

BOOK REVIEW

Advanced Mathematics for Applications. By ANDREA PROSPERETTI. Cambridge University Press, New York 2011. 724 pages. ISBN: 978-0-521-51532-0 (Hardback), ISBN: 978-0-521-73587-2 (Paperback).

This is a really wonderful book. It offers an amazing variety of information, well condensed but nevertheless very clear and easily understandable. It can be used just for learning mathematics, its notions and insights, yet the advanced reader discovers new, unexpected material over and over again, becoming up to date in a broader field, learning special methods, or using the book as an excellent manual when solving problems and exercises.

The author has succeeded in creating a new, highly innovative kind of textbook. It is a ‘cross’ book, i.e. organized quite differently and almost orthogonal to common textbooks, setting a new stage at a time when the internet and symbolic computation programs, such as Mathematica or Maple, and numerical software such as MATLAB, seem to be making textbooks obsolete. This handy book is not meant for sequential reading and study. Rather, it is organized in a ‘modular’ fashion, each module in itself well ordered logically and mathematically, but the modules in turn following a different idea of order. (Here the notion ‘modular order’ is quite appropriate: the reviewer did not read sequentially either.) When using the book, the reader will jump back and forth between the modules, becoming absorbed by ever new aspects, methods, and applications. The author’s style is of the utmost precision, always very clear, amazingly condensed, without unnecessary ornaments. The density of information is high, but since the text is written in a brilliant style, the reading is great pleasure indeed. Hence, even though *Advanced Mathematics for Applications* is not meant as a systematic textbook, the reader is inspired to delve into well-defined subfields, easy to read but full of deep insights, associations, and applications.

Although not organized as a reference book, one can also use it as such, profiting from the masterly presentation. Many of us will (and should!) have ‘Prosperetti’ on their desks: beginners who want to learn; physicists for their everyday needs in research; knowledgeable scientists; teachers for their classes; students for solving practical problems as well as in search of deeper analytical insight. One can expect a rather broad audience coming from various sciences, including mathematics students.

But enough praise for the book and its author: let us have a brief review of its content. A short introductory section, Part 0, under the heading ‘General Remarks and Basic Concepts’, goes *medias in res*, dealing with classical fields and vector analysis, illustrated with the most important equations of mathematical physics, including fluid dynamics (Navier–Stokes equations), electrodynamics (Maxwell’s equations), elasticity, and quantum mechanics (the Schrödinger equation). This is followed by the presentation of common solution methods such as eigenfunction expansions, variational principles, Green’s functions, and of course the ‘ δ -function’, its meaning and proper use.

It is already the next set of modules, Part I, which is devoted to ‘Applications’. The five sub-modules cover Fourier series and transformations, the Laplace transform and its typical use, cylindrical and spherical systems, together with the respective special

functions for these geometries, and many applications. An extensive index will prove to be highly useful in daily scientific work.

The longest section, Part II, is the heart of the book. In eleven subsections it offers ‘Essential Tools’: deeper understanding of the main notions underlying the applications, more sophisticated details, various theorems, and sometimes the main ideas of their proofs. Readers who still want more are referred to Part III, entitled ‘Some Advanced Tools’. Here the audience is led in three sub-modules right through Hilbert space, spectral theory, the theory of operators of various kinds, test functions and distributions, including the precise background of the δ -distribution or ‘ δ -function’, etc. In addition and in closing, the Appendix contains short sections with reference material on sets, measures, functions, integration, curves, bounds, and limits.

Prosperetti’s unique book is a complex network which combines many different mathematical aspects and allows for multi-level use in various contexts. It can be recommended for individual readers as well as for smaller or larger groups who learn and study together. This is supported by a large collection of exercises which can be found via the internet for free use: see www.cambridge.org/prosperetti – a rich treasure trove of examples. Lecturers, take note!

This most valuable modern book hardly needs recommendation, since it captures the reader as soon as she or he starts reading. We shall soon find it in reference lists. The author deserves the highest praise for this wonderful intellectual gift to the community. It certainly involved a huge amount of work combined with an expert’s broad knowledge. Therefore it seems almost petty to point to imperfections – as reviewers usually do – such as the lack of multiple products in curvilinear coordinates or a few remarks on the historical background, etc. Of course, there were limitations to size (and time). Enjoy reading this exciting and enriching masterpiece!

Siegfried Grossmann