

# A NOTE ON THE UNEMPLOYMENT VOLATILITY PUZZLE: IS CREDIBLE WAGE BARGAINING THE ANSWER?

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This paper shows that the ability of the credible wage bargaining model to match the observed unemployment volatility hinges on an unrealistic assumption about disagreement payoffs to the firm. Relaxing this assumption can lead to the substantial wage flexibility. As a consequence, the model is unable to capture the observed unemployment volatility.

**Keywords:** Credible Bargaining, Search Frictions, Unemployment Volatility

## 1. INTRODUCTION

It is well known that the standard search and matching model, pioneered by Diamond (1982), Mortensen and Pissarides (1994), is unable to explain the observed fluctuations in the labor market, which is often referred to as the unemployment volatility puzzle. One of the prominent responses to this puzzle, advanced by Hall and Milgrom (2008), argues that the credible wage bargaining can generate the weak response of the current wage to productivity shocks.<sup>1,2</sup> As a consequence, a firm's profits and recruiting effort are more responsive to productivity shocks. This ensures that job vacancies and unemployment vary strongly with the productivity.

Under the credible bargaining, the firm and the worker take turns making their wage offer. If the wage offer were not accepted by the counterparty, both the worker and the firm have to face disagreement payoffs, which the worker enjoys the flow value of nonwork and the firm faces the cost of delay. This gives the wage as a linear combination of the productivity of the job match and disagreement payoffs. Unlike in the Nash bargaining, the worker's outside option is not a main threat in the wage negotiation. Therefore, the credible bargaining dampens the influence of labor market conditions on the wage. Hall and Milgrom (2008) argue that this is a key reason the model can generate real wage rigidity. This paper, however, shows that real wage rigidity in the credible bargaining model hinges on unrealistic assumptions about disagreement payoffs to the firm. Relaxing this

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assumption can deliver the substantial wage flexibility and destroys the model's ability to match the large volatility of unemployment observed in the data.

In the standard search and matching frictions model of the labor market, the cost of delay to the firm is unspecified. Hall and Milgrom (2008) interpret this cost as the cost of idle capital and assume that it is fixed. We are skeptical about assuming a fixed cost of idle capital. Both in theory and in practice, the firm is able to adjust the capital according to the marginal product of capital. Since the marginal product of capital varies across the business cycle, the capital stock also varies across the business cycle. As does the cost of idle capital. To measure the cyclical nature of the cost of idle capital, we construct the data on capital stock per employee for the time period from 1950 through 2017. We find that the capital stock per employee is about 1.77 times as volatile as the labor productivity. This evidence strongly suggests that the cost of idle capital should be pro-cyclical.

We show that under this more plausible assumption, the wage has substantial flexibility, similar to the Nash bargained wage, and the model is unable to match the observed fluctuations in the labor market. Those results can be obtained even if we assume the fixed disagreement payoff to workers<sup>3</sup> and calibrate a low value to the probability that the wage negotiation breaks down so that the impact of the worker's outside option on the wage bargain is rather limited. Those calibration strategies dampen the influence of labor market conditions on the wage bargain and are commonly used in the literature. The failure of the model to match the volatility data in spite of using those calibration strategies reveals that the cost of delay to the firm is a key determinant for the wage flexibility.

To interpret our results, we consider a positive productivity shock. This increases the cost of delay to the firm, so the firm is eager to reach a wage agreement. The rational worker is aware of this change. Although the delay never occurs in equilibrium, this observation raises the threat point made by the worker and therefore puts pressure on the firm to increase the wage.

The paper contributes to the literature in twofold. We show that the limited influence of labor market conditions on the wage is not a sufficient condition for real wage rigidity. In order to generate real wage rigidity, the cost of delay to the firm has to be fixed. This reveals the fragility of credible bargaining as a solution to the unemployment volatility puzzle.

The paper is organized as follows. The model is laid out in Section 2. In Section 3, we derive an implicit wage solution. In Section 4, we compare the sharing rule obtained from credible bargaining with Nash sharing rule. In Section 5, we present some evidence on both the size and the cyclical nature of the cost of idle capital. In Section 6, we use the calibration strategy similar to Hall and Milgrom (2008) to calibrate and simulate the model. In Section 7, we analyze the wage flexibility. Section 8 concludes.

## 2. MODEL

We use a stochastic discrete time version of search frictions model which retains all the elements in Hall and Milgrom (2008). There is a continuum of identical

individuals on the unit interval. Each individual inelastically supplies one unit of labor in every period and consumes all the income they earn. An individual is either employed and earning a wage  $w$ , or else unemployed and receiving a flow value  $b$ .  $b$  is defined as the flow value of nonwork. If unemployed, an individual finds a job with probability  $f_t$ . At the end of each period, existing job matches are exogenously terminated with probability  $\tau$ . Since we assume that all firms are identical, the value of being employed is thus

$$L_t = w_t + \frac{1}{1+r} E_t[(1-\tau)L_{t+1} + \tau U_{t+1}], \tag{1}$$

where  $\frac{1}{1+r}$  is the discount factor and  $r$  is the real discount rate. The value of being unemployed is

$$U_t = b + \frac{1}{1+r} E_t[f_t L_{t+1} + (1-f_t)U_{t+1}]. \tag{2}$$

There is a continuum of identical firms on the unit interval. Each firm can hire up to one worker who produces an amount  $y_t$ . The value of a filled job is

$$J_t = y_t - w_t + \frac{1}{1+r} E_t[(1-\tau)J_{t+1} + \tau V_{t+1}], \tag{3}$$

where  $V$  is the value of a vacancy. Firms must pay a real per-period cost of  $c$  at the start of each period to post a vacancy. Vacancies are then filled at the start of the next period with probability  $q$ . We follow the timing convention of Gertler et al. (2008) and assume that new job matches become productive immediately. The value of an open vacancy is then

$$V_t = -c + \frac{1}{1+r} E_t[q_{t+1}J_{t+1} + (1-q_{t+1})V_{t+1}]. \tag{4}$$

Employment evolves according to

$$N_t = (1-\tau_t)N_{t-1} + h_t, \tag{5}$$

where  $h_t$  is the number of workers hired. The labor market is characterized by search frictions and so firms must post vacancies in order to hire workers. Aggregate hiring is determined by the matching function  $h_t = m u_t^\alpha v_t^{1-\alpha}$ . Defining  $\theta = \frac{v_t}{u_t}$  as labor market tightness. The probability of a firm filling a vacancy is  $q_t = \frac{h_t}{v_t} = m\theta_t^{-\alpha}$ , and the probability that an unemployed worker finds a job is  $f_t = \frac{h_t}{u_t} = m\theta_t^{1-\alpha} = \theta_t q_t$ .

Imposing the free-entry condition  $V_t = 0$  on (4), we obtain

$$J_{t+1} = E_t \frac{c}{\beta q_{t+1}}. \tag{6}$$

Substituting (6) into (3), we obtain the job creation condition

$$y_t - w_t - \lambda_t = 0, \tag{7}$$

where  $\lambda_t = c[(1+r)\frac{1}{q_t} - (1-\tau)E_t\frac{1}{q_{t+1}}]$  is the real cost of hiring a worker.

### 3. THE CREDIBLE BARGAIN

Under the credible bargaining, the incentive of the parties to reach an agreement depends on the bargainers' time preference and the risk of breakdown of negotiation. The firm and the worker understand they have a strictly higher payoff from reaching an agreement rather than breaking up and accepting the outside options. The firm and the worker take turns making their wage offer denoted by  $w^f$  and  $w^w$ . Time in the bargaining process is divided into several rounds of length  $\sigma$ . In each bargaining round, each party either accepts the counterparty's offer or rejects and proposes a counteroffer in the next bargaining round. After a delay, the firm incurs a cost of delay  $\gamma_t\sigma$ , while the worker enjoys the value of nonwork  $b\sigma$ .

We assume that the cost of delay to the firm is pro-cyclical,  $\gamma_t = \gamma y_t$ . This differs from Hall and Milgrom (2008) in which the cost of delay to the firm is fixed. In Section 5, we present some evidence on this assumption.

There is a probability  $1 - e^{-\delta} \approx \delta$  that the job opportunity is exogenously destroyed between bargaining rounds. In that case, the firm and the worker revert to their outside options.<sup>4</sup> Following Boitier and Lepetit (2018), we rewrite the discount factor as an exponential function (i.e.  $e^{-r} \approx \frac{1}{1+r}$ ) in order to derive a relatively simple solution for the wage.

The optimal wage offer proposed by each party ensures that the counterparty is indifferent between accepting the wage offer or rejecting and waiting until the next round to make a counteroffer. As a result, the initial wage offer will be accepted. This gives the following optimal rule for proposing  $w^f$  and  $w^w$ ,

$$L_t^f = b\sigma + e^{-r\sigma} [(1 - e^{-\delta\sigma})U_t + e^{-\delta\sigma} L_t^w] \tag{8}$$

$$J_t^w = -\gamma_t\sigma + e^{-(r+\delta)\sigma} J_t^f. \tag{9}$$

Combining (8) and (9), using the fact that  $L_t^f + J_t^w = L_t^w + J_t^f = L_t + J_t$ , we obtain the following sharing rule:

$$L_t - \frac{b}{r + \delta} - \frac{\delta U_t}{r + \delta} = J_t + \frac{\gamma_t}{r + \delta}. \tag{10}$$

Equation (10) gives the optimal condition for the wage: the net value of a job match to the worker should be equal to the net value of a job match to the firm. For the worker, the net value of a job match is equal to the value of being employed net of the discounted value of nonwork and net of the discounted value of the outside option. For the firm, the net value of a job match is equal to the value of a filled vacancy plus the discounted cost of delay in wage negotiation.

Using the value functions (1) and (3) to replace the left-hand sides of (8) and (9) and then combining (8) and (9), we have

$$w_t^f + w_t^w + e^{-r} [(1 - \tau)E_t L_{t+1} + \tau E_t U_{t+1}] - \frac{b}{r + \delta} - \frac{\delta U_t}{r + \delta} = y_t + e^{-r} (1 - \tau) E_t J_{t+1} + \frac{\gamma_t}{r + \delta}. \tag{11}$$

Following Hall (2017), we assume that the wage is the average  $1/2(w_t^f + w_t^w)$ . Substituting (10) into (11), we obtain a solution for the wage:

$$w_t = \frac{y_t}{2} + \frac{(r + \tau)(b + \gamma_t)}{2(r + \delta)(1 + r)} + \frac{\delta U_t}{2(r + \delta)} - \frac{(\tau r + \delta)E_t U_{t+1}}{2(r + \delta)(1 + r)}. \tag{12}$$

Wage equation (12) contains three cyclical components: the labor productivity  $y_t$ , the cost of delay  $\gamma_t$ , and the worker’s value of outside option  $U_t$ .

#### 4. COMPARISON WITH NASH SHARING RULE

It is useful to compare sharing rule (10) with Nash sharing rule. The wage under Nash bargaining should satisfy the following sharing rule:

$$\phi J_t = (1 - \phi)(L_t - U_t),$$

where  $\phi$  is the worker’s bargaining power. There is a major difference between the two sharing rules. Under the Nash bargaining, a job-matching surplus is  $J_t + L_t - U_t$ . Whereas in the credible bargaining, since the bargainer’s main threatening point is switched from the outside option payoff to the disagreement payoff, a job-matching surplus is written as  $J_t + L_t + \frac{\gamma_t}{r + \delta} - \frac{b}{r + \delta} - \frac{\delta U_t}{r + \delta}$ . Costs of delay to both parties enter the surplus, and the impact of outside option payoff on the wage now depends on the probability that the wage negotiation breaks down. Hall and Milgrom (2008) assume that costs of delay to both parties are fixed through business cycle, and they calibrate an extremely low value to the probability that the wage negotiation breaks down so that the impact of outside option payoff on matching surplus is rather limited. In doing so, they reduce the flexibility of job-matching surplus. In the next section, we show that when a pro-cyclical cost of delay to the firm is considered, the wage delivers substantial flexibility even if we fix the cost of delay to the worker and calibrate a low weight on  $U$ .

#### 5. THE COST OF IDLE CAPITAL

Before turning to the quantitative assessment, we present some evidence on the size and the cyclicity of the cost of idle capital. We assume that the capital cost of creating a vacancy is  $k_t$ . The production function is  $f(1, k_t) = A_t^\varepsilon k_t^{1-\varepsilon}$ , where  $A_t$  is the labor-augmenting productivity. As Pissarides (2000) points out, incorporating the capital into the standard Diamond-Mortensen-Pissarides model does not change any of the equations in the model and leads to only a reinterpretation of the productivity process. In such case, the labor productivity can be written as  $y_t = A_t^\varepsilon k_t^{1-\varepsilon} - (r + d)k_t$ , where  $r$  is the interest rate of renting the capital and  $d$  is the capital depreciation rate. The firm’s cost of idle capital is  $(r + d)k_t$ .

For simplicity, we assume that  $r$  and  $d$  are unresponsive to a labor productivity change. Thus, the cyclicity of the cost of idle capital solely depends on  $k_t$ . We assume that firms can buy and sell capital in a competitive market, and the optimal amount of capital per vacancy satisfies the equilibrium condition

$$\text{Marginal Product of Capital} = (1 - \varepsilon)A_t^\varepsilon k_t^{-\varepsilon} = r + d. \tag{13}$$

Rearranging (13), we obtain

$$k_t = \left[ \frac{(1 - \varepsilon)A_t^\varepsilon}{r + d} \right]^{\frac{1}{\varepsilon}}. \tag{14}$$

Equation (14) indicates that  $y_t$  is still an exogenous process. This is important since it ensures that the wage bargain is not affected by the presence of capital. Differentiating  $k_t$  with respect to  $A_t$ , we obtain

$$\eta_{k_t, A_t} = \frac{dk_t}{dA_t} \frac{A_t}{k_t} = 1. \tag{15}$$

Using (15), we derive the elasticity of capital per vacancy with respect to the labor productivity,

$$\eta_{k_t, y_t} = \frac{\eta_{k_t, A_t}}{\eta_{y_t, A_t}} = 1. \tag{16}$$

Given (16), we now can assume  $(r + d)k_t = \gamma y_t$ <sup>5</sup>.

To identify the size of the cost of idle capital relative to the labor productivity (namely  $\gamma$ ), we now compute the steady-state value for capital per vacancy. Typical estimates from the national accounts imply a capital income share  $1 - \varepsilon = \frac{1}{3}$ . We assume that the monthly interest rate is 0.00417 (equivalent to 5% of yearly interest rate) and the monthly capital depreciation rate is 0.00833 (equivalent to 10% of yearly depreciation rate). The value for  $A$  is chosen to ensure the steady-state value for labor productivity equals 1. Using equation (14), we obtain  $k = 23.73$ . Thus, the cost of idle capital is  $(0.00417 + 0.00833) \times 23.73 = 0.30$ . So the steady-state cost of idle capital equals 30% of the average labor productivity. Our estimate is smaller than Hagedorn and Manovskii’s estimate (0.474) but larger than Hall and Milgrom’s calibration (0.27).

## 6. SIMULATIONS

In this section, we present two sets of simulation results, allowing the cost of delay to the firm to be fixed ( $\gamma$ ) and then to be pro-cyclical ( $\gamma y_t$ ). Because the cyclicity of the cost of delay does not affect the equilibrium wage and equilibrium unemployment, two simulations have same steady state. We choose the calibration strategy to let the simulation with a fixed cost of delay match the observed volatility data. Then, we use the same calibration strategy to run the simulation with a pro-cyclical cost of delay.

### 6.1. Calibration

Our calibration strategy is slightly different to Hall and Milgrom (2008). Unlike Hall and Milgrom (2008) which solely focuses on the volatility of unemployment, we let the model with a fixed cost match the volatility of four labor market variables. They are unemployment, job vacancies, market tightness and job finding

**TABLE 1.** Values of calibrated parameters

| Parameter | Interpretation                      | Value   |
|-----------|-------------------------------------|---------|
| $\tau$    | Job separation rate                 | 0.0333  |
| $r$       | Monthly interest rate               | 0.00417 |
| $\beta$   | Discount factor                     | 0.99583 |
| $y$       | Labor productivity                  | 1       |
| $b$       | Flow value of nonwork               | 0.68    |
| $c$       | Vacancy cost                        | 0.313   |
| $\gamma$  | Cost of delay to employer           | 0.3     |
| $m$       | Matching coefficient                | 0.77    |
| $\alpha$  | Matching elasticity                 | 0.4     |
| $\delta$  | Probability of bargaining breakdown | 0.0366  |

**TABLE 2.** Values of endogenous variables for calibration

| Parameter | Interpretation         | Data  | Model |
|-----------|------------------------|-------|-------|
| $u$       | Unemployment rate      | 0.055 | 0.055 |
| $\theta$  | Labor market tightness | 0.610 | 0.610 |
| $f$       | Job finding rate       | 0.570 | 0.570 |

rate. We normalize a time period to be one month. Our calibrated parameter values are outlined in Table 1. The monthly real discount rate is set as  $r = 0.417\%$ . The average job separation rate is  $\tau = 0.0333$ . So on average 3.33% of employed workers exit employment every month. We target the job finding rate,  $f(\theta)$ , to be 0.57.<sup>6</sup> These imply the equilibrium unemployment rate of 5.5%, the average post-war U.S unemployment rate. The labor productivity is set equal to one in the stationary equilibrium.

We set the elasticity of the job finding rate with respect to the labor market tightness ratio as  $\alpha = 0.4$ . Petrongolo and Pissarides (2001) find that the range of plausible estimates of  $\alpha$  is 0.3 and 0.5. We target the vacancy/unemployment ratio,  $\theta$ , to be 0.61, the average of the estimate made by Hall and Milgrom (2008) and Pissarides (2009). This requires that the coefficient of matching efficiency is set as  $m = f(\theta)/\theta^\alpha = 0.57/0.61^{0.5} = 0.77$ . The values of targeted variables are shown in Table 2.

We set the cost of delay to the firm in the steady state as  $\gamma = 0.30$  according to our estimate. This is different with Hall and Milgrom (2008) as they choose  $\gamma$  to match the average level of unemployment. To retrieve the freedom to match our calibration targets in Table 2, we choose the vacancy cost,  $c$ , as a free parameter.

We set the opportunity cost of employment as  $b = 0.68$ . This is slightly lower than Hall and Milgrom's calibration but lies in the range of empirical estimates of  $b$ , between 0.47 and 0.96, done by Chodorow-Reich and Karabarbounis (2016). 0.68 is close to the average of their estimate. We find that there is no consensus on

**TABLE 3.** Shimer’s summary statistics, quarterly US data, 1951–2003

|                           | <i>u</i> | <i>v</i> | $\theta$ | <i>f</i> | <i>y</i> |        |
|---------------------------|----------|----------|----------|----------|----------|--------|
| Standard deviation        | 0.190    | 0.202    | 0.382    | 0.118    | 0.020    |        |
| Quarterly autocorrelation | 0.936    | 0.940    | 0.941    | 0.908    | 0.878    |        |
| Correlation matrix        | <i>u</i> | 1        | −0.894   | −0.971   | −0.949   | −0.408 |
|                           | <i>v</i> | –        | 1        | 0.975    | 0.897    | 0.364  |
|                           | $\theta$ | –        | –        | 1        | 0.948    | 0.396  |
|                           | <i>f</i> | –        | –        | –        | 1        | 0.396  |
|                           | <i>y</i> | –        | –        | –        | –        | 1      |

Notes: All variables reported are log deviations from an Hodrick-Prescott trend with smoothing parameter  $10^5$ .

**TABLE 4.** Constant cost of delay to employer in the credible bargaining

|                           | <i>u</i> | <i>v</i> | $\theta$ | <i>f</i> | <i>y</i> |        |
|---------------------------|----------|----------|----------|----------|----------|--------|
| Standard deviation        | 0.128    | 0.140    | 0.252    | 0.151    | 0.020    |        |
| Quarterly autocorrelation | 0.970    | 0.737    | 0.878    | 0.878    | 0.878    |        |
| Correlation matrix        | <i>u</i> | 1        | −0.776   | −0.937   | −0.937   | −0.937 |
|                           | <i>v</i> | –        | 1        | 0.948    | 0.948    | 0.948  |
|                           | $\theta$ | –        | –        | 1        | 1        | 1      |
|                           | <i>f</i> | –        | –        | –        | 1        | 1      |
|                           | <i>y</i> | –        | –        | –        | –        | 1      |

calibrating the vacancy cost in the literature. We choose the vacancy cost to match the average labor market tightness. In doing so, we set  $c = 0.313$ . This value lies in the range of 0.213 in Shimer (2005a) to 0.58 in Hagedorn and Manovskii (2008). Following Hall and Milgrom (2008), we choose the probability that bargaining breaks down,  $\delta$ , to match the observed volatilities of labor market variables by matching wage flexibility. Doing so, we set  $\delta = 0.0366$ .

For the processes driving productivity shocks, we assume  $\rho^s = 0.878$  for the autoregressive component and  $\sigma_s = 0.01$  for the volatilities of the underlying shocks. These values generate shocks that match the autocorrelations and standard deviations of the labor productivity in the US data for 1951–2003 reported in Shimer (2005a).

### 6.2. Volatilities

We use the empirical statistics in Shimer (2005a) as our simulation targets and report them in Table 3. Table 4 describes the simulation results when the cost of delay to the firm is fixed. Under this assumption, the model is able to match 68% of the standard deviation of the labor market variables observed in the data.<sup>7</sup> Our simulation can be seen as a robustness check to Hall and Milgrom (2008) as



**TABLE 5.** Cyclical cost of delay to employer in the credible bargaining

|                           | <i>u</i> | <i>v</i> | $\theta$ | <i>f</i> | <i>y</i> |        |
|---------------------------|----------|----------|----------|----------|----------|--------|
| Standard deviation        | 0.020    | 0.025    | 0.044    | 0.027    | 0.020    |        |
| Quarterly autocorrelation | 0.970    | 0.737    | 0.878    | 0.878    | 0.878    |        |
| Correlation matrix        | <i>u</i> | 1        | -0.776   | -0.937   | 0.937    | -0.937 |
|                           | <i>v</i> | -        | 1        | 0.948    | 0.948    | 0.948  |
|                           | $\theta$ | -        | -        | 1        | 1        | 1      |
|                           | <i>f</i> | -        | -        | -        | 1        | 1      |
|                           | <i>y</i> | -        | -        | -        | -        | 1      |

**TABLE 6.** Wage flexibility

|                                     | A Constant cost of delay | A Cyclical cost of delay |
|-------------------------------------|--------------------------|--------------------------|
|                                     | $\gamma$                 | $\gamma y_t$             |
| Standard deviation of the wage      | 0.0159                   | 0.0202                   |
| Elasticity of the wage $\eta_{w,y}$ | 0.7608                   | 0.9665                   |

we adopt a different calibration strategy and a different simulation approach. Our simulation confirms the credible wage bargaining as a prominent response to the unemployment volatility puzzle when the cost of delay to the firm is fixed.<sup>8</sup>

Table 5 describes the simulation results when the cost of delay to the firm is pro-cyclical. Not surprisingly, the credible bargaining model produces too little volatility of unemployment and vacancies from realistic fluctuations in productivity. Comparing with the simulation results in Table 4, the simulated volatility of unemployment, vacancies, labor market tightness, and job finding rate decreases by approximately 85%. A massive decline of the simulated volatilities reveals the fragility of the credible bargaining model as a solution to the unemployment volatility puzzle.

## 7. ANALYSIS

### 7.1. Wage Flexibility

Our results have strong implications for wage flexibility. Table 6 shows that the elasticity of the wage with respect to the labor productivity is close to 0.97 when the cost of idle capital is pro-cyclical, higher than the wage elasticity under a fixed cost of idle capital by approximately 25%.

To understand the importance of the cost of idle capital in the wage elasticity, we consider the steady-state wage equation. If we assume  $\tau = \delta$ , the wage can be written as

$$w = \frac{1}{2}y + \frac{1}{2(1+r)}(b + \gamma y). \tag{17}$$

**TABLE 7.** The standard deviations of the wage and unemployment under the different levels of  $\eta_{r,y}$

| Elasticity of $\gamma$ | SD of the wage | SD of unemployment |
|------------------------|----------------|--------------------|
| 0.1                    | 0.0163         | 0.1171             |
| 0.2                    | 0.0167         | 0.1065             |
| 0.3                    | 0.0172         | 0.0960             |
| 0.4                    | 0.0176         | 0.0855             |
| 0.5                    | 0.0180         | 0.0750             |
| 0.6                    | 0.0185         | 0.0645             |
| 0.7                    | 0.0189         | 0.0540             |
| 0.8                    | 0.0194         | 0.0435             |
| 0.9                    | 0.0198         | 0.0330             |
| 1.0                    | 0.0202         | 0.0224             |

Thus, the increase in the wage elasticity due to a pro-cyclical cost of idle capital can be measured by

$$\eta_{w,y}|_{pro-cyclical\gamma} - \eta_{w,y}|_{fixed\gamma} = \frac{y}{2(1+r)w}\gamma. \tag{18}$$

Equation (18) provides a lower bound of the estimate of the increase in the wage elasticity as it ignores the impact of  $\gamma$  on the wage via the value of the worker’s outside option. In a standard calibration, the value of  $\frac{y}{(1+r)w}$  is normally above 1. Therefore, the increase in the wage elasticity should be no less than  $\frac{\gamma}{2}$ .

Table 7 reports the standard deviations of the wage and unemployment under the different levels of the elasticity of the cost of idle capital with respect to the labor productivity. For every 0.1 unit increase in the elasticity of the cost of idle capital, the standard deviation of unemployment declines by approximately 0.01 unit, due to a small rise in the wage elasticity. Those results demonstrate that even with a moderate elasticity of the cost of idle capital, the credible bargaining model is unable to generate a large volatility of unemployment.

### 7.2. Market Tightness Elasticity

The comparison between two sets of simulation results reveals that a small increase in wage elasticity can cause a large decrease in the simulated volatility of unemployment, vacancies, labor market tightness, and job finding rate. We explain this by considering the impact of wage elasticity on labor market tightness. Using the labor market free-entry condition, the elasticity of market tightness with respect to labor productivity is written as

$$\eta_{\theta,y} = [1 - (1 - \pi)\eta_{w,y}]\frac{1}{\alpha\pi}, \tag{19}$$

where  $\pi = (y - w)/y$  is the profit ratio.<sup>9</sup> Since two simulations share the steady states, the difference in  $\eta_{\theta,y}$  is

$$\Delta \eta_{\theta,y} = -\frac{w}{\alpha(y-w)} \Delta \eta_{w,y}. \quad (20)$$

The multiplier  $\frac{w}{\alpha(y-w)}$  in (20) shows that the impact of real wage rigidity on the market tightness elasticity is amplified by the size of firm's profits.<sup>10</sup> Real wage rigidity ensures that vacancy creation varies with productivity. Small profits can lead to vacancy variation transferring strongly into unemployment variation. When the firm's profits are small, firms put relatively few resources into recruiting, leading to a relatively low level of vacancies and a relatively high level of vacancy-filling rate. For any vacancy variation, a low level of vacancies is associated with a large percentage change of vacancies. A high vacancy-filling rate implies that any variations in vacancies response to productivity shocks are transmitted strongly into variations in unemployment, leading to a large percentage change of unemployment.

## 8. CONCLUSION

This paper shows the fragility of credible wage bargaining as a solution to the unemployment volatility puzzle. When the firm faces a pro-cyclical cost of delay, the credible bargaining model is unable to match the observed fluctuations in the labor market. Those results can be obtained even if we assume a fixed opportunity cost of employment and calibrate a small probability that the bargaining breaks down. This is because a variable cost of delay to the firm can deliver substantial wage flexibility. Hence, the wage is too volatile and firm's profits are less responsive to the productivity shock. This leads to the limited fluctuations of vacancies and unemployment. Recent studies suggest that the labor market volatility might be driven by other forces, such as the discount rates in the financial market, for example, Hall (2017). It is worth to note that regardless of the driving force behind the labor market volatility, a certain level of wage rigidity is essential for the search frictions model to match the data. The credible bargaining model can deliver this only under questionable assumptions about the disagreement payoffs. So how to model the wage formation is still an unsolved question to the macroeconomists.<sup>11</sup>

## NOTES

1. See also Christiano et al. (2016) who incorporate the credible wage bargaining into the New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with search frictions in the labor market.

2. It is worth to note that the literature has developed other approaches to address the volatility puzzle. Some papers are based on Calvo type wage rigidities, for example, Gertler et al. (2008) and Faccini et al. (2013). Some papers argue that job destruction shocks play a prominent role in generating unemployment volatility, for example, Fujita and Ramey (2009), Pizzinelli et al. (2018). Some

papers emphasize the match-specific productivity shock, for example, Costain and Reiter (2008), and others explore the influence of labor market institutions on aggregate fluctuations, for example, Zanetti (2011a,b).

3. Recently, Chodorow-Reich and Karabarbounis (2016) present evidence that the disagreement payoff to workers ( $b$ ) is in fact strongly pro-cyclical and argue that as a consequence existing explanations of the unemployment volatility puzzle are inadequate. To stress the importance of the cost of delay to the firm on the wage cyclicality, our paper assumes  $b$  to be constant.

4. For the worker, the outside option is unemployment, which has value  $U$ . For the firm, the outside option is to quit the labor market or open a new job vacancy. In equilibrium, those two options have the same value, which is zero.

5. Measuring the cyclicality of the cost of idle capital is a challenging work since there are no data on the capital associated with new vacancies. We construct an indirect measurement by using the data on capital stock at constant national prices for US for the time period from 1950 to 2017. Letting it be divided by total nonfarm payrolls, we obtain the capital stock per employee. After taking the first difference of its logarithm, we find that the standard deviation of the capital stock per employee is about 1.77 times as large as the standard deviation of labor productivity.

6. This is close to Shimer (2005b)'s estimation.

7. According to Hall and Milgrom's estimate, the standard deviation of unemployment driven by productivity is 0.68 percentage points. We see 0.68 percentage points as an upper bound of the estimate as some papers argue that this percentage could be lower, for example, see Balleer (2012).

8. Table 4 reveals a shortcoming of the search frictions model with credible bargaining. The correlation of labor market variables and labor productivity is too high compared to the data. The standard search frictions model suffers the similar problem, see Hagedorn and Manovskii (2008).

9. See Costain and Reiter (2008) for a similar approach.

10. See Mortensen and Nagypal (2007), Zanetti (2011a), and Ljungqvist and Sargent (2017) for the similar arguments.

11. See Martin and Wang (2018) for a recent development of modeling endogenous real wage rigidity.

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