]\theta(r_p - \xi)$ for $\xi \geq 0$. The numerical results are shown in Figure 3. We see that for a given l_p value there exists an optimum r_p which leads to maximum electron acceleration. As l_p increases, the optimum r_p decreases, accompanied by the increase of the electron acceleration efficiency. Our results are consistent with the well-known rule (Tajima & Dawson, 1979) that the most effective electron acceleration occurs when the laser pulse is about a plasma-wavelength long, namely $L_L = c\tau_L \sim \lambda_p/2$ (or $l_p = r_p = 5$ for $\lambda_p = 10\,\lambda_L$ when $n_e = 0.01 n_c$) for symmetric laser pulses, is also roughly valid for asymmetric laser pulses. As expected, for a given pulse-falling length, the steeper the rising segment of the pulse, the more effective the electron acceleration. However, there is a critical point. When the pulse-rising length is shortened to pass the critical point, the accelerated electron energy can drop rapidly, as for the case $l_p = 2.5$ and $r_p = 5$ in Figure 3. However, for sufficiently long pulse falling lengths, it seems that electron acceleration can continue to increase with decrease of the pulse rising length. Figure 4 shows the dependence of $|\varphi_{\min}|$ on l_p for different r_p values. We see that for a given laser pulse rising length r_p, LWFA becomes more effective as l_p increases, but it becomes less effective when l_p passes a critical point. The electron LWFA seems to

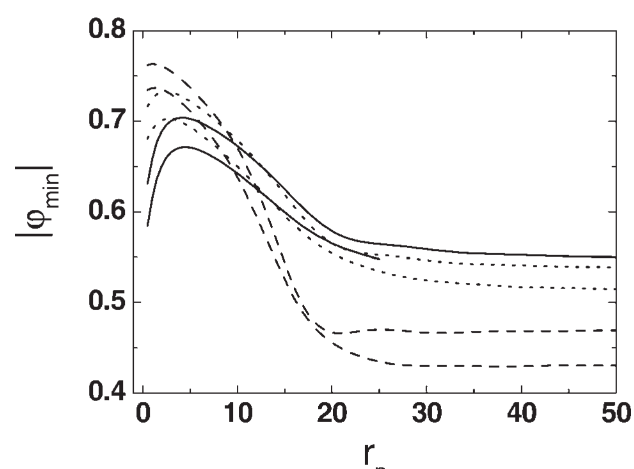

Fig. 3. Relation between $|\varphi_{\min}|$ and r_p for asymmetric Gaussian (Lower lines) and sine (upper lines) laser pulses with different $l_p = 2.5$ (solid line), 5.0 (dotted line) and 10.0 (dashed line). The laser strength is $a = 2$.

https://doi.org/10.1017/S0263034609000044 Published online by Cambridge University Press

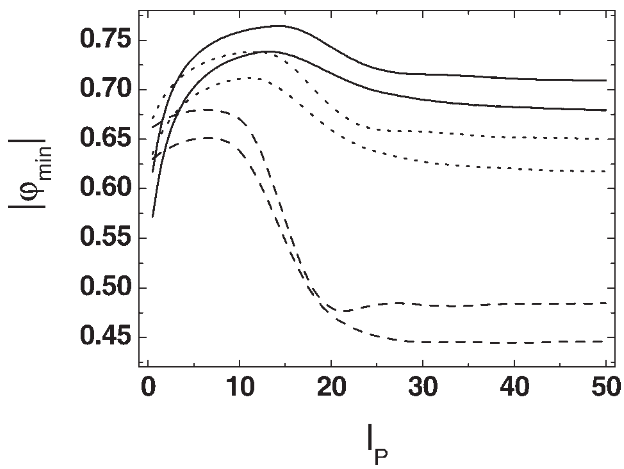

Fig. 4. Relation between $|\varphi_{min}|$ and l_p for asymmetric laser pulses with different $r_p = 2.5$, 5.0 and 10.0. The other system parameters and line types have the same meanings as in Figure 3.

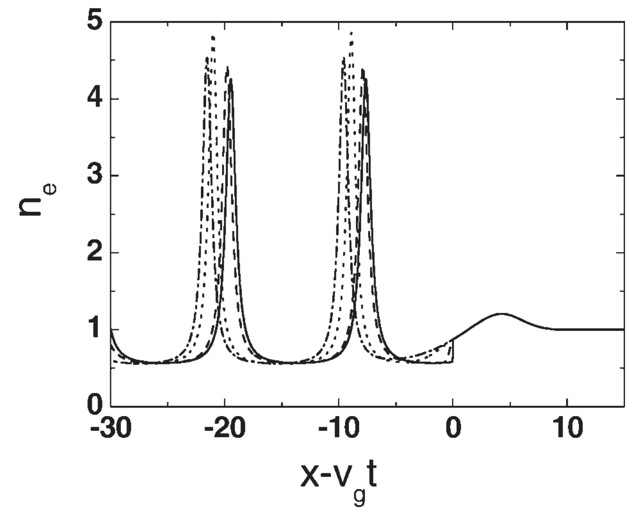

Fig. 6. Electron density profile of the first wake wave cycles for $r_p = 10$, and $l_p = 0.05$ (solid line), 1 (dashed line), 6 (dotted line), and 10 (dot dashed line). The other parameters are the same as in Figure 1.

saturate when the l_p becomes large. On the other hand, for constant r_p, the electron LWFA increases as l_p decreases. In general, under the same laser and plasma conditions, the acceleration efficiency of the sine pulse is somewhat higher than that of the Gaussian pulse. This can be attributed to the fact that a sine pulse has a sharper rising (and trailing) front, which results in a higher ponderomotive force and thus also the space-charge separation field.

The relation between the electron acceleration and the electron density is shown in Figure 5 for fixed r_p (=10) and 6 for fixed l_p (=10). We see that, in general, the higher the peak density, the stronger the electron acceleration. It is reasonable since the wakefield is due to the space charge separation. Therefore, it is possible for us to estimate the electrons' acceleration by simply seeing the height of plasma electron density peak pushed by the asymmetric laser pulse, as shown in Figures 5 and 6.

SCALING LAWS

Figure 7 shows the relationship between l_p and r_p for optimum electron acceleration by an asymmetric laser pulse. By fitting the numerical data, we can see that for a wide density range, namely $n_e \sim (10^{-4} - 10^{-1})n_c$, the scaling law $l_p \sim \lambda_p/2 \propto n_e^{-1/2}$ for optimal electron LWFA holds. The result is also consistent with the expectation that strong resonance between laser and driven wakefield occurs when the pulse width is about half of plasma wavelength (Joshi, 2007).

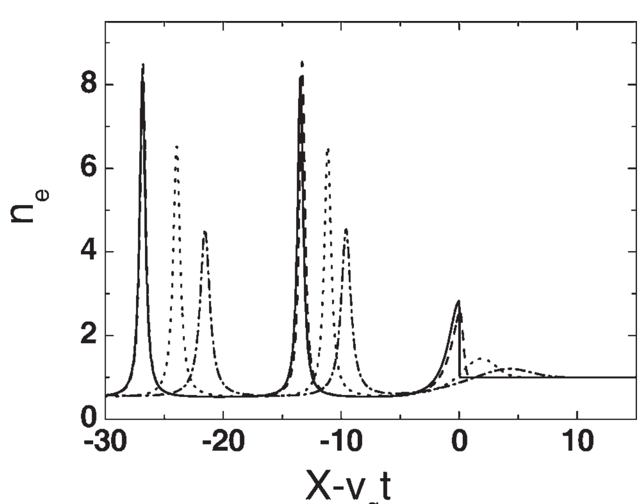

Fig. 5. Electron density profile of the first wake wave cycles for $l_p = 10$, and $r_p = 0.05$ (solid line), 1 (dashed line), 6 (dotted line), and 10 (dot dashed line). The other parameters are the same as in Figure 1.

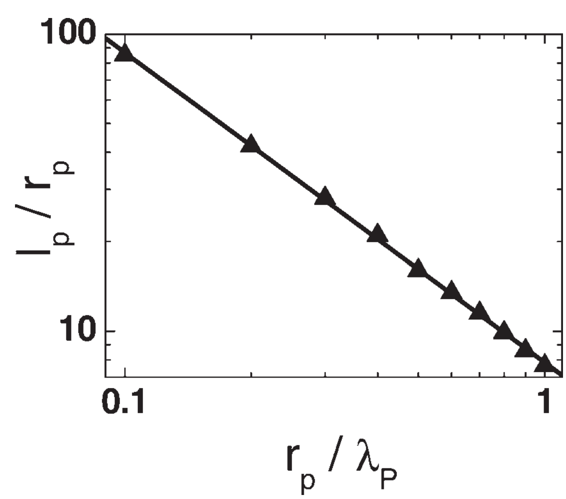

Fig. 7. Relation between l_p and r_p for optimum electron acceleration in the first wakefield cycle for asymmetric laser pulses. The triangles are from the calculated data (for $n = 0.0001 - 0.1\ n_c$) and the straight line is the linear fit. The laser pulse has a sine profile with $a_0 = 2.0$.

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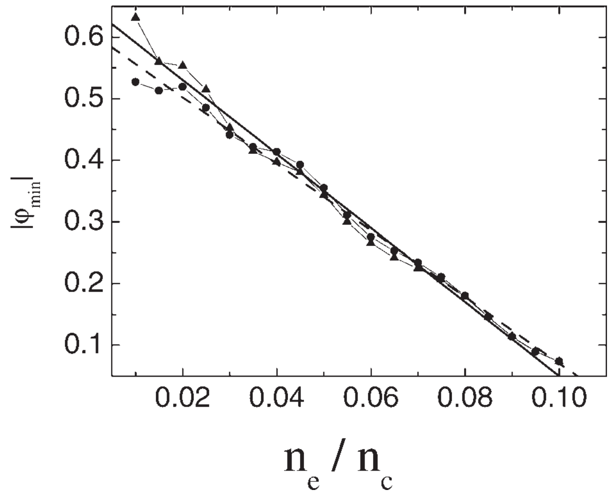

Fig. 8. Relation between the acceleration field $|\varphi_{min}|$ and the plasma density n_e. The triangles represent the numerical result for a laser pulse with $r_p = 5\ \lambda_L$ and $l_p = 15\ \lambda_L$, and the circles are for a pulse with $r_p = 15\ \lambda_L$ and $l_p = 5\ \lambda_L$. The solid and dashed lines are linear fits to the corresponding calculated points. The laser pulse has a sine profile with $a_0 = 2.0$.

It is well known that the de-phasing length of an electron moving in the laser wakefield is $l_d \approx 2\pi c\omega^2/\omega_{pe}^3 \propto \omega^2/n_e^{3/2}$ and the maximum electrostatic field (at the wave breaking limit) of cold electron plasma waves is $E_{\max} \approx 0.96 n_e^{1/2}$. Accordingly, the maximum electron energy gain can be estimated as (Tajima & Dawson, 1979; Mourou *et al.*, 2006; Joshi, 2007) $\Delta\varepsilon \propto l_d E_{\max} \propto n_e^{-1}$. These power laws have been experimentally confirmed (Mangles *et al.*, 2004). For asymmetric laser pulses the behavior is different. Figure 8 shows the scaling between the optimum electron acceleration field $|\varphi_{min}|$ and the plasma density n_e. Here we find a dependence of the form $|\varphi_{min}| = a - bn_e/n_c$, where a and b are positive constants. For example, one finds from Figure 8 that $|\varphi_{min}| \sim 0.65 - 6\ n_e/n_c$ when $r_p = 5\ \lambda_L$ and $l_p = 15\ \lambda_L$, and $|\varphi_{min}| \sim 0.61 - 5.4\ n_e/n_c$ when $r_p = 15\ \lambda_L$ and $l_p = 5\ \lambda_L$. That is, for asymmetrical laser pulses the dependence of the maximum energy gain on the plasma density is linear.

For symmetrical ($l_p = r_p$) laser pulses, we find (not shown) that the maximum wakefield roughly scales as $E_{\max} \approx n_e^{0.3} \sim n_e^{0.5}$ for different laser-pulse lengths. That is, the corresponding index of the density power for the energy gain in the numerical result deviates somewhat with the theoretical value -1. This difference can be attributed to the fact that in the theory, the electron energy gain is at the wave-breaking strength and in the numerical simulation the electrons experienced an electric field which is not yet at wave-breaking.

As mentioned, we found similar negative-indexed power laws for symmetric laser pulses but with a smaller index value. However, for sufficiently asymmetric laser pulses the electron energy gain always scales linearly with the density. This can be attributed to the fact that an asymmetrical pulse, say with steep front, produces a sharp plasma density modulation behind the front, resulting in such a strong and robust charge separation that the first cycles of the wakefield [are] almost unaffected by the much gentler trailing part of the pulse and the background plasma. However, in real applications (Hoffmann *et al.* 2005; Hora, 2007) other external and spontaneous effects such as the injection phase of the accelerated electrons and spontaneous magnetic field effects (Niu *et al.*, 2008) can also be involved in this nonlinear wave-plasma interaction, so that a more detailed investigation may be warranted.

SUMMARY

In this paper, we have studied electron acceleration in the plasma wakefield driven by an asymmetric relativistic laser pulse. It is found that the laser-pulse asymmetry can significantly modify the phase portrait of the electrons' dynamics in the laser and wakefields, and there exists an optimum asymmetry ratio between the lengths of the rising and falling segments of the laser pulse. In contrast to symmetric pulses, for which the maximum electron energy gain has a fractional power law dependence on the plasma density, the maximum electron energy gain for the asymmetric pulse scales linearly with the plasma density. Thus, it should be possible to tailor the profile of a laser pulse for achieving optimum energy gain in LWFA of electrons.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (NSFC) under grant Nos. 10875015, 10834008, 10835003 and 10725521, and by the 973 Program (Project No. 2006CB806004). BSX is also supported by the New Century Excellent Talents in University (NCET) of China.

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