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A Line of Sight Counteraction Navigation Algorithm for Ship Encounter Collision Avoidance

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A new navigation method, called a Line of Sight Counteraction Navigation (LOSCAN) algorithm has been introduced to aid manoeuvre decision making for collision avoidance based on a two-ship encounter. The LOSCAN algorithm is derived from an extension of the basic principle of traditional missile proportional navigation, recognising that the objective of the latter is target capture rather than target avoidance. The basic concept is to derive an acceleration command so as to increase the misalignment between the ships' relative velocity and the line-of-sight. The algorithm includes a risk assessment and the generation of appropriate navigation commands to manoeuvre own ship free of collision if a risk of collision exists. Numerical examples have been used to demonstrate the effectiveness of the algorithm. The relationship between the distance at the closest point of approach with respect to early warning distance, and with the norm of the acceleration, has also been analysed. In operation, the collision avoidance decision making process is a complicated problem, with its solution subject to ship states, practical dynamic constraints, Collision Avoidance Regulations (COLREGS), encountering ship manoeuvre coordination and human decision making factors. The proposed algorithm provides a consistent manoeuvre signal to aid decision-making.

KEY WORDS

1. Marine Navigation. 2. COLREGS. 3. Automation. 4. Safety.

1. INTRODUCTION. The efficient use of collision avoidance manoeuvres when in the vicinity of other vessels is of vital importance for marine safety, especially in confined and crowded waters. In practice, the solution to the problem is a set of worldwide agreed procedures, which have been evolved from historic navigational practice, the Collision Avoidance Regulations (COLREGS). Efficient use of the COLREGS requires a highly trained and experienced officer in charge of the ship manoeuvres when facing a potential collision situation (Smeaton, 1990; Zhao, 1996). In the 1960s, a series of journal papers were published by Calvert and Hollingdale (Calvert, 1960; Calvert, 1969; Hollingdale, 1964; Hollingdale, 1968), which have provided insights into ship collision avoidance problems based on intuitive geometrical case studies. These papers proposed some operational solutions to improve the COLREGS (Calvert, 1960), and emphasised the importance of the radar

display and accurate positioning system's to reduce errors caused by human factors (Calvert, 1969). With the rapid advances in computer signal processing technology, modern control theory, accurate positioning and navigation systems, there is a growing interest in the development of an intelligent ship navigation system, which can provide either an enhanced navigation aid for mariners, or even automate the process of determining collision avoidance manoeuvres for ships.

By comparison to traditional ship navigation, missile engagement guidance was born in the electronic era. A proportional navigation method has proved to be a successful guidance technique for many years (Murtaugh, 1966; Duflos, 1999; Ghose, 1994; Dhar, 1993). Whilst the problem in missile engagement navigation is to attempt to hit rather than avoid an unpredictable and moving dynamic target, exploitation of the basic theory of proportional navigation can provide valuable insights for the development of automated collision avoidance for ships.

A qualitative and quantitative analysis of the necessary conditions for collision is important for risk assessment as well as for the generation of an appropriate navigation command to manoeuvre own ship free of collision when necessary. A simple method is initially introduced for collision risk assessment based on the basic concept of proportional navigation. A new navigation method, called a Line of Sight Counteraction Navigation (LOSCAN) algorithm, is then introduced for the two-ship encounter collision avoidance problem. The derivation of the LOSCAN algorithm is based on an extension and revision of the basic principle of traditional proportional navigation.

The line-of-sight (LOS) between a missile and its target is a basic parameter used in proportional navigation for homing guidance, where an acceleration command is applied to the missile such that the relative velocity between missile and the target is aligned with the LOS, and consequently the rotations rate of the LOS is stabilised. In contrast, the algorithm discussed in this paper aims to avoid a collision, by ensuring that the relative velocity between two ships is not aligned with the LOS. The basic principle is to derive an acceleration command so as to increase the misalignment between two ships' relative velocity and LOS. To achieve this, an acceleration command should be applied to own ship, which is normal to the LOS and the sign of which is counteractive to the sign of the derivative of the LOS rotation rate. The process of achieving this we have called the LOSCAN algorithm. Numerical examples are used later to demonstrate the effectiveness of the algorithm. The relationship of the distance at the closest point of approach (DCPA) with early warning distance, and with the amplitude of the acceleration, has also been analysed with appropriate numerical simulations.

In operation, decision making for collision avoidance manoeuvres is a complicated problem, with its solution subject to ship states, practical dynamical constraints, the COLREGS, encountering ship manoeuvre coordination and human decision making factors. The proposed algorithm is an intelligent information synthesis process based on information from radar and the positioning and computing systems. It can provide consistent manoeuvring information as an aid to decision making; for example, the algorithm can be applied in an onboard simulator to predict the consequences of proposed manoeuvres. The algorithm can also be applied to both ships in order to evaluate and determine which could best achieve the collision avoidance manoeuvre, based on the evaluation of the manoeuvrability of each ship. The LOSCAN's role in collision avoidance decision-making is illustrated in Figure 1. Incorporation of the

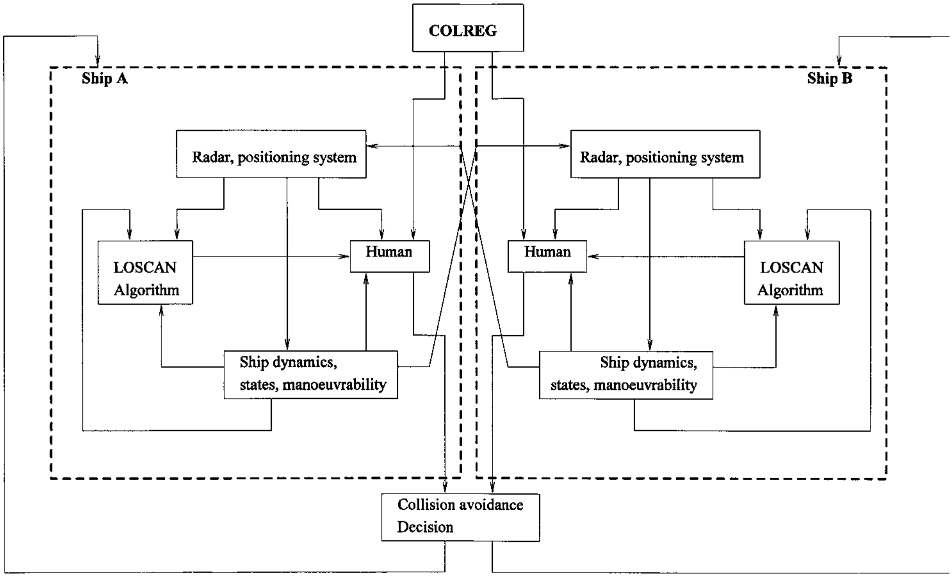

Figure 1. An illustration of LOSCAN as an aid to decision making.

algorithm into the conventional decision making process will be a subject of further research.

2. PROBLEM FORMULATION.

Consider the problem of manoeuvring an own ship (OS) toward a waypoint while encountering a target ship (TS) with which there is a potential collision risk. The starting point of the problem is to assess the risk of collision with the target ship and to determine whether a collision avoidance manoeuvre is required. If a collision avoidance manoeuvre is required, it will be calculated using a certain navigation command law and applied to the own ship dynamics in order to alleviate and diminish the risk. This section presents some heuristic and quantitative analysis for both collision risk assessment and collision avoidance navigation, of which the majority is based on a revision and extension of proportional navigation in the missile engagement problem. As already stated, analysis of the necessary conditions for collision is important for risk assessment as well as for the generation of appropriate navigation commands to avoid collision. For this purpose, the necessary conditions for collision are analysed first.

Consider the movement of two ships on the Earth's surface using a fixed Cartesian coordinate system XY, where it is assumed that the velocity vector of the target ship $\mathbf{v}_{TS}$ is constant relative to the Earth-fixed coordinate, and both ships' positions and velocities are perfectly known at each time t. The two-ship encounter geometry is plotted in Figure 2. The velocity vector of OS is denoted as $\mathbf{v}_{OS}(t)$. The relative velocity of TS to OS is denoted as:

$$\mathbf{v}_R(t) = \mathbf{v}_{TS} - \mathbf{v}_{OS}(t). \tag{1}$$

The vector from TS to OS, denoted as $\mathbf{r}(t)$, is called the line-of-sight (LOS). The norm of $\mathbf{r}(t)$, denoted as $\mathrm{r}(t)$, is the range between the two ships. The course of the target ship speed $\mathbf{v}_{TS}$ in earth-fixed coordinates as η_0, and $\eta(t)$ is denoted as the angle between $\mathbf{v}_{TS}$

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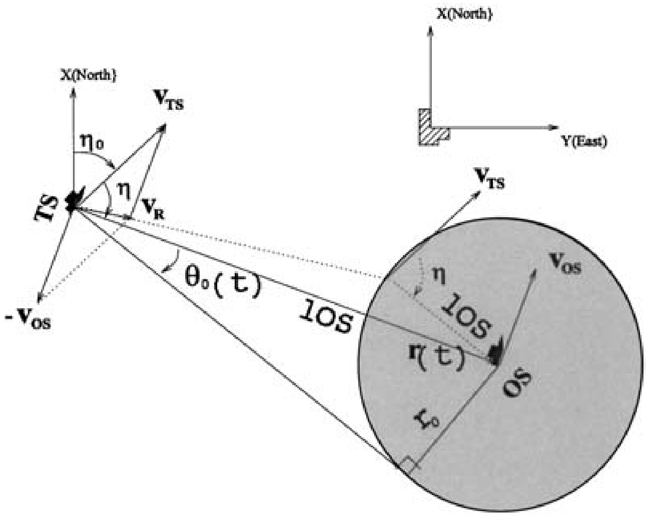

Figure 2. Two-ship encounter geometry.

and the LOS. Since $\mathbf{v}_{TS}$ is assumed to be a constant vector, the angular velocity $\dot{\eta}(t)$ can be regarded as the angular velocity of the orientation of the LOS in an Earth-fixed coordinate system. If $\mathbf{v}_{OS}(t)$ is a constant vector, the necessary conditions for a two-ship collision are that the relative velocity of TS to OS, $\mathbf{v}_R$, is aligned with the LOS, such that the angle $\dot{\eta}(t)$ remains a constant, and the rotation speed of the LOS, $\dot{\eta}(t)$ equals zero.

Suppose that at an initial time, the relative velocity of TS to OS, $\mathbf{v}_R$, is not aligned with the LOS, then the principle of proportional navigation in the missile engagement problem (Murtaugh, 1966) can be extended here as the necessary conditions for collision, which is, an acceleration command is applied to adjust $\mathbf{v}_{OS}(t)$ so as to reduce the discrepancy in alignment. For instance, an acceleration command normal to the LOS and proportional to $\dot{\eta}(t)$, the rotation rate of the LOS, such that $\dot{\eta}(t)$ is reduced to zero at the collision time, would suffice. Thus the three necessary conditions for two-ship encounter collision are:

(a) the range between the two ships is smaller than a finite range r_f, called the warning distance;
(b) the direction of the relative velocity of target ship to own ship $\mathbf{v}_R$ is aligned with direction of $\mathbf{r}(t)$, the LOS from TS to OS;
(c) the turning rate of the relative velocity $\mathbf{v}_R$ equals $\dot{\eta}(t)$, the turning rate of $\mathbf{r}(t)$, the LOS between the two ships. That is, the acceleration of the relative velocity between the two ships is proportional to the rotation rate $\dot{\eta}(t)$ of the LOS.

Note that condition (c) guarantees that condition (b) is consistently satisfied until the collision occurs.

For practical ship collision avoidance, a safety zone or area surrounding OS needs to be kept clear of the target ship. Suppose that the desired collision free region is a circle with radius r_0 around the centre of the ship. The collision free region can be used for practical collision risk assessment and navigation command generation. If the TS encroaches the safety region, then the collision risk assessment is positive. From Figure 2 it can be seen that, if the angle between the direction of the relative velocity of target ship to own ship $\mathbf{v}_R$ and direction of $\mathbf{r}(t)$, and the LOS from TS to OS remains less than $\theta_0(t)$ defined by:

$$\theta_0(t)=\arcsin\left(\frac{r_0}{r(t)}\right), \tag{2}$$

then the TS will encroach on the OS safety region.

2.1. *Lemma 1.* – *Necessary condition for collision.* Denote $g(t)=\dfrac{\langle \mathbf{r}(t), \mathbf{v}_R(t)\rangle}{r(t)\|\mathbf{v}_R(t)\|}$. Consider a finite period of time $t \in [1, N]$. The necessary conditions for target ship entering the collision free region are that $\exists t_1 \in [1,N]$, so that:

$$g(t)=\frac{\langle \mathbf{r}(t), \mathbf{v}_R(t)\rangle}{r(t)\|\mathbf{v}_R(t)\|} \in [\cos\theta_0(t), 1], \tag{3}$$

and

$$r(t)<r_f, \tag{4}$$

where: $\|\bullet\|$ is *Euclidean* norm and $\langle \bullet,\bullet \rangle$ denotes inner product.

3. LINE OF SIGHT COUNTERACTION NAVIGATION ALGORITHM (LOSCAN).

Lemma 1 will be used not only as the risk assessment criteria, but it will also be used in the derivation of a new navigation command law, called line-of-sight counteraction navigation. The LOSCAN algorithm includes this risk assessment and the generation of appropriate navigation commands to manoeuvre OS free of collision when such a risk exists. As stated earlier, the principle is to derive an acceleration command so as to increase the misalignment between the two ships' relative velocity and the LOS; to achieve this, an acceleration command can be applied to the OS, which is normal to the LOS and the sign of the which is counteractive to the sign of the derivative of the LOS rotation rate.

Define a TS fixed Cartesian coordinate **(i,j,k)** with **i** as the unit vector aligned with the LOS, $(\mathbf{i}=\mathbf{r}(t)/r(t))$. The equations of motion of the OS can be derived as,

$$\begin{aligned}
\mathbf{r}(t)&=r(t)\mathbf{i},\\
\frac{d\mathbf{r}(t)}{dt}&=\dot{r}(t)\mathbf{i}+r(t)\dot{\eta}(t)\mathbf{j},\\
\frac{d\mathbf{r}^2(t)}{dt^2}&=[\ddot{r}(t)-r(t)\dot{\eta}^2(t)]\mathbf{i}+[2\dot{r}(t)\dot{\eta}(t)+r(t)\ddot{\eta}(t)]\mathbf{j}.
\end{aligned} \tag{5}$$

As $\mathbf{v}_R(t)=-\dfrac{d\mathbf{r}(t)}{dt}$, and from (1), it can be shown that the acceleration command $\mathbf{a}_{os}(t)$ over the own ship (OS) is:

$$\mathbf{a}_{OS}(t)=\frac{d\mathbf{v}_{OS}(t)}{dt}=\frac{d^2\mathbf{r}(t)}{dt^2}. \tag{6}$$

Based on Lemma 1, it can be shown that a proper acceleration command can be applied to the collision avoidance problem. The objective of control here is to reduce $g(t)$ until $g(t)<\cos\theta_0$(t). Suppose the acceleration command $\mathbf{a}_{os}(t)$ is normal to the LOS, such that $\mathbf{a}_{os}(t)=a_{os}(t)\mathbf{j}$ then,

$$\begin{aligned}
\ddot{r}(t)-r(t)\dot{\eta}^2(t)&=0\\
2\dot{r}(t)\dot{\eta}(t)+r(t)\ddot{\eta}(t)&=a_{OS}(t)
\end{aligned} \tag{7}$$

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If no acceleration is applied, that is, $a_{os}(t)=0$, then using $\mathbf{v}_R(t)=-\frac{d\mathbf{r}(t)}{dt}$, and substituting Equation (5) into (3), the $g(t)$ without an acceleration command, denoted as $g_1(t)$, is,

$$g_1(t)=-\frac{\dot{r}(t)}{\sqrt{\dot{r}^2(t)+(r(t)\dot{\eta}(t))^2}}. \tag{8}$$

If $a_{os}(t)\neq 0$, denote the variation in $\mathbf{v}_R(t)$ as $\Delta\mathbf{v}_R(t)=T_s a_{os}(t)\mathbf{j}$, then the new $\mathbf{v}_R(t)$ (denoted as $\mathbf{v}'_R(t)$) is $\mathbf{v}'_R(t)=\mathbf{v}_R(t)+\Delta\mathbf{v}_R(t)=-r(t)\mathbf{i}-[r(t)\dot{\eta}(t)-\lambda a_{os}(t)]\mathbf{j}$, where: T_s is a positive constant denoting sampling rate, it can be derived from (3) that the $g(t)$ with an acceleration command, denoted as $g_2(t)$, is:

$$g_2(t)=-\frac{\dot{r}(t)}{\sqrt{\dot{r}^2(t)+(r(t)\dot{\eta}(t)-\lambda a_{os}(t))^2}}. \tag{9}$$

To ensure that $g_2(t)<g_1(t)$, it is therefore necessary to ensure that $(r(t)\dot{\eta}(t)-\lambda a_{OS}(t))^2 > (r(t)\dot{\eta}(t))^2$. This can be accomplished by setting,

$$\begin{aligned} a_{OS}(t)>0 \quad &\text{if} \quad \dot{\eta}(t)<0 \\ a_{OS}(t)<0 \quad &\text{if} \quad \dot{\eta}(t)>0 \end{aligned} \tag{10}$$

The navigation command given in Equation (10) is called the line-of-sight counteraction navigation (LOSCAN) law. It is interesting to note that, in proportional navigation for the missile engagement problem, the objective is to increase $g(t)$ until it reaches the maximum value 1 (the $\mathbf{v}_R$, is then aligned with the LOS), such that $(r(t)\dot{\eta}(t)-T_S a_{OS}(t))^2 \rightarrow 0$, yielding that the acceleration command is proportional to the LOS rotation rate $\dot{\eta}(t)$, and the $\dot{\eta}(t)$ is reduced to 0 at the collision time. From Equation (1), $\Delta\mathbf{v}_{OS}(t)=-\Delta\mathbf{v}_R(t)$, and denoting $\mathbf{v}_{OS}(t)=[v_{OSx}(t), v_{OSy}(t)]^T$ as the own ship velocity components in earth fixed coordinates, it can be shown that the:

$$\begin{vmatrix} \Delta v_{OSx}(t) \\ \Delta v_{OSy(t)} \end{vmatrix} = \begin{vmatrix} \cos(\eta_0+\eta(t)) - \sin(\eta_0+\eta(t) \\ \sin(\eta_0+\eta(t)) \cos(\eta_0+\eta(t) \end{vmatrix} \bullet \begin{vmatrix} 0 \\ -\lambda a_{OS}(t) \end{vmatrix}. \tag{11}$$

Equation (11) can be used to compute the desired OS velocity in earth fixed coordinates:

$$\mathbf{v}_{OS}(t+1)=\mathbf{v}_{OS}(t)+\Delta\mathbf{v}_{OS}(t). \tag{12}$$

It is noted that the collision avoidance manoeuvre algorithm is designed to reduce the risk of collision, and the success of the manoeuvre is dependent on predetermined system parameters, ship initial velocity and importantly manoeuvrability of the ships. One of the most widely used criteria in collision avoidance is the distance at closest point of approach (DCPA). The success of a collision avoidance manoeuvre can be determined if $DCPA>r_0$ is ultimately achieved. Some insights can be drawn from an analysis of the system performance with respect to the critical parameters, such as the warning distance r_f and the norm of the acceleration $\|a_{os}\|$. Firstly, the manoeuvre time is in practice limited by the warning distance r_f, as $\frac{dr(t)}{dt}<0, r(t)\rightarrow 0$ (if a collision risk exists). A large r_f makes it possible for the collision avoidance manoeuvre to be performed over a longer time, thus allowing the requirement on high manoeuvrability to be alleviated. Appropriate values of r_f for different encounter scenarios, which are dependent on the relative velocity of the two ships, can be obtained from offline

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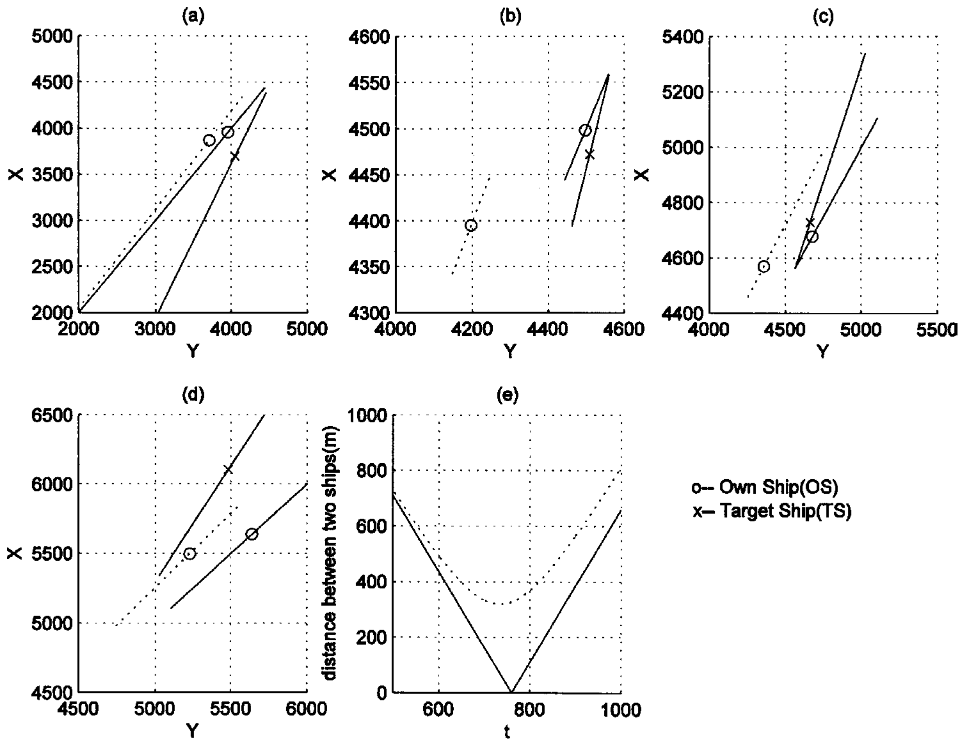

Figure 3. Numerical results in Example 1; (a) ship trajectory ($t=1\sim739$ s); (b) ship trajectory ($t=739\sim759$ s); (c) ship trajectory ($t=759\sim850$ s); (d) ship trajectory ($t=850\sim1000$ s); (e) distance between the two ships; (Dotted Line: with LOS counteraction navigation, and Solid Line: without LOS counteraction navigation).

computation. Secondly, in the case that r_f is fixed, a larger $\|a_{os}\|$ is needed to perform a faster collision avoidance manoeuvre. This means that a large $\|a_{os}\|$ is required if r_f is small. A direct and logical implication is that when two ships engage in a serious collision risk (r_f is small), the ship with greater manoeuvrability should perform the collision avoidance manoeuvre. The manoeuvrability of both ships differ due to their physical characteristics, initial velocity and encounter situation, but generally one ship will have greater manoeuvrability than the other. For instance, assume both ships' (A, B) speeds are in the direction of their headings; if LOS happens at right angles to the velocity of A, such that A has to make a positive acceleration on heading by increasing speed, this would be a difficult manoeuvre for most commercial ships. But this situation only occurs to ship A if it is slower than B. Meanwhile B's heading would be at a smaller angle to the LOS, so it can easily make larger accelerations normal to the LOS by altering course. This suggests that B should be OS, undertaking the collision avoidance manoeuvre.

Note that the direction of the navigation acceleration command normal to the LOS is constantly changing as the LOS rotates; in practice, this generally means that ship velocity and heading will be changing during the manoeuvre process, which is subject to the manoeuvrability of ship, therefore the norm of $\|a_{os}\|$ can be practically limited, suggesting that the early warning distance r_f should be set as large as possible.

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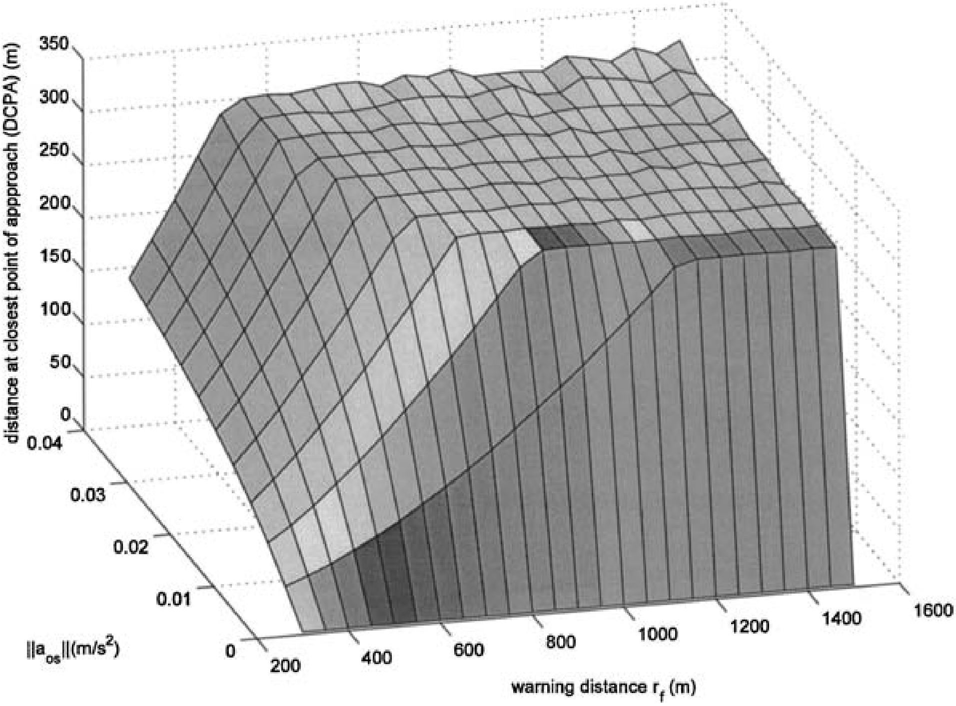

Figure 4. System performance study in Example 1 (DCPA as a function of warning distance r_f, and norm of the acceleration command $\|a_{os}\|$).

The two-ship encounter collision avoidance navigation algorithm can be simply summarised as: (i) suppose both ships are moving with a constant velocity with respect to the Earth-fixed coordinates, and both ships' positions and velocities are precisely known at each time instant t. Set a warning distance r_f; (ii) at each time t, the collision risk assessment is carried out using the necessary condition for collision (Equations (3) and (4) in Lemma 1). If a risk report exists, (iii) derive the norm of the acceleration command $\|a_{os}\|$ achievable by each ship based on the ship constraints of manoeuvrability (depends on encounter situations, regulations, ship parameters and velocity status), (iv) the largest $\|a_{os}\|$ will be set as the OS, the LOSCAN command $\|a_{os}\|$ (normal to the LOS and in the direction given in (10)) will be applied to the own ship. It is assumed that that there is understanding between two ships for the procedure to accomplish this task. If no collision risk is reported (i.e. necessary condition for collision is not satisfied), no manoeuvre applies to the own ship.

4. NUMERICAL EXAMPLES.

4.1. *Example 1.* Consider the movement of two ships with respect to Earth-fixed coordinates. The initial conditions (when $t=1$) in Earth coordinates for the target ship (TS) are position $[-1953{\cdot}2, 679{\cdot}3]^T(m)$ and velocity $[v_{TSx}, v_{TSy}]^T=[8{\cdot}59, 5{\cdot}12]^T$(m/s), and for own ship (OS) are position $[10, 10]^T$ and velocity $[v_{OSx}, v_{OSy}]^T=[6, 6]^T$. Set the risk free region around OS as a circle of radius $r_0=300$ m. The sampling rate $T_s=1$ s. If the two ships are moving with constant velocity with the above initial conditions, the ship trajectories and the distance between them can be calculated. The distance at the

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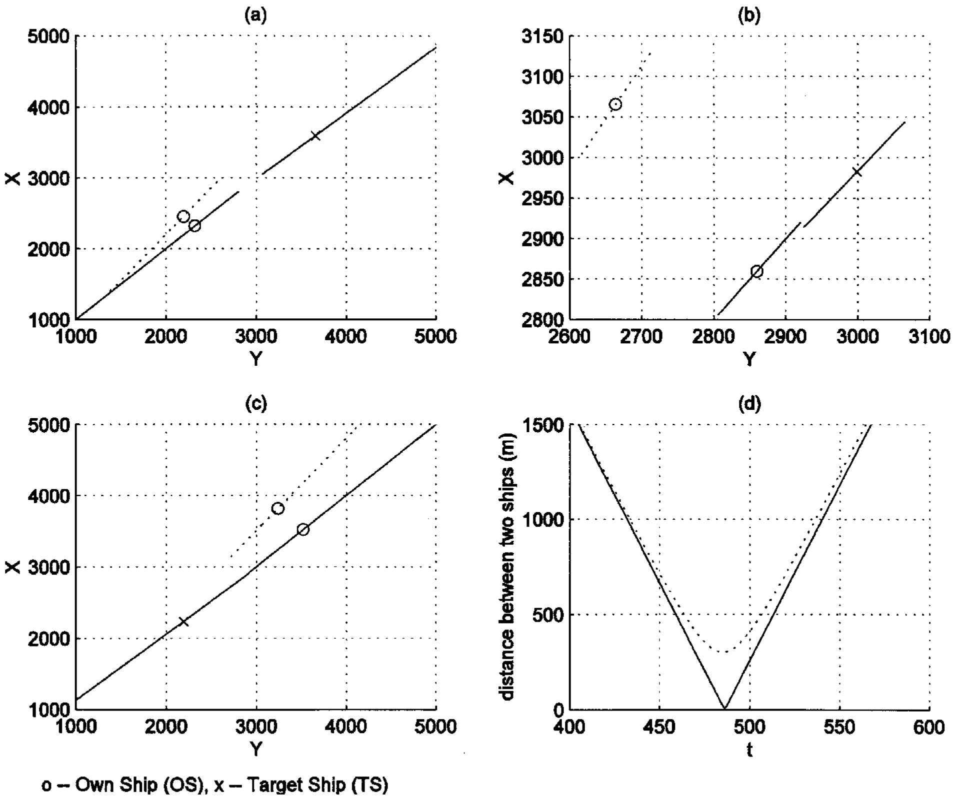

Figure 5. Numerical results in Example 2; (a) ship trajectory ($t=1\sim466$ s); (b) ship trajectory ($t=466\sim486$ s); (c) ship trajectory ($t=486\sim1000$ s); (d) distance between the two ships; (Dotted Line: with LOS counteraction navigation, and Solid Line: without LOS counteraction navigation).

closest point of approach (DCPA) is 2·54 m when $t=759$ s. This means that collision would occur if no collision manoeuvre were performed by either ship.

Initially set the warning distance $r_f = 1500$ m, and the norm of acceleration as $\|a_{os}\| = 0{\cdot}02$ m/s^2. Both ships' trajectories can be calculated for each time t while the collision risk is assessed using Lemma 1; (3) and (4). Unless there is a risk of collision between two ships (the necessary condition is satisfied), own ship maintains its velocity. If a collision risk is detected, the acceleration command with norm $\|a_{os}\| = 0{\cdot}02$ m/s^2 is applied normal to the LOS between two ships, and the sign is determined in accordance with (10).

The effectiveness of such collision avoidance navigation can be shown using a comparison of the ship trajectories, with and without collision avoidance, as plotted in Figure 3(a–d). Figure 3(e) demonstrates the distances between the two ships, with and without collision avoidance, indicating the manoeuvre is successful in avoiding the potential collision, since the DCPA with a manoeuvre (319·4 m $> r_0$) is reached when t = 734 s.

To provide a quantitative analysis of the system performance with respect to the warning distance r_f and the norm of the acceleration $\|a_{os}\|$, the DCPA is measured as a function of r_f and $\|a_{os}\|$. The simulation was carried out with the same initial conditions, but allowing the warning distance r_f to vary from 300 m to 1500 m, and the

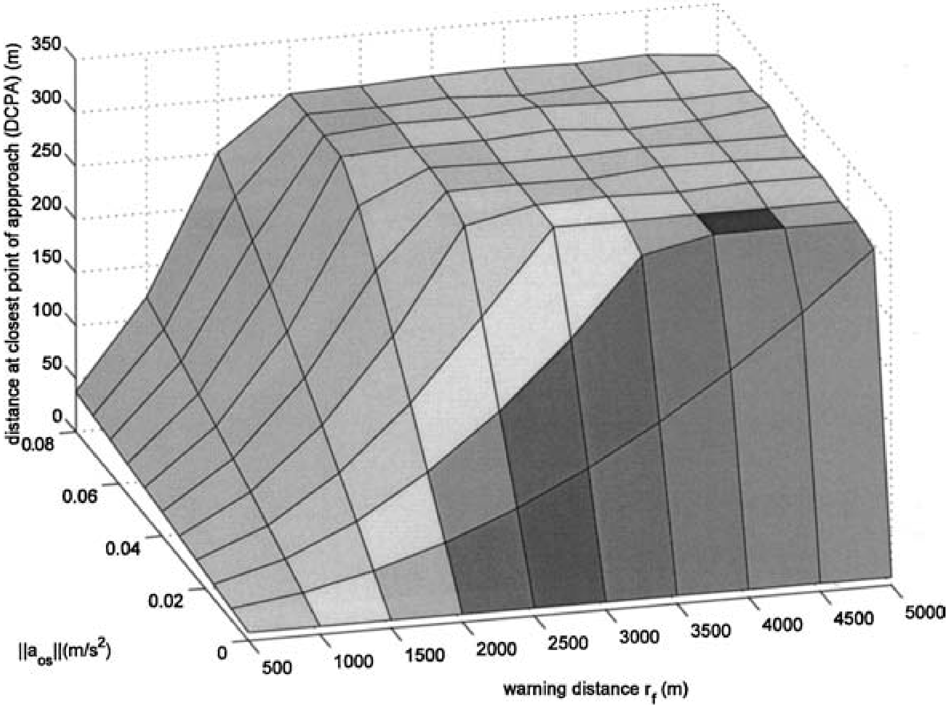

Figure 6. System performance study in Example 2 (DCPA as a function of warning distance r_f, and norm of the acceleration command $\|a_{os}\|$).

norm of the acceleration command $\|a_{os}\|$ to vary from 0 to 0·04 m/s². The results are shown at Figure 4, providing some visual insights, as illustrated in Section 3, on system performance with respect to the system parameters.

4.2. *Example 2.* Again consider the movement of the two ships with respect to Earth-fixed coordinates. The initial conditions (when $t=1$) for the target ship (TS) are position $[6208{\cdot}0, 6485{\cdot}5]^T(m)$ and velocity $[v_{TSx}, v_{TSy}]^T=[-6{\cdot}79, -7{\cdot}34]^T$(m/s), and for own ship (OS) position $[10, 10]^T(m)$ and velocity $[v_{OSx}, v_{OSy}]^T=[6, 6]^T$. Again set the risk free region around OS as the a circle of radius $r_0=300$ m and a sampling rate of $T_s=1$ s. The distance at the closest point of approach (DCPA) was 7·57 m when $t=486$ s. Again, this indicates that collision would occur if no collision manoeuvre were performed by either ship.

Initially set the warning distance r_f to 5000 m, and the norm of acceleration as $\|a_{os}\|=0{\cdot}04$ m/s². Both ships' trajectories are calculated for each time t as before and the collision risk is assessed. Unless there is a risk of collision, own ship maintains its velocity. If a collision risk is detected, the acceleration command with norm $\|a_{os}\|=0{\cdot}04$ m/s² is applied to own ship (OS) normal to the LOS between two ships, and the sign determined in accordance with (10). As before, the effectiveness of such collision avoidance navigation can be shown using a comparison of the ship trajectories with and without collision avoidance navigation, as plotted in Figure 5(a–c). Figure 5(d) demonstrates the distances between the two ships, with and without collision avoidance, indicating that manoeuvre is successful in avoiding the potential collision, since the DCPA with a manoeuvre (302·1 m $> r_0$) is reached when $t=485$ s.

Again, to provide a quantitative analysis of the system performance with respect to the warning distance r_f and the norm of the acceleration $\|a_{os}\|$, the distance at the closest point of approach (DCPA) is measured as a function of r_f and $\|a_{os}\|$. The simulation was carried out with the same initial conditions, but allowing the warning distance r_f to vary from 500 m to 5000 m, and the norm of the acceleration command $\|a_{os}\|$ to vary from 0 to 0·08 m/s². The results are shown in Figure 6.

Note that the numerical examples have shown that the proposed algorithms can be successfully used in collision avoidance navigation provided that the values of early warning distance and norm of the acceleration are appropriately chosen. These values can vary with the encounter situation, such as Example 1 for an overtaking encounter and Example 2 for an oncoming encounter. The analysis of the relationship between a distance at the closest point of approach (DCPA) with respect to early warning distance, and with the norm of the acceleration in Section 3 has been shown to be consistent using simulation results from Examples 1 and 2, despite the different encounter scenarios are involved.

5. CONCLUSIONS.

This paper presents a new navigation method, called a Line-of-Sight Counteraction Navigation (LOSCAN) algorithm for two-ship encounter collision avoidance. The algorithm includes risk assessment and the generation of an appropriate navigation command to manoeuvre own ship free of collision if a risk of collision exists. The principle is to derive an acceleration command that increases the misalignment between the two ships' relative velocity and the LOS between them. The algorithm is an extension of traditional proportional navigation used in missile guidance; whilst the objective of the latter is target capture rather than target avoidance, reverse logic has been used to develop an algorithm for two-ship encounter collision avoidance. Numerical examples have demonstrated the effectiveness of the algorithm. The relationship between a distance at the closest point of approach (DCPA) with respect to early warning distance, and with the norm of the acceleration have also been analysed using the numerical simulations. In operation, the proposed algorithm can be applied to assist collision avoidance manoeuvre decision-making.

REFERENCES

Calvert, E. S. (1960). Manoeuvres to ensure the avoidance of collision. This *Journal*, **13**, 2, pp. 127–137, April.

Calvert, E. S. (1969). Human factors and the collision problem. This *Journal*, **22**, 2, pp. 48–137, April.

Dhar, A. and Ghose, D. (1993). Capture region for a realistic TPN guidance law. *IEEE Tran. on Aerospace and Electronic Systems*, **29**, 3, pp. 995–1003, July.

Duflos, E., Penel, P. and Vanheeghe, P. (1999). 3D-guidance law modelling. *IEEE Tran. on Aerospace and Electronic Systems*, **35**, 1, pp. 72–83, January.

Ghose, D. (1994). On the generalisation of true proportional navigation. *IEEE Tran. on Aerospace and Electronic Systems*, **30**, 2, pp. 545–555, April.

Hollingdale, S. H. (1964). The effect of observational errors on the avoidance of collision at sea. This *Journal*, **17**, 4, pp. 345–357, April.

Hollingdale, S. H. (1968). The mathematics of collision avoidance in two dimensions. This *Journal*, **14**, 3, pp. 48–137, April.

Murtaugh, S. A. and Criel, H. E. (1966). Fundamentals of proportional navigation. *IEEE Spectrum*, **3**, pp. 75–85, December.

Smeaton, G. P. and Coenen, F. P. (1990). Developing an intelligent marine navigation system. *Computing and Control Engineering Journal*, pp. 95–103, March.

Zhao, J. (1996). *Maritime collision and liability*. PhD thesis, Dept of Ship Science, University of Southampton.