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ARTICLE

Bankruptcy costs, idiosyncratic risk, and long-run growth†

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Abstract

This paper analyzes how idiosyncratic risk, measured by the variance of firm-level idiosyncratic shocks, affects long-run growth when bankruptcy costs are present. These costs are incurred by creditors during the bankruptcy procedure of failing firms. In an endogenous growth model with bankruptcy costs where firms privately observe the outcome of idiosyncratic shocks, an increase in idiosyncratic risk reduces long-run growth. This happens because, when bankruptcy costs are present, higher idiosyncratic risk enlarges the wedge between the rental rate of capital and its marginal product, thereby slowing down capital accumulation. This growth-reducing effect of idiosyncratic risk is stronger when bankruptcy costs are higher. Empirical support for these propositions is provided in a growth regression that exploits cross-country variations in the dispersion of firms' real sales growth as a proxy for idiosyncratic risk along with recovery rates as a measure that proxies the inverse of bankruptcy costs.

Keywords: Bankruptcy Costs; Idiosyncratic Risk; Endogenous Growth Model; Firm-level Data

1. Introduction

Creditors incur significant costs during the bankruptcy procedure of failing firms around the world [World Bank (2019)]. These costs include legal, accounting, and administrative fees, and the lost value of the insolvent firm's assets through depreciation amid the possibly lengthy bankruptcy procedure. Previous research suggests that these institutional costs, called *bankruptcy costs* hereafter, play an important role in economic development and associated outcomes such as investment and productivity. For instance, Djankov et al. (2008), who propose a comprehensive measure of the efficiency of debt enforcement with respect to an insolvent firm, show that this measure is strongly associated with per capita income across the world. Ponticelli and Alencar (2016) show that a bankruptcy reform that increased creditors' protection in Brazil increased firms' access to finance and investment when the quality of court enforcement was high. Erashin (2020) shows that the strengthening of creditor rights in bankruptcy increased firms' productivity in the USA by prompting them to invest in new capital and information technology.

Bankruptcy costs can also play a key role in business cycle fluctuations, particularly when these costs are interacted with *idiosyncratic risk* debtors are subject to. At the firm level, idiosyncratic risk can be measured by the variance of cross-sectional idiosyncratic shocks firms are subject to.

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For example, Christiano et al. (2014), using a medium-scale dynamic general equilibrium model with bankruptcy costs, show that shocks to idiosyncratic risk, denoted as "risk shocks", critically drive business cycles. Further, Chugh (2016), using US mirco data to estimate the risk shock process, finds that risk shocks account for a notable share of Gross domestic product (GDP) fluctuations even in a small-scale model with bankruptcy costs that nests a real business cycle model.[1,2] Intuitively, idiosyncratic risk can have significant business cycle effects, because in the presence of bankruptcy costs, the initial effects of risk shocks on the contractual terms between firms and creditors and thus on output dynamics are *amplified* through an endogenous change in borrowing firms' balance sheet condition, known as the financial accelerator mechanism [e.g., Bernanke and Gertler (1989) and Bernanke et al. (1999)].

Acknowledging the insights from those strands of the literature on bankruptcy costs, this paper examines how idiosyncratic risk affects *economic growth* (a commonly used indicator of economic development) particularly when these costs are present. Specifically, we study how, both theoretically and empirically, the effects of idiosyncratic risk on growth might vary across different levels of bankruptcy costs. This investigation into the interaction effects on economic growth can be motivated by the following two observations. First, firm-level idiosyncratic risk exhibits not only a cyclical pattern, but also a long-term trend [e.g., Campbell et al. (2001), Comin and Mulani (2006), and Hamao et al. (2007)].[3] Second, bankruptcy costs, that closely reflect the quality of legal institutions of an economy, differ substantially across countries with different income levels [e.g., Djankov et al. (2008) and World Bank (2019)].

To proceed, we first build an AK (endogenous growth) model that embeds the financial contracting problem of Carlstrom and Fuerst (1997, 1998). In the model, firms face idiosyncratic productivity shocks, and the outcome of shocks is their private information. An increase in idiosyncratic risk is modeled as a mean-preserving spread of idiosyncratic shocks. The model is similar to the aforementioned business cycle study of Chugh (2016), except that our focus is on the balanced-growth path. Firms that are hit by unfavorable shocks may default, and financial intermediaries incur costs during the bankruptcy procedure of failing firms. In equilibrium, the degree of idiosyncratic risk is reflected in the contractual terms between firms and financial intermediaries. An increase in idiosyncratic risk reduces long-run growth, because bankruptcy costs create a wedge between the real rental rate of capital and its marginal product, and an increase in idiosyncratic risk enlarges it. Further, when bankruptcy costs are higher, a given increase in idiosyncratic risk has a more adverse growth effect through the capital accumulation channel.

We then test these propositions empirically. First, using the World Bank's Enterprise Surveys (WBES), which originally cover 161,977 firms from 144 countries, we estimate for a large number of countries the dispersion of firms' real sales growth as a measure of idiosyncratic risk of the country.[4] Our methodology on the estimation of the measure of idiosyncratic risk at the country level is an application of the methodology of Castro et al. (2009), who estimate idiosyncratic risk for different sectors within the USA. As a measure of bankruptcy costs, we utilize World Bank estimates of the recovery rate [World Bank (2019)], that is recorded as cents on the dollar recovered from failing businesses by secured creditors through procedures such as judicial reorganization, asset liquidation, or debt enforcement. This measure is thus inversely related to the bankruptcy costs considered in the theoretical model. Then, exploiting the cross-country variations of the dispersion of firms' sales growth and recovery rates, empirical results suggest that, consistent with the theory, an increase in idiosyncratic risk has a more adverse growth effect when recovery rates are lower (i.e., bankruptcy costs are higher).

To clarify from the outset, the use of the AK model imposes an important limitation, in that it disregards technological progress as a key factor in understanding the process of economic growth. Indeed, to examine how idiosyncratic risk interacts with bankruptcy costs to affect economic growth, our focus is solely on the capital accumulation channel. Acknowledging this limitation, however, the evidence below suggests that in fact the capital accumulation channel plays a key role in the interaction effect on growth, offering support for the use of the AK model as a theoretical guidance. Besides, the tractability of the AK model helps us to clarify how the effect occurs. Thus, while it is likely that a model with endogenous technological progress would reveal a new channel through which such interaction affects growth, we believe that the use of the AK model is justifiable in the current context.[5]

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 develops the AK model with bankruptcy costs. Section 4 estimates a measure of idiosyncratic risk for a large number of countries. Section 5 provides empirical support for the theoretical predictions. Finally, Section 6 provides concluding remarks.

2. Related literature

Broadly speaking, this paper is related to empirical studies that analyze the legal determinants of financial and economic development. For example, based on the seminal works of La Porta et al. (1997, 1998), Djankov et al. (2007) analyze the extent to which creditor rights are protected as a determinant of the amount of credit extended to the private sector in 129 countries. If creditors are paid first out of the proceeds of liquidating bankrupt firms, for instance, they might extend more credit than otherwise. The degree of law enforcement has also been studied as an important determinant for private credit [e.g., Jappelli et al. (2005), Safavian and Sharma (2007), and Djankov et al. (2007)], since even when laws on creditor rights protection are established on books, if their enforcement is time-consuming and costly, creditors may not be encouraged to lend. In this regard, Levine (1998, 1999), running cross-country regressions, highlight the importance of creditor rights and the efficiency of law enforcement in banking development, and more broadly in financial development, respectively.[6] More recently, Araujo et al. (2012), exploiting the bankruptcy law reforms in Brazil, show that an increase in creditor protection and an improvement in the efficiency of the bankruptcy system increase lenders' willingness to supply credit and improve firms' access to external finance. However, Vig (2013), exploiting the introduction of a reform in India that allowed for faster seizure and liquidation of the assets of the defaulting firm by secured creditors, shows that stronger creditor rights led to a reduction in secured debt and total debt.[7]

Regarding the legal determinants of economic outcomes in development, Djankov et al. (2008), as mentioned, indicate that there is a strong association between the efficiency of bankruptcy procedures and per capita income. Further, the aforementioned empirical works of Ponticelli and Alencar (2016) and Erashin (2020) illustrate that the strengthening of creditor protection in bankruptcy procedures could have a positive impact on investment and productivity, respectively.[8] However, other empirical studies such as Acharya and Subramanian (2009) and Acharya et al. (2011) show that stronger creditor rights could have an adverse effect by reducing firms' risk-taking and investment into innovation. Acknowledging the apparently opposite views on the effects of creditor rights on firm lending and investment, Thapa et al. (2020) examine the role of firm's access to internal capital, an important firm characteristic, as a conditioning factor for the effects of creditor rights.

This paper is also related to studies that examine long-run and cross-sectional variations in firm-level idiosyncratic risk and their effects on economic development. As mentioned, several papers report that idiosyncratic risk exhibits a long-run trend within a country. For example, Campbell et al. (2001) and Comin and Mulani (2006), examining firms' stock returns and sales growth rates, respectively, show that there was an increasing trend in idiosyncratic risk in the USA during the late 20th century. In contrast, Hamao et al. (2007) find a decreasing trend in idiosyncratic risk in Japan in the 1990s after the stock market crash. Motivated by the existence of a trend in idiosyncratic risk, Oikawa (2010) shows theoretically that a rise in idiosyncratic risk increases productivity growth via a learning-by-doing mechanism in the research sector. Meanwhile, Castro et al. (2009) and Castro et al. (2015) highlight sectoral heterogeneity in idiosyncratic risk within the USA. The theoretical model of Castro et al. (2009), in turn, suggests that when idiosyncratic risk is higher for firms producing investment goods relative to those producing consumption goods, weaker investor protection lowers the investment rate and thus national income. Michelacci and Schivardi (2013) show that in Organization for Economic Cooperation and Development (OECD) countries with low level of risk diversification opportunities, sectors characterized by high idiosyncratic risk perform worse in terms of productivity and investment.[9]

Having reviewed the related literature, a key contribution of the present paper is to integrate some of the insights discussed there. Specifically, we bring a novel dimension to the debate on bankruptcy costs and development, by highlighting the interaction effect on economic growth of these costs and idiosyncratic risk firms are subject to. We do this both from theoretical and empirical perspectives. Theoretically, we present an AK model that reveals a mechanism of the interaction effect through the capital accumulation channel. Empirically, we show that the cross-country variation in idiosyncratic risk is substantial, and we exploit such variation to provide evidence consistent with the theoretical predication.[10]

3. Theory

To shed light on how bankruptcy costs might interact with idiosyncratic risk in long-run growth, we build an endogenous growth (AK) model that embeds the financial contracting problem of Carlstrom and Fuerst (1997, 1998). The model is similar to Chugh (2016) who investigates the business cycle effects of idiosyncratic risk in the presence of bankruptcy costs, except that our focus is on the balanced-growth path equilibrium of the model.

3.1 The AK model with bankruptcy costs

3.1.1 Households

A representative and infinitely living household obtains utility from the consumption of the final good, C_t^h. Lifetime utility takes the following form:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \log C_s^h, \tag{1}$$

where β is a subjective discount factor. The budget constraint is given by

$$C_t^h + K_{t+1}^h = r_t^k K_t^h + (1-\delta)K_t^h, \tag{2}$$

where K_t^h is the household's capital stock held at the beginning of period t. The resources available from the net return from capital, $r_t^k K_t^h$, and undepreciated capital, $(1-\delta)K_t^h$ are used to purchase the final good, C_t^h, and to invest in next period capital, K_{t+1}^h. Solving this problem yields the consumption Euler equation, which indicates that, at the optimum, households are indifferent between consuming and investing into capital:

$$\frac{C_{t+1}^h}{C_t^h} = \beta(1 + r_{t+1}^k - \delta).^{11} \tag{3}$$

3.1.2 Contracting problem

There is a continuum of firms in the interval $(0, 1)$. Each firm, represented by the subindex j, produces the final good, $Z_{j,t}$, with the following technology:

$$Z_{j,t} = \omega_{j,t} A_t K_{j,t}, \tag{4}$$

where A_t is an aggregate technology parameter (common to all firms) and $K_{j,t}$ is capital used by firm j, constituting an AK model. Importantly, production is subject to idiosyncratic shocks, as captured by an iid stochastic productivity parameter, $\omega_{j,t}(>0)$, with $E\left(\omega_{j,t}\right)=1$. A change in idiosyncratic risk is modeled as a change in the dispersion of idiosyncratic shocks. Specifically, a rise in risk corresponds to a mean-preserving spread of shocks.

Firm j has access to external finance. With a given level of net worth (internal funds), $N_{j,t}$, the firm requires a loan of size, $r_t^k K_{j,t}-N_{j,t}$, where r_t^k is the rental rate of capital. External funding is provided by households through financial intermediaries.[12] Following Townsend (1979)'s costly state verification (CSV) setting, the realized value of the idiosyncratic shock, $\omega_{j,t}$, is firm $j's$ private information. If financial intermediaries want to observe it, they need to pay monitoring costs. Importantly, since the optimal contract is a standard debt contract under the CSV setting [Townsend (1979)], monitoring costs are incurred only when firms fail to repay. These costs are thus interpretable as *bankruptcy costs*, incurred by financial intermediaries (creditors) during the bankruptcy procedure of failing firms.[13] In practice, these costs could include various legal, accounting, and administrative fees associated with debt enforcement, state verification, and asset liquidation, and also the lost value of the insolvent firm's assets through depreciation.

As in Carlstrom and Fuerst (1997, 1998), we consider an intra-period contract, which is renewed every period. In any period t, before the idiosyncratic shock is realized, the contact determines repayment amount as $\Psi_{j,t}(r_t^k K_{j,t}-N_{j,t})$, where $\Psi_{j,t}$ is the gross interest rate on the loan. If the firm does not default after the shock is realized, it pays back the pre-determined amount within the period, whereas if the firm defaults, the financial intermediary pays bankruptcy costs, seizing whatever remains from the insolvent firm. There is thus a cut-off value, $\overline{\omega}_{j,t}$, such that

$$\overline{\omega}_{j,t}A_tK_{j,t}=\Psi_{j,t}(r_t^kK_{j,t}-N_{j,t}), \tag{5}$$

where if $\omega_{j,t}\geq\overline{\omega}_{j,t}$, a firm repays the predetermined amount; otherwise it defaults. Since net worth, $N_{j,t}$, is predetermined, the contract with firm j effectively sets the amount of capital rented, $K_{j,t}$ and the cut-off value, $\overline{\omega}_{j,t}$.

Firms are risk neutral, maximizing their expected profits, whereas perfectly competitive financial intermediaries, which we assume do not incur any operation cost, diversify risks by lending to a large number of firms and recoup the amount they lend within a period. Therefore, the contracting problem is written as

$$\max_{\{K_{j,t},\overline{\omega}_{j,t}\}} f(\overline{\omega}_{j,t})A_tK_{j,t}$$

subject to

$$g(\overline{\omega}_{j,t})A_tK_{j,t}=r_t^kK_{j,t}-N_{j,t}, \tag{6}$$

where $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ are the expected shares of firm's production going to the firm and the financial intermediary, respectively. They are expressed as

$$f(\overline{\omega}_{j,t})\equiv\int_{\overline{\omega}_{j,t}}^{\infty}\omega\phi(\omega)d\omega-(1-\Phi(\overline{\omega}_{j,t}))\overline{\omega}_{j,t}, \tag{7}$$

$$g(\overline{\omega}_{j,t})\equiv\int_0^{\overline{\omega}_{j,t}}\omega\phi(\omega)d\omega+(1-\Phi(\overline{\omega}_{j,t}))\overline{\omega}_{j,t}-\mu\Phi(\overline{\omega}_{j,t}). \tag{8}$$

In equations 7 and 8, $\phi(\omega)$ and $\Phi(\omega)$ are the probability and cumulative density functions of idiosyncratic shocks. The sum of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ is less than unity due to the payment of bankruptcy costs: $f(\overline{\omega}_{j,t})+g(\overline{\omega}_{j,t})=1-\mu\Phi(\overline{\omega}_{j,t})$, where μ is the bankruptcy costs parameter, and $\Phi(\overline{\omega}_{j,t})$ is the default rate. The implicit assumption here is that in case firm i defaults, a financial intermediary pays a fixed fraction μ of the *expected* production outcome, $A_tK_{j,t}$, during the bankruptcy procedure. In a robustness check, we also consider the alternative setup where bankruptcy costs are a proportion of the *realized* production, $\omega_{j,t}A_tK_{j,t}$.

Solving the problem above, and aggregating the first-order condition across firms yields

$$r_t^k = (1 - \Theta(\overline{\omega}_t))A_t.^{14} \tag{9}$$

This indicates that $\Theta(\overline{\omega}_t)$ is a wedge between the rental rate, r_t^k, and the marginal product of capital within the AK framework, A_t, which is expressed as

$$\Theta(\overline{\omega}_t) \equiv \mu\Phi(\overline{\omega}_t) + \left(1 - \frac{1}{\lambda\,(\overline{\omega}_t)}\right) f(\overline{\omega}_t), \tag{10}$$

where $\lambda(\overline{\omega}_t)$ is the Lagrange multiplier:

$$\lambda(\overline{\omega}_t) = \frac{1}{1 - \mu\frac{\phi(\overline{\omega}_t)}{1-\Phi(\overline{\omega}_t)}}. \tag{11}$$

Equation 10 shows that the investment wedge consists of the two components. The first one of $\mu\Phi(\overline{\omega}_t)$ represents the part of production lost during the bankruptcy procedure, interpretable as a deadweight loss. The second component can be interpreted as economic rents accruing to firms in the presence of bankruptcy costs, since the Lagrange multiplier, $\lambda\,(\overline{\omega}_t)$, which represents the shadow price of firms' net worth, exceeds unity only when bankruptcy costs are present, i.e., $\mu > 0$. This component of the wedge becomes larger as the share of output going to firms, $f(\overline{\omega}_t)$, increases. When bankruptcy costs are absent ($\mu = 0$), both components are zero and the wedge disappears.

3.1.3 Endogenizing net worth

While the preceding optimization problem treats firm j's net worth, $N_{j,t}$, as exogenous, it needs to be endogenized in the dynamic context. Specifically, $N_{j,t}$ is determined as the gross return of the capital the firm holds at the beginning of the period, $K_{j,t}^e$, i.e., $N_{j,t} = (r_t^k + 1 - \delta)K_{j,t}^e$.[15] Aggregating this relation across firms yields

$$N_t = (r_t^k + 1 - \delta)K_t^e, \tag{12}$$

where N_t and K_t^e are aggregate net worth and firms' capital. To avoid a situation in which firms ultimately become self-financed, we follow Bernanke et al. (1999) and assume that a constant fraction υ of randomly-selected firms exit each period, and consume all the accumulated wealth just before their exit (think of firms as entrepreneurs).[16] The population of firms remains constant at unity, with new firms replacing those exiting the economy. Thus, K_t^e follows

$$K_{t+1}^e = (1 - \upsilon)A_t f(\overline{\omega}_t)K_t. \tag{13}$$

Aggregate consumption by firms, C_t^e, is given as $C_t^e = \upsilon A_t f(\overline{\omega}_t)K_t$.

3.1.4 Market clearing

Denoting total aggregate consumption and capital as $C_t(= C_t^h + C_t^e)$ and $K_t(= K_t^h + K_t^e)$, respectively, the final goods market clearing condition becomes

$$Y_t = C_t + K_{t+1} - (1 - \delta)K_t, \tag{14}$$

where

$$Y_t = (1 - \mu\Phi(\overline{\omega}_t))A_tK_t. \tag{15}$$

In words, the sum of total consumption and investment equals aggregate output net of bankruptcy costs. Equation 15 thus embodies the assumption that the bankruptcy costs are a deadweight loss in general equilibrium. This assumption is motivated by the observation that in reality bankruptcy costs contain a type of costs that arise when capital is kept idle for some period of time during the bankruptcy procedure (e.g., lost sales and profits, and the depreciation of capital).[17]

3.1.5 Balanced growth equilibrium

Our focus is on the balanced-growth path equilibrium of the model, where the aggregate productivity parameter, A_t, and the threshold value of idiosyncratic shock, $\overline{\omega}_t$, are assumed to be constant. Along the path, variables grow at the constant rate of η, except for the rental rate, $r^k = (1 - \Theta(\overline{\omega}))A$, which is also constant. Here, we only sketch the solution process for brevity, leaving the details to Appendix A.

First, acknowledging that households' budget constraint (equation 2) is a first-order linear difference equation, we can rewrite it as

$$K_t^h = \sum_{j=0}^{\infty} \left(\frac{1}{r^k + 1 - \delta} \right)^{j+1} C_{t+j}^h + \lim_{T \to \infty} \left(\frac{1}{r^k + 1 - \delta} \right)^T K_{t+T}^h. \tag{16}$$

In the balanced-growth equilibrium where K_t^h grows at the constant rate of η, the limit of the present value of the terminal value of households' capital holding, $\lim_{T \to \infty} \left(1/(r^k + 1 - \delta) \right)^T K_{t+T}^h$ is zero. This is because we know from the Euler equation (equation 3) that:

$$\eta = \beta(r^k + 1 - \delta), \tag{17}$$

and the subjective discount factor is less than unity, i.e., $\beta < 1$. Thus, equation 16 becomes

$$K_t^h = \frac{1}{(1 - \beta)(r^k + 1 - \delta)} C_t^h, \tag{18}$$

a static relation between K_t^h and C_t^h, which holds along the balanced growth path. Next, remember from equation 9 that bankruptcy costs create the wedge, $\Theta(\overline{\omega})$, between the real rental rate, r^k, and the marginal product of capital, A:

$$r^k = (1 - \Theta(\overline{\omega}))A. \tag{19}$$

By substituting equation 19 into equation 18, and noticing that K_t^h and C_t^h can both be expressed as a linear function of K_t (explained in Appendix A), we obtain

$$\lambda(\overline{\omega}) = \frac{\beta}{1 - \upsilon}. \tag{20}$$

In words, the steady-state shadow price of net worth, $\lambda(\overline{\omega})$, is increasing in the discount factor, β, and decreasing in firms' marginal propensity to invest into capital, $1 - \upsilon$.[18]

Equation 20 makes it clear that once the distribution function of idiosyncratic shocks is specified, the long-run threshold value of $\overline{\omega}$ can be solved for given parameter values. Specifically, denoting the dispersion parameter of idiosyncratic shocks as ρ, $\overline{\omega}$ is tied down as a function of four parameters:

$$\overline{\omega} = \overline{\omega}(\mu, \rho, \beta, \upsilon). \tag{21}$$

Subsequently, we can solve the balanced growth rate, η, as (see equations 17 and 19):

$$\eta = \beta \left[(1 - \Theta(\mu, \rho, \beta, \upsilon))A + 1 - \delta \right]. \tag{22}$$

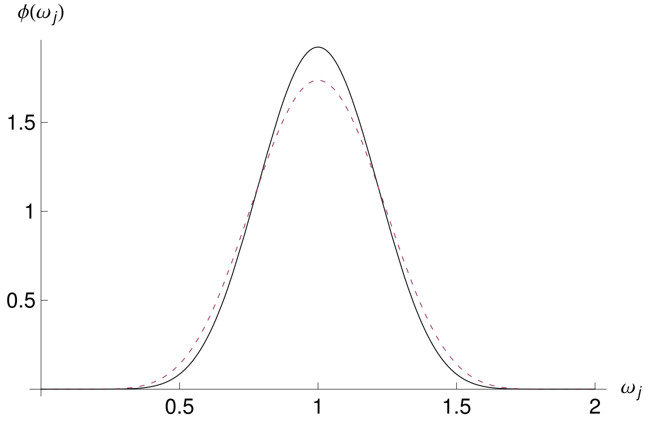

Figure 1. Mean-preserving spread using beta distribution.
Note: Mean-preserving spread corresponds to a shift from the solid to the dashed line.

Equation 22 indicates that an increase in the investment wedge between the rental rate of capital and its marginal product, Θ, reduces the long-run growth rate, η. This happens because the wedge, reducing the rental rate of capital, makes investment less attractive for households, thereby slowing down capital accumulation. In the present AK setup, evaluating the growth effect of idiosyncratic risk boils down to evaluating the effect of the dispersion parameter of idiosyncratic shocks on the wedge. Acknowledging this, our aim now is to analyze how the dispersion parameter interacts with the bankruptcy costs parameter, μ, to determine the wedge, and thus growth.

3.2 Analysis

3.2.1 Distribution function of idiosyncratic shocks

We model an increase in idiosyncratic risk as a mean-preserving spread of idiosyncratic shocks, ω. The following analysis considers a beta distribution and a log-normal distribution of idiosyncratic shocks. The former distribution enables us to model a mean-preserving spread without an effect on its skewness, whereas with the latter, a mean-preserving spread necessarily changes skewness. We regard the former as a reference distribution function for the sake of simplicity and check the robustness of results using the latter. A beta distribution exhibits unit mean and zero skewness when the probability density function, $\phi(\omega)$, takes the form:

$$\phi(\omega) = \frac{1}{Beta(\rho, \rho)} \frac{\omega^{\rho-1}(2-\omega)^{\rho-1}}{2^{2\rho-1}}, \tag{23}$$

where $Beta(\rho, \rho) = \int_0^1 \omega^{\rho-1}(1-\omega)^{\rho-1} d\omega$. Figure 1 illustrates a mean-preserving spread as a shift of distributions from the solid to the dotted lines, corresponding to a *fall* in the dispersion parameter of the function, ρ.[19] In what follows, the lack of analytical solution prompts us to proceed numerically. Nonetheless, our primary focus is on the *qualitative* effect of the interaction between idiosyncratic risk and bankruptcy costs on growth.

3.2.2 Calibration

We now calibrate parameter values that determine the investment wedge and growth: β, μ, υ, ρ, A, and δ (see equation 22). The time unit is a quarter. The discount factor, β, is set as 0.99. Regarding the bankruptcy costs parameter, μ, previous works for advanced economies such as

Table 1. Reference parameter values and targeted financial values

Parameter/target values	Description
Parameter values	
$\beta = 0.99$	Subjective discount factor
$\mu = 0.31$	Bankruptcy costs
$\upsilon = 0.072$	Firms' exit probability
$\rho = 6.38$	Dispersion parameter of idiosyncratic shocks (beta distribution)
$A = 0.048$	Aggregate productivity
$\delta = 0.031$	Depreciation rate of capital
Targeted values	
$\varpi/g(\varpi) - 1 = 0.015$	Credit spread: loan rate minus risk-free rate
$g(\varpi)/(1 - \mu\Phi(\varpi)) = 0.444$	Credit-to-GDP ratio
$\beta\left[(1 - \Theta(\varpi))A + 1 - \delta\right] = 1.005$	(Gross) steady-state growth rate

Note: Time unit is a quarter.

Bernanke et al. (1999) and Chugh (2016) set $\mu = 0.12$ and 0.15, respectively, while Christiano et al. (2014) estimate it as $\mu = 0.21$ for the USA. However, as illustrated below (Figure 4), there is a clear indication that bankruptcy costs are generally higher in lower-income countries. Since the subsequent empirical analysis mainly covers non-advanced economies (associated with the coverage of the World Bank's Enterprise Surveys used there), we use $\mu = 0.31$ as a reference value. This is the value used for Argentina by Aysun and Honig (2011) to investigate the role of bankruptcy costs in the output loss following sudden stops of capital flows. We set the depreciation rate of capital, δ, as 0.031, based on the annual depreciation rate of 0.1255 used by García-Cicco et al. (2010) for Argentina.

Next, we set firms' exit probability $\upsilon = 0.072$, corresponding to firms' expected survival periods of 7 quarters (and their aggregate saving rate of 0.93). Setting this value of υ enables us to target an empirically plausible value of the credit-to-GDP ratio, defined as total credit divided by aggregate output, $\frac{g(\varpi(\mu,\rho,\beta,\upsilon))}{1-\mu\Phi(\varpi(\mu,\rho,\beta,\upsilon))}$ (see equations 6 and 15). To tie down the dispersion parameter of idiosyncratic shocks for the beta distribution, ρ, we focus on the credit spread, measured by the premium in firms' gross intra-period interest rate on loan over the corresponding risk-free interest rate (which is unity), $\frac{\varpi(\mu,\rho,\beta,\upsilon)}{g(\varpi(\mu,\rho,\beta,\upsilon))} - 1$ (equation 5). Specifically, targeting the spread of 0.015 (1.5%), and using also the equilibrium shadow price of net worth (equation 20), the value of ρ and the threshold value of ϖ can be tied down simultaneously as $\rho = 6.38$ and $\varpi = 0.45$.[20] Subsequently, the model yields the credit-to-GDP ratio of 0.444 (44.4%) in line with the empirical counterpart.[21] Last, to tie down the value of the aggregate productivity parameter, A, in the reference equilibrium, we target the quarterly *net* growth rate of 0.005 (0.5%), based on the median annual GDP growth rate of around 2% obtained from the IMF's WEO over the 1983-2017 period for all the countries available.[22] We can then obtain from equation 22 that $A = 0.048\left(= \frac{1}{1-\Theta(\mu,\rho,\beta,\upsilon)}\left(\frac{1+0.005}{\beta} - 1 + \delta\right)\right)$. Table 1 summarizes the parameter values, together with the values used as a target.

3.2.3 Interaction effect of idiosyncratic risk and bankruptcy costs on growth

Turning to the analysis, we first examine how an increase in idiosyncratic risk affects growth for given bankruptcy costs. Next, we illustrate how the effects of idiosyncratic risk on growth vary across different levels of bankruptcy costs.

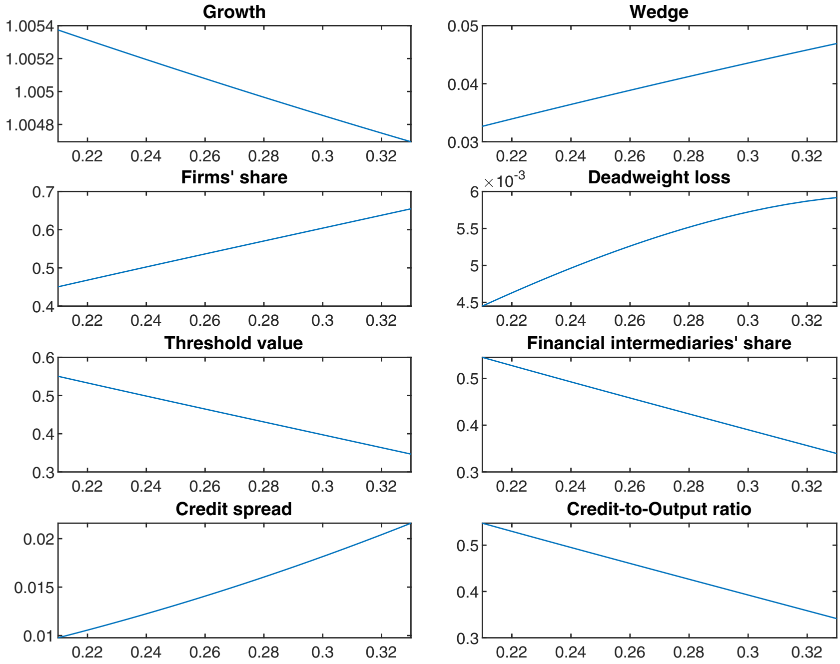

Figure 2. Effects of idiosyncratic risk on growth and related variables.
Notes: The horizontal axis is the standard deviation of idiosyncratic shocks, s, around the reference value of $s = 0.27$ (corresponding to $\rho = 6.38$). Shocks follow the beta distribution. Parameter values other than ρ are kept constant at the reference values with $\mu = 0.31$.

Effect of idiosyncratic risk on growth for given bankruptcy costs. Here, as a direct measure of idiosyncratic risk, we use the standard deviation, s, of idiosyncratic shocks rather than the dispersion parameter of the beta distribution, ρ. Note that for this distribution, s is *inversely* related to ρ. Figure 2 reports how the growth rate and various financial variables (including the wedge) change over a range of idiosyncratic risk around $s = 0.27$, which corresponds to the reference dispersion parameter of $\rho = 6.38$ (as in Table 1). The other parameters are kept constant at the reference values, including the bankruptcy costs parameter: $\mu = 0.31$.

Figure 2 shows that an increase in idiosyncratic risk reduces growth by increasing the wedge. In turn, to see how the wedge increases, remember from equation 10 that it can be decomposed into the deadweight loss and rents that accrue to firms. Observe that as idiosyncratic risk increases, the rents become larger due to an increase in the share of output going to firms.[23] Intuitively, because the outcome of idiosyncratic shocks is firms' private information, the larger dispersion of shocks gives firms a greater informational advantage and thus the larger output share. Although the deadweight loss also increases due to the higher default rate, this result, as shown below, is not robust due to the two opposing forces. On the one hand, larger idiosyncratic risk makes the left tail of the distribution fatter (see Figure 1) and thus increases the default rate at the given level of the threshold value, but on the other hand, it decreases the threshold value endogenously. Regarding the remaining financial variables, a rise in idiosyncratic risk increases the credit spread, and reduces the credit-to-output ratio. That is, when the dispersion of idiosyncratic shocks is larger, financial intermediaries charge higher interest rates on loan, and reduce the supply of credit relative to output.

For robustness checks, Figures A1, A2, A3, and A4 in Appendix B consider the alternative cases where (1) idiosyncratic shocks follow a log-normal distribution, (2) the bankruptcy cost

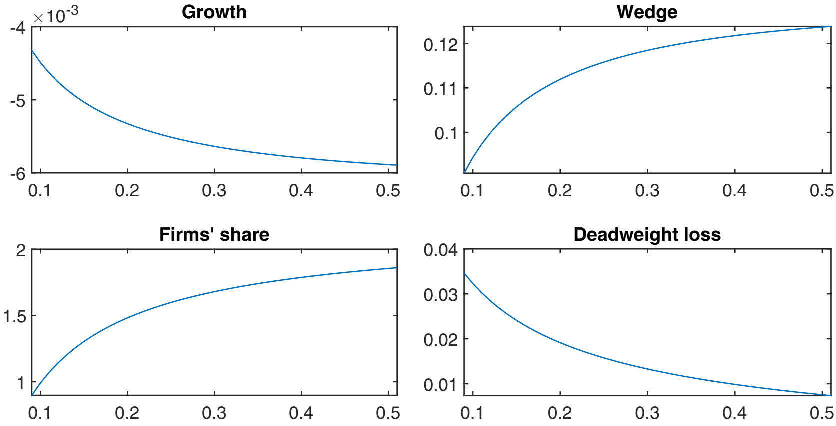

Figure 3. Marginal effects of idiosyncratic risk on growth across different bankruptcy costs.
Notes: The horizontal axis is the bankruptcy costs parameter, μ, and the vertical axis is the marginal effect of the standard deviation of idiosyncratic shocks on the respective variable. $\beta = 0.99$, $\upsilon = 0.072$, $\rho = 6.38$ ($s = 0.27$), $\delta = 0.031$, and $A = 0.048$ throughout.

parameter takes different values of $\mu = 0.1$ and 0.5 (with idiosyncratic shocks following the beta distribution), and (3) bankruptcy costs are modeled as a proportion of the realized production outcome (with $\mu = 0.31$ and the beta distribution).[24] For each case, the reference dispersion parameter (thus the standard deviation) of shocks, firms' exit probability, υ, and aggregate productivity, A are set, by targeting the same levels of the credit spread (0.015), credit-to-output ratio (0.44), and steady-state growth rate (1.005). Then, the growth rate and the financial variables including the wedge are plotted around the reference value of the standard deviation, while keeping the other parameter values constant.[25] As in Figure 2, a rise in idiosyncratic risk reduces growth while increasing the wedge, with the latter driven by an increase in the output share going to firms, but not necessarily by an increase in the deadweight loss (see Figures A1 and A2). Again, a rise in idiosyncratic risk increases the credit spread and decreases the credit-to-output ratio.

Effects of idiosyncratic risk on growth across different bankruptcy costs. We now consider the effects of idiosyncratic risk on growth across different values of bankruptcy costs parameter, μ. Specifically, assuming that idiosyncratic shocks follow the beta distribution, Figure 3 plots the marginal effect of the standard deviation of idiosyncratic shocks, s, on growth, i.e., $\partial\eta/\partial s$, around the reference value of $\mu = 0.31$ between 0.1 and 0.5. For this exercise, the parameter values other than μ are set at the reference values as in Table 1. The idea is thus to compare the adverse growth effects of a given increase in idiosyncratic risk across economies where the only difference is in terms of the size of bankruptcy costs.

At the reference equilibrium where $\mu = 0.31$, we obtained $\partial\eta/\partial s = -0.0057$. This means that a rise in idiosyncratic risk by 0.1 standard deviations reduces the steady state *annual* growth rate by about $0.228 (= 0.057 * 4)$ percentage points.[26] Figure 3 shows that as the bankruptcy costs parameter rises, a given increase in idiosyncratic risk has a greater growth-reducing effect, driven by a greater impact on the wedge. The figure also shows that as bankruptcy costs increase, a rise in idiosyncratic risk has a greater (smaller) positive impact on the output share going to firms (the deadweight loss). Thus, the reason why a given increase in idiosyncratic risk has a greater distortionary effect when bankruptcy costs are higher is that it has a greater impact on the firms' share in output and thereby economic rents accruing to them. Figures A5 and A6 in Appendix B indicate that the results are robust to the use of the log-normal distribution function and the alternative assumption that bankruptcy costs are proportional to the realized (rather than expected) production outcome.

4. Estimation of idiosyncratic risk

We now test the theoretical predictions on the growth effects of idiosyncratic risk conditional on bankruptcy costs. We proceed in two steps. First, the present section estimates for a large number of countries a measure of country-specific idiosyncratic risk, proxied by the conditional dispersion of firms' real sales growth rates. Second, the subsequent section runs a cross-country growth regression where the idiosyncratic risk measure is interacted with World Bank's estimates of recovery rates as an inverse measure of bankruptcy costs.

4.1 Methodology

We use the following model to estimate idiosyncratic risk at a country level:

$$sales\_growth_{i,j,k,(t,t-2)} = \alpha + \beta sales_{i,j,k,t-2} + \sum_{l=1}^{n} \gamma_l z_{i,j,k,l,t} + \delta_{k,t} + \eta_{j,k} + \varepsilon_{i,j,k,t}, \tag{24}$$

where $sales\_growth_{i,j,k,(t,t-2)}$ is real annual sales growth of firm i in sector j in country k over the 3 years between years t and $t-2$. In the right-hand side, the log of the initial sales level, $sales_{i,j,k,t-2}$, is controlled for. $z_{i,j,k,l,t}$ includes variables specific to individual firm i: size, age, types of ownership (domestic vs foreign and private vs public), access to bank finance, and export status. $\delta_{k,t}$ and $\eta_{j,k}$ are country-time and country-sector fixed effects, respectively. The former controls for any time-varying economy-wide shock common to all firms of a country, whereas the latter controls for sector-specific factors within a country, capturing, for instance, differences in productivity growth rates across sectors which may differ across countries. Last, $\varepsilon_{i,j,k,t}$ reflects the part of the firm's annual growth rate which is not accounted for by the aforementioned factors.

Here, our main focus is on the estimated residual of $\hat{\varepsilon}_{i,j,k,t}$. Specifically, we obtain the country-specific standard deviation of the residual as a measure of firm-level idiosyncratic risk for the country. Our methodology is an application of Castro et al. (2009), who estimate idiosyncratic risk for different *sectors* within the USA. We first consider equation 25, whereby the log of the country-specific variance of the residual can be estimated as a coefficient on the country dummy:

$$\ln \hat{\varepsilon}^2_{i,j,k,t} = \theta_k + \nu_{i,j,k,t}. \tag{25}$$

Subsequently, we obtain the country-level standard deviation of $\varepsilon_{i,j,k}$ as $\sqrt{\exp\left(\hat{\theta}_k\right)}$. This is the measure of dispersion of firms' real sales growth rates, which we use as a proxy of firm-level idiosyncratic risk in country k.[27]

4.2 Data

Our main data source is the WBES, that offer an expansive array of firm-level data on 161,977 firms from 286 surveys spanning 144 countries over the 2006–19 period.[28] For each country, 1 to 4 surveys are available. The surveys mostly cover developing countries, though some advanced economies, such as Sweden and Israel, are also included. The data were collected by private contractors hired by the World Bank via face-to-face interviews, and confidentiality of the survey respondents and the sensitive information is maintained to facilitate the highest degree of survey participation. The surveys are answered by owners and top managers of businesses, while questions about the sales are sometimes answered by company accountants. The surveys cover a representative sample of an economy's manufacturing, retail and other services, which correspond to the three sectors considered in the model.

The real sales growth rate is calculated from deflated values of sales typically over the 3 years between the year preceding the survey and 3 year before (denoted in equation 24 as t and $t-2$, respectively).[29] Previous studies such as Şeker and Yang (2014) and Chauvet and Ehrhart (2018) use the WBES to examine the determinants of firms' sales growth.[30] We checked outliers for the initial as well as final sales (in US dollars), and excluded, for each survey, the logs of the initial and final sales that are further more than three standard deviations from the respective means [as in Şeker and Yang (2014)]. The firm's annual real sales growth rate is relatively scarce, because this is often missing for a very young firm, and some firms simply do not grasp the sales level. Information on firm characteristics, used as a control, are also sometimes missing. Thus, to mitigate the possible bias from a representation problem caused by these missing values, the subsequent analysis only uses a survey in which at least 50% of firms, and at least 100 firms, provide information on all the variables considered in the estimation of idiosyncratic risk.

After taking the aforementioned measures, the dataset reduces to 100,640 observations spanning 227 surveys from 127 countries with the average of 1.79 surveys per country.[31] Specifically, 1 (2,3,4) surveys are available for 59 (38, 28, 2) countries over the 2006–19 survey period. The survey years are not synchronized across the countries. When multiple surveys are available for a country, repeated observations at the firm level can potentially be used. However, because (1) the availability tends to be limited (with the coverage of firms often being different across the waves), and (2) our subsequent analysis makes use of the cross-country variation of the key variables, it is important to estimate the dispersion of firms' real sales growth rate for as many countries as possible.[32] This is the main reason why equation 24 forgoes firm-fixed effects and covers all the countries including the 59 countries for which only one survey is available throughout the sample period.

4.2.1 Descriptive statistics

Table 2 presents descriptive statistics for the sample. The mean of the real annual sales growth rate, the dependent variable in equation 24, is 1.96%, while the standard deviation is substantial. Turning to the firms' characteristics used as a control, the mean of the log of annual sales level in the initial year is 13.29, corresponding to 591,253 US dollars.[33] Firm size, which takes the value of either 1, 2, or 3 (corresponding to the number of workers less than 20, between 20 and 99, and more than 100, respectively), is 1.76 on average.[34] The average age is 19.6 years. Regarding the ownership structure, on average, 1.25% of firms are owned by government or state (in the sense that it holds at least 10 percent of ownership), while 10.1% are foreign-owned (i.e., foreign agents own at least 10%). 39.5% have access to bank loans, and 15.8% are exporters (in that at least 10% of firm's annual sales come from direct exports). 58 (18, 24)% of firms belong to manufacturing (retail, other services) sectors.

4.3 Results

Table 3 presents the estimation results of equation 24. Column (1) (Column (2)) shows results for models without (with) country-time and country-sector fixed effects. In both cases, the coefficient on the log of initial sales level is negative and statistically significant. The coefficient of −2.84 in Column (1) means that an increase in initial sales level of 1% is associated with a decrease in the sales growth rate of 0.028 percentage points. The relations of size and age with sales growth rate are significantly positive and negative, respectively.[35] The coefficient on government ownership is positive, but insignificant, whereas a foreign ownership is associated with higher sales growth. Access to bank loans and export status are both significantly associated with higher growth. Using the country-time and country-sector fixed effects increases the model's explanatory power greatly, as seen from a rise in the adjusted R-squared from 0.06 to 0.27.

We then use equation 25 to estimate idiosyncratic risk as the country-specific standard deviation of the estimated residual from the model with the country-time and country-sector fixed effects. Table 4 summarizes the result. When considering the whole 127 countries, the mean value of idiosyncratic risk is 8.71%. Importantly, there is a substantial variation as reflected by

Table 2. Descriptive statistics: For estimation of idiosyncratic risk

Variable	Mean	Std. Dev.	Min.	Max.
Real annual sales growth	1.96	25.11	−99.96	100
Log of initial sales level (US dollars)	13.29	2.49	1.05	27.45
Size	1.76	0.77	1	3
Age (years)	19.61	15.48	3	214
Govt/state ownership	1.25	11.11	0	100
Foreign ownership	10.13	30.17	0	100
Access to bank loan	39.46	48.88	0	100
Exporter	15.77	36.45	0	100
Manufacturing	0.58	0.49	0	1
Retail	0.18	0.38	0	1
Other services	0.24	0.43	0	1

Notes: 100,640 observations, spanning 227 surveys from 127 countries over the 2006–19 survey period. Annual sales growth rate is in real terms (using GDP deflator from World Bank's WDI) typically over the 3-year period, expressed in percent. Initial sales level is converted to US dollars, and then log-transformed. Exchange rate data is from WDI, complemented by IMF's International Financial Statistics. Size takes the value of 1 (2, 3) if the number of workers is less than 20 (between 20 and 99, more than 100). Government/state (Foreign) ownership takes the value of 100 when at least 10% of ownership is held by government/state (foreigners), 0 otherwise. Access to bank loan takes the value of 100 if a firm has access to it. Exporter takes the value of 100 if at least 10% of a firm's annual sales is derived from direct exports, 0 otherwise. Manufacturing takes the value of 1 if a firm belongs to the business sector, 0 otherwise. Retails and other services are defined similarly.

Table 3. Determinants of firms' real sales growth

Regressors	(1)	(2)
Log of initial sales level	-2.84^{***}	-5.23^{***}
	(−4.40)	(−17.42)
Size	6.24^{***}	9.81^{***}
	(5.46)	(16.42)
Log of firm age	-2.68^{***}	-2.46^{***}
	(−4.92)	(−11.21)
Access to bank loan	0.04^{***}	0.03^{***}
	(5.18)	(10.31)
Government/state ownership	0.03	0.01
	(1.27)	(0.91)
Foreign ownership	0.03^{***}	0.03^{***}
	(3.49)	(7.36)
Exporter	0.02^{***}	0.02^{***}
	(3.83)	(6.56)
Country-time fixed effects	No	Yes
Country-sector fixed effects	No	Yes
Observations	100,640	100,640
Adj. R2	0.0640	0.268

Notes: OLS estimations. Dependent variable is firms' annual real sales growth rates (in percent). See notes for Table 2 for the description of independent variables. Constant and coefficients on country-time and country-sector fixed effects are not shown for brevity. t-statistics are in parentheses. Clustered standard errors are used to adjust for correlation of error terms within country. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 4. Cross-country variation in idiosyncratic risk

Variable	Mean	Std. Dev.	Min.	Max.
Idiosyncratic risk	8.71	2.61	4.36	17.01

Notes: 127 countries (observations). Idiosyncratic risk (in percent) is the country-specific standard deviation of an unexplained component of firms' sales growth rates estimated using equations 24 and 25.

the standard deviation of 2.61.[36] The subsequent growth regression will exploit the cross-country variation in idiosyncratic risk to test the theoretical predictions above.

5. Cross-country growth regression

Having estimated idiosyncratic risk for a large of countries using the WBES, we now run a simple cross-country growth regression to shed light on an interaction effect between idiosyncratic risk and bankruptcy costs. Regarding bankruptcy costs, we utilize World Bank estimates of the recovery rate. This measure is *inversely* related to the bankruptcy costs considered in the theoretical model.

5.1 Recovery rate

In what follows, we utilize World Bank estimates of recovery rates, a key sub-component of "resolving insolvency" from Doing Business indices [World Bank (2019)]. The estimates use a methodology developed by Djankov et al. (2008). Doing Business defines the recovery rate as cents to the dollar recovered by secured creditors through reorganization, liquidation, or debt enforcement proceedings, thereby *inversely* related to bankruptcy costs considered in the theory. Specifically, the calculation takes account of cost as well as time of the proceedings. The cost includes various fees, such as court fees, government levies, and fees of auctioneers and lawyers. Time is associated with the value lost through depreciation of the insolvent company's assets. The calculation also considers the outcome of whether the company's business emerges as a going concern (i.e., as an operating business) or the assets are sold piecemeal.[37] The recovery rate is obtained as the remaining proceeds per dollar in present value terms.

To obtain the flavor of how the recovery rate differs across time and countries, Figure 4 plots the evolution over the 2004–2017 period of yearly averages within the different income groups of low-income, lower-middle-income, upper middle-income, and high-income countries (LICs, Lower_MICs, Upper_MICs, and HICs, respectively).[38,39] Among countries that are included in the reference regressions analysis below (104 countries, see Appendix D for the list), countries for which the recovery rate is available throughout the 14 years are used to calculate the yearly averages in the figure (90 countries). Two observations are in order. First, the recovery rate shows a *substantial cross-country variation*, associated positively with income levels. Second, recovery rates tend to exhibit little time variation, except that HICs show a somewhat distinct upward trend. This tendency of lacking time variation is consistent with the fact that the recovery rate closely reflects institutional features of a country, particularly its legal systems.[40] In what follows, the cross-country variation of the recovery rate is exploited to estimate its interaction effect with idiosyncratic risk on long-run growth.

5.2 Methodology

Our methodology is a simple modification of Barro (1991) who investigates growth in a cross-section of countries:

$$\overline{\triangle \ln y_k} = \alpha + \beta\, risk_k + \gamma\, \overline{rec}_k + \delta\, risk_k * \overline{rec}_k + \sum_{l=1}^{n} \eta_l\, z_{k,l} + \varepsilon_k, \tag{26}$$

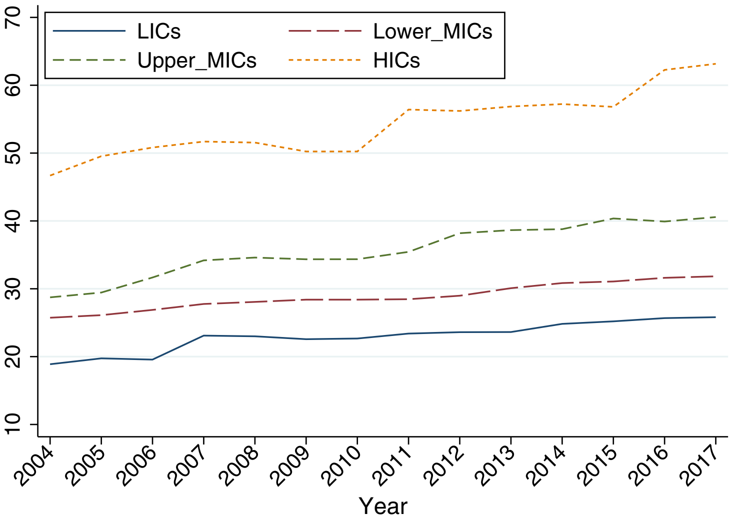

Figure 4. Evolution of the recovery rate across income levels: 2004–2017.
Notes: The vertical axis represents cents to the dollar recovered by secured creditors. LICs (Lower_MICs, Upper_MICs, HICs) are countries whose income levels in terms of real GDP per capita, PPP adjusted were within the 4th (3rd, 2nd, 1st) quarter in 2003 among countries covered by IMF's World Economic Outlook (WEO). Among countries which are included in the regression below (104 countries), countries for which the recovery rate is available throughout the 14 years (90 countries) are included to calculate the yearly average within each income group.

where $\overline{\triangle \ \ln y_k}$ is an average annual growth rate of per capita real GDP in PPP terms in percent in country k over the 2004–17 period, calculated as $(\ln y_{k,2017} - \ln y_{k,2004}) * 100/14$.[41] The time period is a period with the 2-year lag from the 2006–19 period over which the WBES was reported. The use of the lagged period is motivated by the fact that (1) there is usually a 1-year lag between the time the firm's final sales figure was recorded and the time the survey was reported, and (2) the annual sales growth is calculated typically over 3 years. $risk_k$ is the log of idiosyncratic risk estimated for country k, while $\overline{rec}_k$ is the average of the log of the recovery rate over the 2004–2017 period.[42] Acknowledging the inverse relationship between the recovery rate and bankruptcy costs, the preceding theoretical model predicts that the interaction coefficient, δ, should be *positive*.

$z_{k,l}$ is a vector of variables. Based on Barro (1991), we routinely control for the log of initial real per capita GDP (PPP adjusted) as well as initial human capital stocks. Initial income captures the convergence effect, expected to have a negative effect, whereas initial human capital stocks should have a positive effect, say, by facilitating an introduction of new goods and thereby technological progress. Initial human capital is proxied by the percentage of the population of relevant age enrolled in primary school as of 1994, assuming that those enrolled in primary school will add to growth as workers with the substantial lag of at least 10 years.[43] The population growth rate, averaged over the 2004–17 period, is also included routinely. As a robustness check, we also control for aggregate (macroeconomic) volatility, proxied by the standard deviation of the annual growth rate of real per capita GDP over the 2004–17 period.

Regarding the channel through which idiosyncratic risk affects growth in the presence of bankruptcy costs, the preceding theory suggests the relevance of the distortion to households' investment decisions (equation 22). To take account of this insight in estimation, the reference regression equation of equation 26 does *not* control for the ratio of investment to GDP (investment, for short). To shed light on the role of the capital accumulation channel, however, we also estimate the model with investment controlled for. The idea is that, to the extent that the capital accumulation channel is important, we expect that the interaction coefficient of δ becomes less

Table 5. Descriptive statistics: For growth regression

Variables	Mean	Std. Dev.	Min.	Max.
Growth rate of real GDP pc	2.86	1.95	−3.75	8.35
Idiosyncratic risk	8.74	2.43	4.36	15.98
Recovery rate	33.12	15.73	7.33	75.91
Initial real GDP pc	8978.21	8324.59	610.82	38, 922.54
Initial enrollment rate, primary level	81.63	17.78	20.74	99.7
Population growth rate	1.29	1.17	−1.2	4.19
Aggregate volatility	2.95	1.8	0.6	10.31
Investment	23.89	6.09	10.59	44.69
Bank finance	38.6	16.65	6.52	76.88

Notes: 104 observations (countries), corresponding to Table 6. Growth rate of real GDP pc (per capita) is in percent, the average of the annual rates over the 2004–2017 period. Idiosyncratic risk is the country-specific conditional standard deviation of an unexplained component of firms' sales growth rates in percent. Recovery rate is cents to the dollar recovered by secured creditors through reorganization, liquidation, or debt enforcement proceedings. Initial real GDP pc is in PPP terms (in international dollars). Initial enrollment rate is the percentage of the population of relevant age enrolled in primary school. Population growth rate is in percent. Aggregate volatility is the standard deviation of annual growth rate of real GDP pc (in percent) over the 2004–17 period. Investment is the ratio of total (i.e., private as well as public) investment to GDP, in percent. Bank finance is the proportion of firms with bank loans, calculated using the WBES firm-level data during the sample period.

significant when investment is controlled for. In what follows, we compare the models with and without investment.

5.3 Data

Table 5 presents descriptive statistics for 104 observations covered in the reference analysis. Appendix C details the data sources. Appendix D, that lists the 104 countries, clarifies that the sample consists largely of low- and medium-income countries. This is because the WBES, which has been used to estimate idiosyncratic risk, predominantly covers those countries. The average annual growth rate of real GDP per capita is 2.86%. Idiosyncratic risk, estimated above as the conditional dispersion of firms' real sales growth rate, is 8.74% on average. The recovery rate is averaged at 33.12 cents, with a substantial variation across countries (as in Figure 4). The maximum (minimum) of 75.91 (7.33) cents corresponds to Sweden (Burundi). While the initial real GDP per capita (in PPP terms) is 8978 international dollars on average, it also exhibits a large variation. The mean of the initial human capital, measured by the enrollment rate at the primary level, is 81.63%. The mean population growth rate is 1.29%. The aggregate volatility, proxied by the standard deviation of annual growth rate of real GDP per capita, is 2.95% on average. The mean of investment rate as the ratio of total investment to GDP is 23.9%. Since the theory above assumes the environment where all the firms have access to bank financing, the robustness check below takes account of the proportion of firms with bank loans. On average, 38.6% of firms have a bank loan, while the standard deviation is substantial.

5.4 Results

Table 6 presents the estimation results. The dependent variable is annual growth of real per capita GDP (PPP adjusted) in percent, averaged over the 2004–17 period. Column (1) examines the association between the log of idiosyncratic risk and growth, controlling only for the log of initial real GDP, initial human capital, and population growth rates. The coefficient on idiosyncratic risk of 0.9 indicates that an increase in idiosyncratic risk of 1% is associated with an increase in the

Table 6. Idiosyncratic risk and growth: the role of bankruptcy costs

				Bank finance			
Sample	All	All	All	High	Low	All	All
Regressors	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Idiosyncratic risk (log)	0.90	0.90	−11.36***	−14.62**	−10.58*	−12.01***	−6.09*
	(1.40)	(1.40)	(−2.99)	(−2.29)	(−1.80)	(−3.31)	(−1.73)
Recovery (log)		−0.01	−7.52***	−9.82***	−6.61	−7.99***	−4.40**
		(−0.04)	(−3.22)	(−2.76)	(−1.65)	(−3.53)	(−2.02)
Idiosyncratic risk*Recovery			3.44***	4.39**	3.20*	3.59***	1.89*
			(3.35)	(2.62)	(1.89)	(3.59)	(1.97)
Initial real GDP pc (log)	−1.20***	−1.20***	−1.27***	−0.96***	−1.68***	−1.18***	−1.00***
	(−4.21)	(−3.91)	(−4.58)	(−3.12)	(−3.58)	(−4.39)	(−4.27)
Initial human capital	0.03	0.03	0.04**	0.06***	0.03*	0.03**	0.02
	(1.64)	(1.64)	(2.13)	(2.71)	(1.74)	(2.00)	(1.65)
Population growth	−0.83***	−0.83***	−0.79***	−0.66***	−0.94***	−0.88***	−0.69***
	(−4.85)	(−4.84)	(−4.77)	(−3.18)	(−3.26)	(−4.77)	(−4.60)
Aggregate volatility (log)						−0.57	
						(−1.34)	
Investment							0.14***
							(5.73)
Constant	10.22***	10.24***	37.03***	39.45***	37.89***	39.10***	21.39**
	(3.16)	(3.16)	(4.11)	(2.78)	(2.77)	(4.55)	(2.50)
Observations	104	104	104	52	52	104	104
Adj. R-squared	0.294	0.286	0.337	0.414	0.291	0.355	0.520

Notes: OLS estimations. Dependent variable is annual growth of real per capita GDP (PPP adjusted) in percent, averaged over the 2004–17 period. Idiosyncratic risk (log-transformed) is the country-specific standard deviation of an unexplained component of firms' sales growth rates in percent. Recovery rate (log-transformed) is cents to the dollar recovered by secured creditors through reorganization, liquidation, or debt enforcement proceedings, averaged over the 2004–17 period. Initial real GDP per capita (PPP adjusted) is the value in 2003, log-transformed. Initial enrollment rate is the percentage of the population of relevant age enrolled in primary school, averaged over the 1993–95 period. Population growth rate and investment (the ratio of total investment to GDP) are both in percent, averaged over the 2004–17 period. To divide the sample into countries with the proportion of firms with bank loans being larger (and smaller) than the median in Columns (4) (and (5)), we use the value based on the firm-level data from the WBES for the whole sample period. Aggregate volatility is measured by the standard deviation of annual growth rates of real per capita GDP over the 2004–17 period. Robust t-statistics are in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

annual growth rate of 0.009 percentage points, although the coefficient is not statistically significant. Column (2) adds (log of) recovery rates as a separate control, showing that idiosyncratic risk and recovery rates are both insignificant.[44,45] Column (3) adds the interaction term between idiosyncratic risk and recovery rates, showing that the interaction coefficient is *significantly positive*.[46] Given that the recovery rate is inversely related to bankruptcy costs, this result is consistent with the theoretical result illustrated in Figure 3 above, where as bankruptcy costs become higher, a given increase in idiosyncratic risk has a greater growth-reducing impact.

Columns (4) and (5) conduct the same analysis for the sub-sample of countries characterized by the proportion of firms with bank loans being larger and smaller than the median, respectively. Because the theory considers the interaction effect between bankruptcy costs and idiosyncratic risk in the environment where all the firms have bank loans (although the size of the loans differs across firms), we might expect that the result on the interaction effect does not necessarily holds for countries where the proportion of firms with bank loans is rather low. The positive and significant interaction coefficient in Column (4), together with the smaller and less significant coefficient

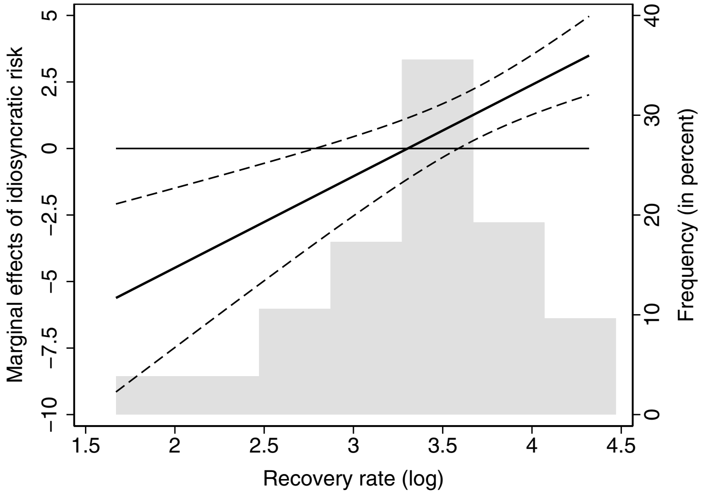

Figure 5. Marginal growth effects of idiosyncratic risk across different recovery rates.
Notes: Corresponds to Column (3) of Table 6. Solid line represents a marginal effect. Dashed line represents a 90% confidence interval. Histogram shows the distribution of the log of recovery rates across 104 countries.

in Column (5), is somewhat in line with this expectation. Next, Column (6), using the whole sample, indicates that even when the macroeconomic volatility variable is controlled for, the result on the interaction coefficient remains the same.[47] The coefficient on macroeconomic volatility is negative, albeit not statistically significant.

To shed further light on the result on the interaction, Figure 5 plots the estimated marginal effect of idiosyncratic risk across different recovery rates using the model of Column (3). The figure also shows the 90% confidence intervals and the distribution of the recovery rates (across the 104 countries). Consistent with the positive interaction coefficient, the marginal effect is positively sloped. The additional insight here is that the marginal effect is negative and significant particularly when the recovery rate is sufficiently low, indicating the possibility that idiosyncratic risk can be growth *reducing* under such an environment.[48] However, the figure also shows that the effect becomes significantly positive when the recovery rate is sufficiently high. This suggests that there might be unaccounted factors (in the theoretical model) that make idiosyncratic risk growth *promoting*. Oikawa (2010), for example, shows theoretically that a rise in firm-level idiosyncratic risk increases productivity growth via a learning-by-doing mechanism in research sector.

Last, Column (7) adds investment to the reference result in Column (3), and shows that the coefficient on the interaction term between idiosyncratic risk and recovery rate is much smaller than the equivalent coefficient in Column (3) (1.89 vs 3.44). The indication is that a change in the recovery rate (thus bankruptcy costs) plays a less substantial role in the growth effect of idiosyncratic risk when investment is controlled for. This is in line with the theory that highlights the relevance of the capital accumulation channel to growth, thus offering some support for the use of the AK model above. To note, the other variables considered in the table (initial real GDP, initial human capital, and population growth) are associated with growth as expected, although initial human capital is marginally insignificant in some columns. Regarding coefficients on initial real GDP per capita (in log), they tend to be slightly lower than −1 (e.g., −1.27 in Column 3), suggesting that the annual convergence rate is slightly over 1% per year.[49]

5.5 Further robustness check: estimating two-period model

The exercise above assumes implicitly that idiosyncratic risk is time invariant. However, it is well-known that idiosyncratic risk can show a time trend [e.g., Campbell et al. (2001), Comin and

Mulani (2006), and Hamao et al. (2007)]. The recovery rate can also change over time, although Figure 4 indicates that the variation is relatively small in LICs and MICs.

To take account of these possible time variations, we first estimate idiosyncratic risk for two separate periods using the data reported in the WBES over the 2006–12 and 2013–19 periods. The estimation methodology is based on equations 24 and 25. With the 2-year lag as before, the log of the recovery rate for the 1st (2nd) period is the average over the 2004–10 (2011–17) period. Then, adjusting the other variables of the growth regression accordingly, we use OLS on the pooled data that uses the variation between the 1st and 2nd period of 2004–10 and 2011–17.[50] While we add a period dummy, which takes the value of 1 if the period is the 2nd, we do not consider a model with country-fixed effects. The reason is twofold. First, since only one observation is available for many countries, running a fixed effects model loses a substantial number of observations. Second, because the model essentially becomes dynamic with the log of initial real GDP per capita as a control, it would suffer an endogeneity problem due to the fixed effects being correlated with the lagged dependent variable.[51]

Table 7 replicates Table 6 for this two-period setup. Again, the coefficients on idiosyncratic risk and the recovery rate, when considered separately, are not statistically significant (Columns (1) and (2)). However, the coefficient on the interaction term between those variables is positive and significant, suggesting that the role of bankruptcy costs in the growth effect of idiosyncratic risk still holds even when taking account of variations of idiosyncratic risk over time (Column (3)). In line with the theory, this is particularly the case when we consider the sub-sample of observations with the proportion of firms with bank loans being larger than the median [compare Columns (4) and (5)]. The result is also robust to controlling for aggregate volatility (Column (6)). Last, in Column (7) with investment as a control variable, the interaction term becomes insignificant, which again suggests the relevance of the capital accumulation channel to the growth effects.

5.6 Interaction effect on investment rate

As emphasized, the theory, based on the AK model, examined the interaction effect between bankruptcy costs and idiosyncratic risk through the capital accumulation channel. To recap, the theoretical insight was that with bankruptcy costs present, an increase in idiosyncratic risk enlarges the wedge between the marginal product of capital and its rental rate, making investment less attractive for households and thus slowing down capital accumulation.[52] Acknowledging this, Tables 6 and 7 provided supporting evidence for the relevance of the capital accumulation channel, by controlling for the investment rate. Here, we offer additional evidence by estimating the interaction effect between bankruptcy costs and idiosyncratic risk directly on investment. This further helps justify the focus on capital accumulation using the AK model.

Specifically, we consider the following model similar to equation 26:

$$\overline{priv\_inv}_k = \alpha + \beta\, risk_k + \gamma\, \overline{rec}_k + \delta\, risk_k * \overline{rec}_k + \sum_{l=1}^{n} \eta_l\, z_{k,l} + \varepsilon_k, \tag{27}$$

where $\overline{priv\_inv}_k$ is an annual *private* investment rate (defined as private investment/GDP, in percent) in country k, averaged over the 2004–17 period. We use the private investment to be consistent with the lack of explicit modeling of public investment in the theory. The idiosyncratic risk measure, $risk_k$, and the recovery rate, $\overline{rec}_k$, are as defined for equation 26, and our focus is again on the interaction coefficient, δ. Based on prior works of Servén (2003) and Cavallo and Daude (2011) on private investment, $z_{k,l}$, a vector of control variables, includes domestic credit extended to the private sector (% of GDP, log transformed), the price level of capital formation (log transformed), and general government investment (% of GDP). To check robustness, we also control for real interest rate, although the number of observations falls substantially.[53] Further,

Table 7. Robustness check: Two-period model (2004–10 and 2011–17)

				Bank finance			
Sample	All	All	All	High	Low	All	All
Regressors	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Idiosyncratic risk (log)	1.05	1.07	−5.47*	−5.84**	−4.61	−4.81	−1.70
	(1.43)	(1.45)	(−1.75)	(−2.37)	(−0.86)	(−1.45)	(−0.57)
Recovery (log)		0.15	−4.20**	−4.90**	−3.20	−3.84*	−1.79
		(0.43)	(−2.05)	(−2.67)	(−0.86)	(−1.81)	(−0.91)
Idiosyncratic risk*Recovery			1.94**	2.03**	1.73	1.75*	0.74
			(2.31)	(2.51)	(1.16)	(1.95)	(0.89)
Initial real GDP pc (log)	−0.93***	−0.98***	−0.96***	−0.88**	−1.31***	−0.87***	−0.72***
	(−3.43)	(−3.37)	(−3.48)	(−2.68)	(−2.89)	(−3.05)	(−3.06)
Initial human capital	0.04**	0.04**	0.04***	0.08***	0.03	0.04**	0.03**
	(2.24)	(2.21)	(2.65)	(3.82)	(1.61)	(2.22)	(2.54)
Population growth	−0.71***	−0.70***	−0.68***	−0.65**	−0.85***	−0.77***	−0.59***
	(−3.77)	(−3.67)	(−3.62)	(−2.52)	(−2.79)	(−3.48)	(−3.58)
Investment							0.15***
							(4.97)
Aggregate volatility (log)						−0.46	
						(−1.02)	
Period dummy	−0.94***	−0.97**	−0.97**	−0.72*	−1.12*	−1.16***	−0.88**
	(−2.66)	(−2.52)	(−2.57)	(−1.92)	(−1.74)	(−2.72)	(−2.53)
Constant	6.97**	6.74**	20.96***	18.93***	21.28	19.92**	8.76
	(2.18)	(2.04)	(2.72)	(3.29)	(1.67)	(2.56)	(1.13)
Observations	152	152	152	76	76	152	152
Adj. R-squared	0.200	0.196	0.210	0.330	0.137	0.221	0.379

Notes: OLS estimations. Dependent variable is the annual growth of real per capita GDP (PPP adjusted) in percent, averaged over the 7-year periods of 2004–10 and 2011–17. Period dummy takes the value of 1 if the period is 2 (i.e., 2011–17). Independent variables are adjusted accordingly (see footnote 50 for details). To divide the sample into observations with the proportion of firms with bank loans lager (smaller) than the median, we use, as in Table 6, the value for each country based on the firm-level data from the WBES for the whole sample period. t-statistics are in parentheses. Clustered standard errors are used to adjust for correlation of error terms within countries. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

we consider a two-period model in line with Table 7. Data sources and descriptive statistics for the analysis are in Appendix E.

Table 8 summarizes the results.[54] Columns (1) to (4) are based on the 1-period (2004–17) model. The punchline is that while the idiosyncratic risk and recovery rate are not significantly associated with private investment rate on their own, the coefficient on the interaction term is positive and significant in Column (3), suggesting that bankruptcy costs also play a key role in the association between idiosyncratic risk and private investment. To gauge this interaction better, Figure 6 visualizes the marginal effects of idiosyncratic risk on private investment across different recovery rates. It suggests that the effect on private investment is negative and significant only when the recovery rate is relatively small, i.e., bankruptcy costs are large. This is consistent with the theoretical insight above.[55] The result on the interaction effect is robust to the addition of real interest rates as an independent variable [Column (4)] and the use of the two-period setup [Columns (5) and (6)]. Among the controls, the private credit to GDP ratio shows a positive association with private investment.[56] Overall, the results offer additional evidence for the focus on the capital accumulation channel in the present paper's context.

Table 8. Bankruptcy costs, idiosyncratic risk, and private investment

Model	One-period				Two-period	
Regressors	(1)	(2)	(3)	(4)	(5)	(6)
Idiosyncratic risk (log)	−0.04	0.01	−35.27***	−29.15**	−18.87**	−15.22*
	(−0.02)	(0.01)	(−3.05)	(−2.15)	(−2.41)	(−1.86)
Recovery (log)		0.18	−22.21***	−18.31**	−12.25**	−10.12
		(0.17)	(−2.92)	(−2.01)	(−2.19)	(−1.60)
Idiosyncratic risk*Recovery			10.13***	8.29**	5.73**	4.77*
			(3.03)	(2.10)	(2.46)	(1.87)
Private credit (log)	1.85***	1.80**	1.82**	2.16**	1.72***	2.25***
	(2.89)	(2.54)	(2.50)	(2.63)	(2.63)	(3.13)
Price level of investment (log)	−3.06	−3.15	−2.10	−2.21	−2.94	−3.94
	(−1.57)	(−1.55)	(−1.02)	(−0.88)	(−1.43)	(−1.56)
Government investment rate	−0.20	−0.20	−0.28	−0.23	−0.30	−0.35
	(−1.01)	(−1.01)	(−1.30)	(−1.02)	(−1.51)	(−1.50)
Real interest rate				−0.02		−0.05
				(−0.23)		(−0.54)
Period dummy					−0.37	0.57
					(−0.47)	(0.55)
Constant	10.11*	9.53	88.89***	74.35**	51.76***	41.27*
	(1.91)	(1.53)	(3.31)	(2.26)	(2.70)	(1.93)
Observations	100	100	100	80	141	107
Adj. R-squared	0.0629	0.0532	0.115	0.0951	0.0725	0.102

Notes: OLS estimations. Dependent variable for Columns (1) to (4) (Columns (5) and (6)) is an annual private investment rate (private investment/GDP in percent), averaged over the 2004–17 period (2004–10 and 2011–17 periods). Independent variables are adjusted accordingly. Period dummy takes the value of 1 if the period is 2 (i.e., 2011–17). Robust t-statistics are in parentheses. Clustered standard errors are used to adjust for correlation of error terms within countries for Columns (5) and (6). *** $p < 0.01$, **$p < 0.05$, *$p < 0.1$.

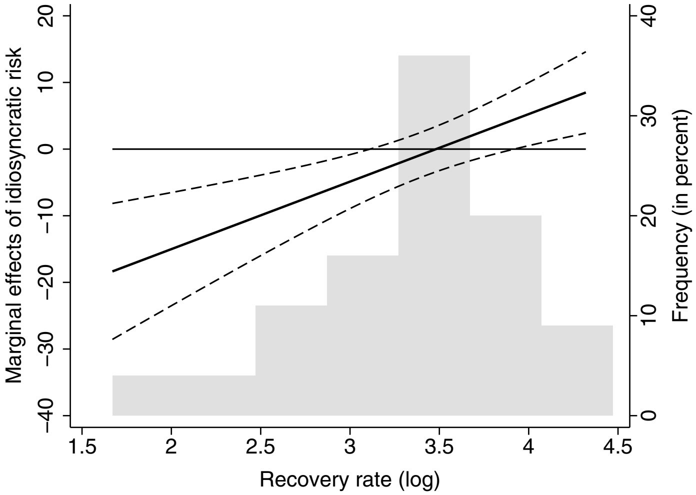

Figure 6. Marginal effects on investment of idiosyncratic risk across different recovery rates.
Notes: Corresponds to Column (3) of Table 8. Solid line represents a marginal effect. Dashed line represents a 90% confidence interval. Histogram shows the distribution of the log of recovery rates across 100 countries.

6. Concluding remarks

We examined how bankruptcy costs interact with idiosyncratic risk in the context of economic growth. The AK model with bankruptcy costs developed in this paper suggests that an increase in idiosyncratic risk reduces growth. When bankruptcy procedures are costly for financial intermediaries, and debtors (firms) privately observe idiosyncratic shocks they are subject to, an increase in idiosyncratic risk, modeled by a mean-preserving spread of idiosyncratic shocks, distorts households' investment/saving decision as ultimate savers, thereby lowering growth. The growth-reducing effect of idiosyncratic risk is stronger when bankruptcy costs are greater. Empirically, we estimated for a large number of countries the conditional dispersion of firms' real sales growth as a proxy for country-specific idiosyncratic risk. A cross-country growth regression that uses the idiosyncratic risk measure along with World Bank's estimates of recovery rates suggests that, in line with our theoretical model, an increase in idiosyncratic risk has a more adverse growth effect when bankruptcy costs are higher. Further, evidence demonstrates that the capital accumulation channel plays a key role in the interaction effect on growth, supporting the use of the AK model.

As stated in the Introduction, the limitation of the AK model is that it disregards technological progress as a key factor in understanding the process of economic growth. While we provided evidence that suggests the relevance of the capital accumulation channel within the present paper's context, it is clearly a fruitful exercise to consider the growth effect of the interaction between bankruptcy costs and idiosyncratic risk in a model with endogenous technological progress. That said, this paper's fundamental contribution is still to highlight the importance of considering the interaction effect between bankruptcy costs and idiosyncratic risk on long-run outcomes. In this regard, the novel insight from this paper is that an institutional reform that reduces the costs incurred by creditors during the bankruptcy procedure might have an overlooked benefit. That is, while it is inevitable that debtors face idiosyncratic shocks, such a reform might help an economy withstand better the possible long-run adverse effects of an increase in idiosyncratic risk.

Notes

1 Christiano et al., (2014) and Chugh (2016) highlight "monitoring costs" as a source of financial frictions. These costs are incurred by creditors to monitor firm-level idiosyncratic shocks that are realized after the firms receive financing. However, as explained below, monitoring costs are interpretable as bankruptcy costs in the standard debt contract environment they consider.

2 Other papers also examine the business cycle effects of risk shocks in the presence of bankruptcy costs [e.g., Williamson (1987), Dorofeenko et al. (2008), and Letendre and Wagner (2018)]. Among them, Letendre and Wagner (2018), using an open-economy model with bankruptcy costs, show that risk shocks have a rather limited impact on the model's dynamics. Their results suggest that modeling choices could affect the relevance of risk shocks to business cycles in a significant manner.

3 It is well-known that idiosyncratic risk exhibits a counter-cyclical pattern [e.g., Higson et al. (2004), Bachmann and Bayer (2013), and Bloom et al. (2018)].

4 The WBES data used for this research was downloaded in May 2020.

5 One possible theoretical extension is to consider a framework where investors face the choice between investing into capital and patents [in line with Romer (1990)]. In such a framework, the interaction between idiosyncratic risk and bankruptcy costs might affect growth through the no-arbitrage condition for investment.

6 Levine (1998) measures banking development by the value of loans made by commercial banks and other deposit taking banks to the private sector divided by GDP. Levine (1999), following King and Levine (1993), uses four indicators of financial intermediary development, including the ratio of liquid liabilities of the financial system to GDP.

7 Relatedly, Haselmann et al. (2010) find that collateral law (regarding what type of assets can be pledged as collateral) is more relevant to the supply of bank credit than bankruptcy law (regarding the systematic and organized implementation of liquidation process) in transition economies. More recently, using reforms across Eastern European countries, Campello and Larrain (2016) demonstrate that enlarging the menu of assets legally accepted as collateral increases firm borrowing and investment.

8 Relatedly, Rodano et al. (2016) find that the Italian bankruptcy law reform that improved creditor rights in liquidation reduced interest rates and increased firm investment. Meanwhile, Neira (2017) shows theoretically that as the bankruptcy costs increase, lenders shift their portfolio of loans toward smaller and less productive firms, and lend less.

9 To note, a large number of studies investigate the relevance of aggregate (macro-level) volatility to growth [e.g., Ramey and Ramey (1995), Jones et al. (2005), Imbs (2007), Koren and Tenreyro (2007), and Choi et al. (2018)], as opposed to idiosyncratic (micro-level) volatility which the present paper focuses on.

10 As emphasized above, our focus here, with the use of an AK model, is essentially on the endogenous capital accumulation rather than the technological progress. In the context of the latter, however, there is another important strand of the literature worth mentioning. In particular, several recent papers incorporate financial frictions and/or idiosyncratic uncertainty into a model with endogenous technological changes to understand issues surrounding the business cycle-growth nexus [e.g., Anzoategui et al. (2019), Bianchi et al. (2019), Ikeda and Kurozumi (2019), Sedláček (2020), Cozzi et al. (2021), and Vinci and Licandro (2021)]. One key difference relative to the present paper is that those works do not highlight bankruptcy costs as a source of financial frictions - with Vinci and Licandro (2021) being an exception, although the paper does not consider their interaction with idiosyncratic risk.

11 The assumption here is that the intertemporal elasticity of substitution of consumption is unity. To check robustness, we also considered the case where the elasticity is less than one, which is considered to be more empirically plausible. We confirmed that all the key results of the theoretical model shown below are robust to the alternative use of the elasticity of 0.5, the mean value of the elasticity from the meta-analysis conducted by Havranek et al. (2015). Thus, to avoid unnecessary complications in the analytical exposition associated with the use of CRRA functional form in equation 1, we stick to the logarithmic form (and assume that the elasticity is unity). All the algebraic derivations and results for the alternative case are available from the authors upon request.

12 In the households' budget constraint (equation 2), the return from lending to firms through financial intermediaries does not appear. This is because when lending takes place within a period (as clarified below), the deposit rate is zero in equilibrium.

13 In related literature, Aysun and Honig (2011), for instance, interpret monitoring costs as bankruptcy costs explicitly, and examine the role of these costs in the output loss following sudden stops of capital inflows. Their model shows that there is an inverted U-shaped relationship between bankruptcy costs and the output loss.

14 The first-order condition for firm j is $r_t^k = (1 - \Theta(\overline{\omega}_{j,t}))A_t$. Since the rental rate, r_t^k, and the aggregate productivity parameter, A_t, are common across entrepreneurs, $\overline{\omega}_{j,t}$ is also common across them.

15 Strictly speaking, each firm's net worth includes a small exogenous transfer, $D_{j,t}$, from firms which exited in the previous period (mentioned below), i.e., $N_{j,t} = (r_t^k + 1 - \delta)K_{j,t}^e + D_{j,t}$. This is to ensure that all firms, including the ones who went bankrupt in the previous period (and those newly entered to replace ones exited), have non-zero initial net worth. However, by assuming that this transfer is infinitesimally small, this transfer element can be ignored.

16 Another possible strategy to avoid a situation in which firms become self-financed is to assume that they maximize their inter-temporal utility with a lower discount rate than households [as in Carlstrom and Fuerst (1997, 1998)].

17 Carlstrom and Fuerst (1997) (page 900) express the view that part of bankruptcy costs is a deadweight loss. Admittedly, not all the bankruptcy costs are necessarily a deadweight loss, because some type of costs such as wage payments by financial intermediaries to workers (e.g., in accounting sector) during the bankruptcy process are not a true loss within the economy. Thus, the assumption embodied in equation 15 is essentially for simplification. In the theoretical literature, it is common to treat bankruptcy costs as a deadweight loss [e.g., Bernanke et al. (1999)].

18 Intuitively, a higher discount factor promotes households' capital accumulation, pushes down its rental rate, and thus increases firms' return on internal funds, whereas a higher firms' propensity to invest makes their net worth more abundant and thus reduces its shadow price.

19 The general form is given by $\phi(\omega, \alpha, \beta, a, b) = \frac{1}{Beta(\alpha,\beta)} \frac{(\omega-a)^{\alpha-1}(b-\omega)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}$, where $Beta(\alpha, \beta) = \int_0^1 \omega^{\alpha-1}(1-\omega)^{\beta-1} d\omega$. The distribution exhibits symmetry and has a unit mean by setting $\alpha = \beta (= \rho)$ and $a = 0$ and $b = 2$.

20 The quarterly spread of 1.5% is based on the cross-country average of the World Bank's World Development Indicator, "risk premium on lending (prime rate minus treasury bill rate, %)". The annual average of all the countries for which the figure is available for 1983–2017 is 5.95%, consistent with a quarterly spread of 1.5%.

21 The World Bank's World Development Indicators, "Domestic credit to private sector (% of GDP)" is available for 1983–2017, providing cross-country averages of 44.4% for those countries where the figure is available.

22 The choice of the 1983–2017 period is to be in line with the period over which the targeted values of the credit spread and the credit-to-GDP ratio are obtained.

23 The shadow price, $\lambda(\overline{\omega})$ is purely determined by the discount rate, β, and firms' propensity to invest, $1 - \upsilon$, both of which are kept constant here (see equation 20).

24 When bankruptcy costs are a proportion of the realized, not expected, outcome, the output share going to financial intermediaries changes to: $g(\overline{\omega}_{j,t}) \equiv \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega + (1 - \Phi(\overline{\omega}_{j,t}))\overline{\omega}_{j,t} - \mu \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega$ (cf. equation 8).

25 The probability density function of a log-normal distribution with $E(\omega) = 1$ is $\phi(\omega) = \exp\left[\frac{-0.5(\ln \omega + 0.5\sigma^2)^2}{\sigma^2}\right]\frac{1}{\omega\sigma\sqrt{2\pi}}$, where the dispersion parameter is σ with the larger value associated with the larger dispersion of the distribution. With the log-normal distribution, hitting the targets yields $\sigma = 0.35$, $\upsilon = 0.099$, and $A = 0.049$. With $\mu = 0.1$ and 0.5 (with beta distribution), $\rho = 4.64$, $\upsilon = 0.041$, and $A = 0.047$, and $\rho = 7.16$, $\upsilon = 0.085$, and $A = 0.049$, respectively. When bankruptcy costs are a proportion of realized production, $\rho = 4.82$, $\upsilon = 0.052$, and $A = 0.047$. The discounted factor and depreciation rate of capital are $\beta = 0.99$ and $\delta = 0.031$ throughout.

26 The effect might appear to be economically sizable. However, the model, as clarified previously, is constructed to show how idiosyncratic risk affects growth via its interaction with bankruptcy costs through the capital accumulation channel. In a richer endogenous growth model with technological progress, idiosyncratic risk is likely to affect growth through other potential channel(s) that are *not modeled* in the present paper. In particular, to the extent that there is a channel through which a rise in idiosyncratic risk increases growth (e.g., as illustrated by Oikawa (2010) using a model with endogenous technological change), the quantitative growth effect of idiosyncratic risk is likely to be smaller.

27 In our theory above, firm-level stochastic disturbances are modeled as idiosyncratic productivity shocks. Thus, it would be interesting to estimate the dispersion of the growth rates of firms' total factor productivity (TFP). However, particularly in the present paper's context where a large number of countries are considered to exploit the cross-country variations in the idiosyncratic risk and bankruptcy costs measures (as seen below), this alternative approach is difficult due to the limited data availability of firm-level TFP. For example, if Solow residuals are used to calculate the TFP growth, it would require international comparable firm-level data that contain information on labor as well as physical capital stocks, which is difficult to obtain.

28 The data used in this paper were downloaded in May 2020.

29 GDP deflator from the World Bank's Word Development Indicators (WDI) was used to calculate the real sales growth.

30 The former (latter) highlights bribery (foreign aid) as a possible determinant.

31 We also removed the sales levels of less than 1 US dollars per year, assuming that they are possibly due to a reporting error or an exchange rate conversion error.

32 In particular, the estimation below exploits the cross-country variation of the recovery rate as an inverse measure of bankruptcy costs (explained in Section 5.1).

33 The choice of many of firm characteristics used here, including the log of initial sales level, is fairly similar to Şeker and Yang (2014) and Chauvet and Ehrhart (2018).

34 The number of temporary workers is adjusted by the average number of months worked in a year.

35 To note, the previous findings on the coefficient on firm size are mixed. Some works find that firms' growth declines with a firm size [e.g., Evans (1987) and Hall (1987)]. Meanwhile, works that examine determinants of firms' sales growth using the WBES such as Şeker and Yang (2014) and Chauvet and Ehrhart (2018) find that the firm size in terms of the number of employees is positively associated with firms' sales growth.

36 The highest (lowest) value of idiosyncratic risk is obtained for Liberia (Montenegro). In general, we observe a negative and significant association between the country's idiosyncratic risk and income level, measured by real GDP per capita (PPP adjusted). Details are available from the authors upon request.

37 Doing business assumes that in the former case, 100% of the company's value is maintained, but in the latter case, 70% of the value can be recovered at maximum.

38 The focus on the 2004–17 period is to be in line with the reference growth regression below (Table 6). There, we consider the average of the annual growth rates of real GDP per capita over the 2004–17 period.

39 Countries are classified into different income groups based on real GDP per capita (PPP adjusted) in 2003, taken from IMF's World Economic Outlook. We categorize countries that belong to the 4th (3rd, 2nd, 1st) quarter of the income level as LICs (Lower_MICs, Upper_MICs, HICs), respectively.

40 Various macro studies on institutions also do not attempt to exploit their (seemingly small) time variation. For example, Keefer and Knack (2007) and Alfaro et al. (2008), in their estimation of the role of institutions in the level of public capital spending and in capital flows, respectively, use cross-country, rather than panel, regressions.

41 In the related literature of volatility and growth, Fatás and Mihov (2013), for example, use a similar framework based on Barro (1991) to estimate the effect of fiscal policy volatility on growth.

42 $\overline{rec}_k$ is calculated even when the recovery rate is not available for some years over the 14 years period.

43 This lagged measure of the primary school enrollment rate follows Fatás and Mihov (2013). To be exact, we take the average of the three-year period of 1993–95. This variable is from the World Bank's World Development Indicators (see Appendix C for detail). When there is a gap in some years in the series, we use interpolated values, and in case the variable becomes available only after 1993, we use the latest value available as a replacement for the preceding years.

44 To note, in line with Djankov et al. (2008), recovery rates are positively correlated with income *levels*: the correlation between the log of recovery rates and the log of initial real per capita GDP is 0.54.

45 While Column (2) shows that the association between recovery rates and growth is not significant, the theory above indicates that at the calibrated reference steady state (see Table 1), the marginal effect of bankruptcy costs on growth is negative, suggesting that a rise in recovery rates promotes growth (the detailed exposition is available from the authors upon request). We leave a thorough analysis of the unconditional effect of bankruptcy costs (or recovery rates) on growth as a relevant topic for a future research.

46 Commenting on the statistically significant coefficient of −7.52 on the log of recovery rate in Column (3), what this means is that when the log of idiosyncratic risk measure is *zero* (i.e., when idiosyncratic risk is 1%), an increase in the recovery rate by 1% is associated with a decrease in the annual GDP growth rate of about 0.075 percentage points. However, because no country in the sample has idiosyncratic risk as low as 1% (see Table 5), this coefficient itself is not relevant.

47 To report, we considered a model with more additional controls (the degree of openness as the sum of the shares of export and import in GDP, and CPI inflation rates), but the result on the interaction coefficient remains the same.

48 To illustrate, when the log of the recovery rate is 2.5 (corresponding to the recovery rate of 12.18 cents), an increase in idiosyncratic risk by 1% is associated with a decrease in the annual GDP growth rate of about 0.028 percentage points.

49 To be exact, the coefficient on initial GDP per capita (in log) is equal to $-(1-e^{-\beta*14})/14$, where β is the convergence rate [see, e.g., Barro and Sala-i Martin (2004) (page 517)]. When focusing on Column 3, where the coefficient is −1.27, the convergence rate β is equal to 1.4% per year, i.e., $-\frac{\ln(1-1.27/100*14)}{14}*100$.

50 To be precise, the annual growth rate of real GDP per capita (PPP adjusted) for the 1st (2nd) period is the average over the 2004-10 (2011-17) period. Initial real per capita GDP for the 1st (2nd) period is the value in 2003 (2010). Initial human capital for the 1st (2nd) period is the average of primary school enrollment rates over the 1993–95 (2000–02) period. Population growth rate and investment for the 1st (2nd) period are the respective averages over the 2004–10 (2011–17) period. Last, the macroeconomic volatility for the 1st (2nd) period is measured by the standard deviation of annual growth rates over the 2004–10 (2011–17) period.

51 It is difficult to mitigate this problem, because there are only two points in time, prohibiting the use of lagged variables as instruments.

52 It is actually straightforward to formalize what happens to investment itself. In the balanced-growth path equilibrium, the investment rate, defined as investment, $I_t(=K_{t+1}-(1-\delta)K_t)$ divided by output Y_t, is obtained as: $\frac{\eta-1+\delta}{A(1-\mu\Phi(\overline{\omega}))}$. Given that the growth rate, η is a function of μ, ρ, β, υ, δ, and A, and the threshold value of idiosyncratic risk, $\overline{\omega}$ is a function of μ, ρ, β and υ (equations 21 and 22), we can obtain the marginal effects of idiosyncratic risk on the investment rate across different levels of bankruptcy costs around the reference value of 0.31 (Table 1). We confirmed that in the presence of bankruptcy costs, a rise in idiosyncratic risk reduces the investment rate, and indeed, the effect becomes more negative as bankruptcy costs increase. These results are available from the authors upon request.

53 Servén (2003) and Cavallo and Daude (2011) also consider the role of real exchange rate uncertainty measured by the conditional variance of the residuals from estimating a GARCH (1,1) model for the variance and AR(1) in the conditional mean equation of the log of real exchange rate by country. We omit this variable because we only consider the relatively short period consistent with the idiosyncratic risk measure based on the WBES.

54 We here focus on countries which are covered in Table 6, to ensure comparability with the growth regression.

55 The fact that there is a hint of a positive impact of idiosyncratic risk on private investment when recovery rate is high is in line with Figure 5, suggesting again that there might be channel(s) that are not modeled in the theory.

56 Price level of investment and real interest rate have expected negative signs, albeit insignificant. The negative coefficient on government investment, though again insignificant, is consistent with Cavallo and Daude (2011) who find that private investment is crowded out by public investment in developing countries.

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Appendix A: Balanced-growth equilibrium

Here, we solve for the balanced-growth equilibrium of the AK model. For simplicity, we assume that the aggregate productivity parameter, A_t, and the cut-off value of idiosyncratic shock, $\overline{\omega}_t$, are constant. The other variables grow at the rate of η, except for the rental rate, r_t^k, which is constant. The model is characterized by the following relations.

$$\frac{C_{t+1}^h}{C_t^h} = \beta\left(r^k + 1 - \delta\right), \tag{A1}$$

$$C_t^h + K_{t+1}^h = (r^k + 1 - \delta)K_t^h, \tag{A2}$$

$$r^k = (1 - \Theta(\overline{\omega}))A, \tag{A3}$$

$$\Theta(\overline{\omega}) = \mu\Phi(\overline{\omega}) + \left(1 - \frac{1}{\lambda\,(\overline{\omega})}\right) f(\overline{\omega}), \tag{A4}$$

$$\lambda(\overline{\omega}) = \frac{1}{1 - \mu\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}}, \tag{A5}$$

$$f(\overline{\omega})\, AK_t = \lambda\,(\overline{\omega})\, N_t, \tag{A6}$$

$$N_t = (r^k + 1 - \delta)K_t^e, \tag{A7}$$

$$C_t^e = \upsilon A f(\overline{\omega})K_t, \tag{A8}$$

$$K_{t+1}^e = (1-\upsilon)Af(\overline{\omega})K_t, \tag{A9}$$

$$C_t = C_t^h + C_t^e, \tag{A10}$$

$$K_t = K_t^h + K_t^e, \tag{A11}$$

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t, \tag{A12}$$

$$Y_t = (1 - \mu\Phi(\overline{\omega}))AK_t. \tag{A13}$$

First, the first-order difference equation of equation A2 can be iterated forward to obtain

$$K_t^h = \sum_{j=0}^{\infty}\left(\frac{1}{r^k+1-\delta}\right)^{j+1} C_{t+j}^h + \lim_{T\to\infty}\left(\frac{1}{r^k+1-\delta}\right)^{T} K_{t+T}^h.$$

Since C_t^h grows at the rate of η in the equilibrium, the Euler equation of equation A1 gives

$$\eta = \beta\left(r^k + 1 - \delta\right). \tag{A14}$$

K_t^h also grows at the rate of η, indicating that the limit of the present value of the terminal value of households' capital holding, $\lim_{T\to\infty}\left(\frac{1}{r^k+1-\delta}\right)^T K_{t+T}^h = 0$. This means that

$$K_t^h = \sum_{j=0}^{\infty}\left(\frac{1}{r^k+1-\delta}\right)^{j+1} C_{t+j}^h.$$

This, in turn, gives the static relation which holds along the equilibrium path:

$$K_t^h = \frac{1}{(1-\beta)(r^k+1-\delta)}C_t^h. \tag{A15}$$

In equation A15, both K_t^h and C_t^h can be expressed as a linear function of K_t. First, equations A6, A7, and A11 yield

$$K_t^h = \left(1 - \frac{Af(\overline{\omega})}{\lambda(\overline{\omega})(r^k+1-\delta)}\right)K_t. \tag{A16}$$

Second, Eqs. A8, A10, A12, and A13 lead to

$$C_t^h = \Big((1-\mu\Phi(\overline{\omega}))A - \beta(r^k+1-\delta) + (1-\delta) - \upsilon Af(\overline{\omega})\Big)K_t, \tag{A17}$$

where $K_{t+1} = \eta K_t$ is used. Then, substituting equations A16 and A17 into equation A15, eliminating K_t, and using equation A3 to express r^k as a function of the cut-off value, $\overline{\omega}$, we obtain

$$\lambda\left(\overline{\omega}\right) = \frac{\beta}{1-\upsilon}.$$

This is equation 20 in the main text. As stated there, once the distribution function of idiosyncratic shocks is specified, the threshold value, $\overline{\omega}$ can be be solved. Then, using equations A3 and A14 gives the solution for the growth rate of η.

Appendix B. Effect of idiosyncratic risk on growth: robustness checks

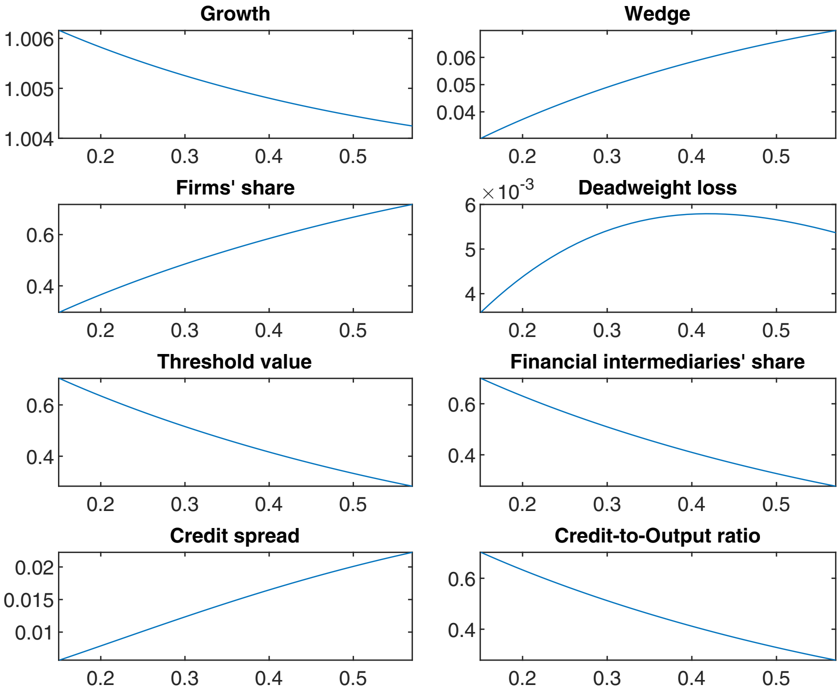

Figure A1. Effects of idiosyncratic risk on growth: Log-normal distribution.

Notes: Idiosyncratic shocks follow the log-normal distribution. The horizontal axis is the standard deviation of idiosyncratic shocks, *s*, around the reference value of $s = 0.36$, corresponding to the dispersion parameter of $\sigma = 0.35$ (see footnote 25 for details). σ is positively associated with the standard deviation by design. Parameter values other than σ are kept constant at the reference values with $\mu = 0.31$.

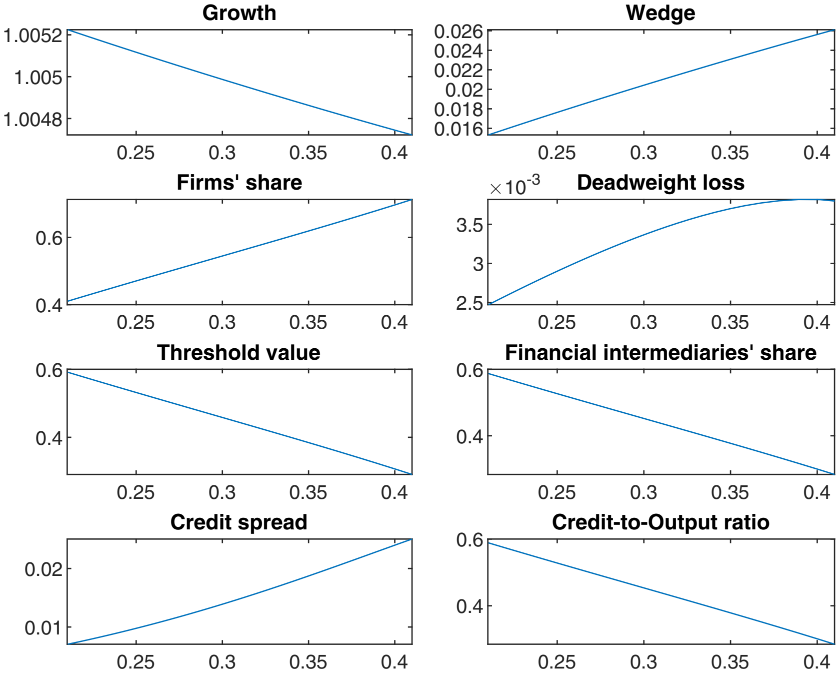

Figure A2. Effects of idiosyncratic risk on growth: $\mu = 0.1$.

Notes: The horizontal axis is the standard deviation of idiosyncratic shocks, *s*, around the reference value of $s = 0.31$, corresponding to the dispersion parameter of $\rho = 4.64$. Shocks follow the beta distribution. The dispersion parameter of the function, ρ is negatively associated with the standard deviation by design. Parameter values other than ρ are kept constant at the reference values with $\mu = 0.1$.

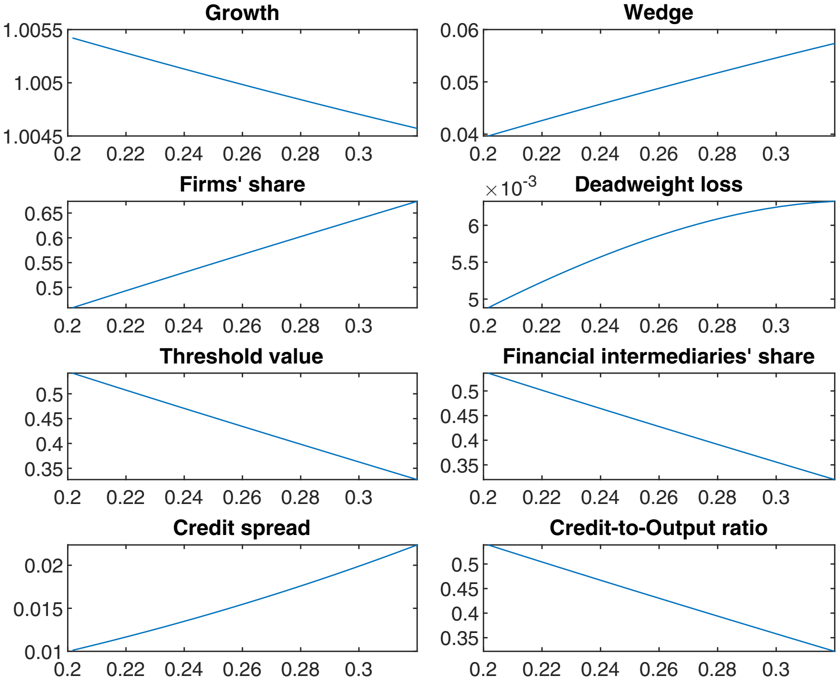

Figure A3. Effects of idiosyncratic risk on growth: $\mu = 0.5$.

Notes: The horizontal axis is the standard deviation of idiosyncratic shocks, s, around the reference value of $s = 0.26$, corresponding to the dispersion parameter of $\rho = 7.16$. Shocks follow the beta distribution. The dispersion parameter of the function, ρ is negatively associated with the standard deviation by design. Parameter values other than ρ are kept constant at the reference values with $\mu = 0.5$.

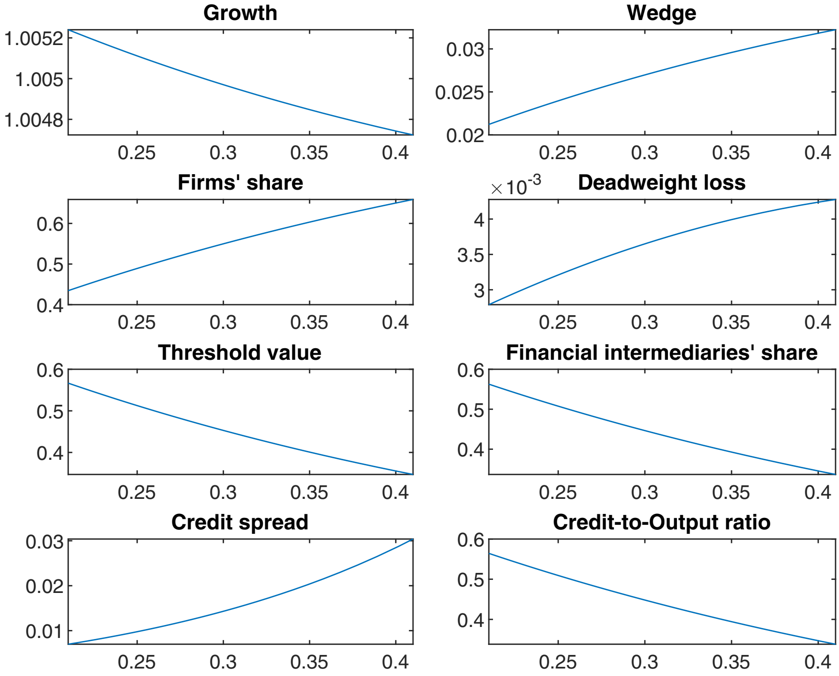

Figure A4. Effects of idiosyncratic risk on growth: Alternative way to model bankruptcy costs.
Notes: The horizontal axis is the standard deviation of idiosyncratic shocks, s, around the reference value of $s = 0.31$, corresponding to the dispersion parameter of $\rho = 4.82$. Shocks follow the beta distribution. The dispersion parameter of the function, ρ is negatively associated with the standard deviation by design. Parameter values other than ρ are kept constant at the reference values with $\mu = 0.31$. Bankruptcy costs are modeled as a proportion of the realized, rather than expected, production outcome.

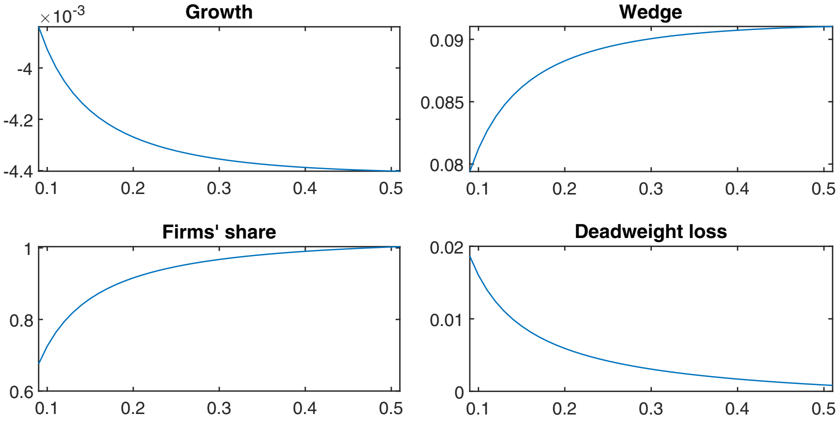

Figure A5. Marginal effects of idiosyncratic risk on growth across different bankruptcy costs: Log-normal distribution.
Notes: Idiosyncratic shocks follow the log-normal distribution. The horizontal axis is the bankruptcy costs parameter, μ and the vertical axis is the marginal effect of the standard deviation of idiosyncratic shocks on the respective variable. $\beta = 0.99$, $\upsilon = 0.099$, $\sigma = 0.35$ ($s = 0.36$), $\delta = 0.031$, and $A = 0.049$ throughout. σ is the dispersion parameter specific to the log-normal distribution.

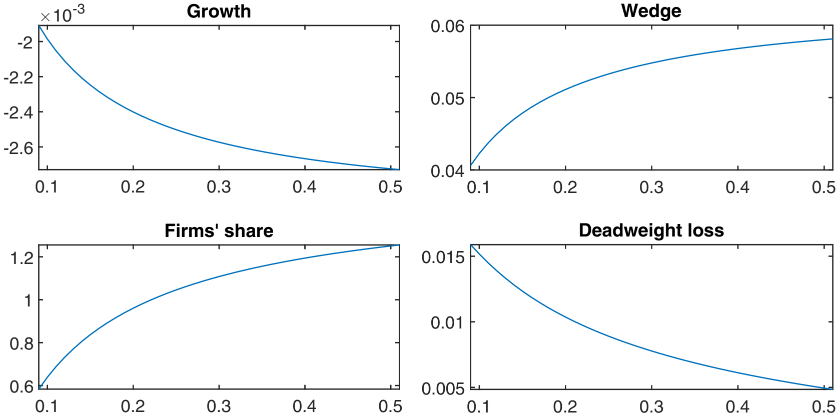

Figure A6. Marginal effects of idiosyncratic risk on growth across different bankruptcy costs: Bankruptcy costs as a proportion of realized production outcome.

Notes: Idiosyncratic shocks follow the beta distribution. The horizontal axis is the bankruptcy costs parameter, μ and the vertical axis is the marginal effect of the standard deviation of idiosyncratic shocks on the respective variable. Bankruptcy costs are modeled as a proportion of the realized, rather than expected, production outcome. $\beta = 0.99$, $\upsilon = 0.052$, $\rho = 4.82$ ($s = 0.31$), $\delta = 0.031$, and $A = 0.047$ throughout. ρ is the dispersion parameter specific to the beta distribution.

Appendix C. Data description for growth regression

Table A1 summarizes data sources of variables relevant for the growth regression (cf. equation 26).

Table A1. Data sources for growth regression

Data	Sources
Growth rate of real per capita GDP, PPP adjusted[a]	IMF's World Economic Outlook (WEO)
Idiosyncratic risk	Our own estimation based on World Bank's Enterprise Surveys (WBES)
Recovery rate[b]	World Bank's Doing Business
Initial level of real per capita GDP, PPP adjusted	IMF's WEO
Initial enrollment rate at primary school level[c]	World Bank's World Development Indicators
Population growth rate[d]	United Nations
Aggregate volatility[e]	IMF's World Economic Outlook (WEO)
Investment ratio[f]	IMF's WEO
Bank finance[g]	WBES

Notes: (a): Total population data (used to convert the real GDP to per capita terms) is from United Nations. Average over the 2004-17 period. (b) A subcomponent of "resolving insolvency". (c) Human capital proxy. We use "school enrollment, primary (% net)", defined as "the ratio of children of official school age who are enrolled in school to the population of the corresponding official school age." (d) Log difference of total population. (e) Standard deviation of annual growth rates of real per capita GDP (PPP adjusted) over the 2004-17 period. (f) The ratio of gross fixed capital formation to GDP, covering both private and public investment. (g) The proportion of firms with bank loans.

Appendix D. List of 104 countries in the sample for growth regression

The following is the list of 104 countries corresponding to Tables 5 and 6. Countries are classified into different income groups based on real GDP per capita (PPP adjusted) in 2003 from IMF's World Economic Outlook. Countries that belong to the 4th, 3rd, 2nd, and 1st quarter of the income level are low-income countries (LICs), lower medium-income countries (lower MICs), upper medium-income countries (upper MICs), and high-income countries (HICs), respectively.

D.1. LICs (28)

Bangladesh; Benin; Burkina Faso; Burundi; Cambodia; Cameroon; Ethiopia; Gambia, The; Guinea; Kenya; Kyrgyz Republic; Lesotho; Madagascar; Malawi; Mali; Mozambique; Myanmar; Nepal; Rwanda; Senegal; Sierra Leone; Solomon Islands; Tajikistan; Tanzania; Togo; Uganda; Uzbekistan; Zambia.

D.2. Lower MICs (29)

Albania; Armenia; Azerbaijan; Bolivia; China; Cote d'Ivoire; El Salvador; Eswatini; Georgia; Ghana; Guatemala; Guyana; Honduras; India; Indonesia; Moldova; Mongolia; Morocco; Namibia; Nicaragua; Nigeria; Pakistan; Peru; Philippines; Serbia; Sri Lanka; Ukraine; Vietnam; Yemen, Rep.

D.3. Upper MICs (36)

Antigua and Barbuda; Argentina; Barbados; Belarus; Belize; Botswana; Brazil; Bulgaria; Chile; Colombia; Costa Rica; Croatia; Dominican Republic; Ecuador; Egypt, Arab Rep.; Estonia; Jamaica; Jordan; Kazakhstan; Latvia; Lithuania; Malaysia; Mauritius; Mexico; Montenegro; Panama; Paraguay; Poland; Romania; Russian Federation; Slovak Republic; South Africa; Thailand; Tunisia; Turkey; Uruguay

D.4. HICs (11)

Bahamas, The; Cyprus; Czech Republic; Greece; Hungary; Israel; Italy; Malta; Portugal; Slovenia; Sweden

Appendix E. Data description for investment regression

Table A2 summarizes data sources of variables relevant for the investment regression (cf. equation 27). Table A3 provides descriptive statistics.

Table A2. Data sources for investment regression

Data	Sources
Private investment rate (% of GDP)	IMF Investment and Capital Stock Dataset
Domestic credit extended to the private sector (% of GDP)	World Bank's WDI
Price level of investment (Price level of US output-side real GDP in 2011= 1)	Penn World Trade, version 9.1 (Feenstra et al. (2015))
General government investment rate (% of GDP)	IMF Investment and Capital Stock Dataset
Real interest rate (%)	World Bank's WDI

Table A3. Descriptive statistics for investment regression

Variables	Mean	Std. Dev.	Min.	Max.
Private investment/GDP (in percent)	17.39	5.02	5.95	29.56
Idiosyncratic risk	8.79	2.45	4.36	15.98
Recovery rate	33.04	15.63	7.33	75.91
Private credit/GDP (in percent)	46.38	35.51	5.43	212.6
Price level of investment	0.56	0.15	0.26	1.13
General government investment/GDP (in percent)	5.34	3.06	0	18.33
Real interest rate (in percent)	6.04	6.58	−6.07	35.25

Notes: 100 observations (countries), corresponding to Columns (1) to (3) of Table 8. The availability of real interest rate is limited (80 observations). All the variables (other than idiosyncratic risk) are an average over the 2004–17 period.

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