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Condition for magnetic insulation of the electron beam in a rod-pinch diode

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Abstract

A condition for the transition of the electron beam produced in a coaxial rod-pinch diode to the mode of magnetic insulation has been established from the law of conservation of particle and field momentum fluxes. The magnetic field of the external current has been shown to contribute twice as much to magnetic insulation of the beam as the magnetic field of the electron beam self-current. Based on the relations derived, a model has been constructed for magnetic insulation of the electron flow in high-current rod-pinch diodes, which are used for radiography of high speed phenomena. The obtained theoretical results agree well with the results of numerical calculations and with experimental data gained at the Naval Research Laboratory (USA).

Keywords: Electron beam, Magnetic insulation, Rod-pinch diode

1. INTRODUCTION

The rod-pinch diode incorporates a thin annular cathode of internal radius R_C, and thickness l and a rode anode of radius R_A that extends through a distance L beyond the cathode plane (Fig. 1). This extension L can be much longer than the anode-cathode (A-C) spacing $D = R_C - R_A$. The anode rod, as a rule, is connected to the positive high-voltage center conductor of a pulsed power generator and the rod-pinch diode normally has a large aspect ratio so that $R_C/R_A \gg 1$. This results in a high density of the absorbed electron energy for a small-radius anode, which produces the anode plasma, and allows for the ion current flow in the diode. In the presence of ions, the e-beam rapidly propagates along the rod from the A-K gap region to the rod tip, where it pinches ensuring a high-power X-ray source with a small ($\sim$1 mm) focal spot.

Pioneering experimental studies of coaxial rod-pinch diodes were conducted in Russia in 1962 (Zelensky *et al.*, 1968). The objective of these studies was to design efficient X-ray sources. In this article investigations were performed on a diode with an "inverse" cathode where the electrons acted upon by the magnetic field were shown to be forced out of the cathode section to the tip of the anode rod that improves the beam focusing.

Systematic experimental and theoretical studies of the characteristic of rod-pinch diodes (RPDs) got underway at the Naval Research Laboratory (NRL) in 1978 (Mahaffey *et al.*, 1978). Based on the results of these experimental investigations, the first phenomenological model which described the RPD operation was constructed. According to this model, the charged-particle flow in a rod-pinch diode may occur in three regimes: the space-charge-limited regime (the SPL regime), the transition regime, and the magnetically limited regime (the ML regime).

Numerical simulation thereafter supported the existence of three regimes and allowed introduction of criteria of the mode-to-mode transition of the diode (Swanekamp *et al.*, 2000). According to this model, the diode first operates in the SCL regime and the diode current is therewith primarily radial. For a cylindrical geometry, the electron current I_e proportional to the Langmuir–Blodgett current is

$$I_{LB} = (2e/m)^{1/2}(2U^{3/2}l_{eff}/9R_A k^2), \quad (1)$$

where k^2 is a function of $R_C/R_{A.}$, e and m are the charge and the rest mass of an electron, U is the voltage across the diode, and $l_{eff} = l + 2D$ and is the effective cathode length,

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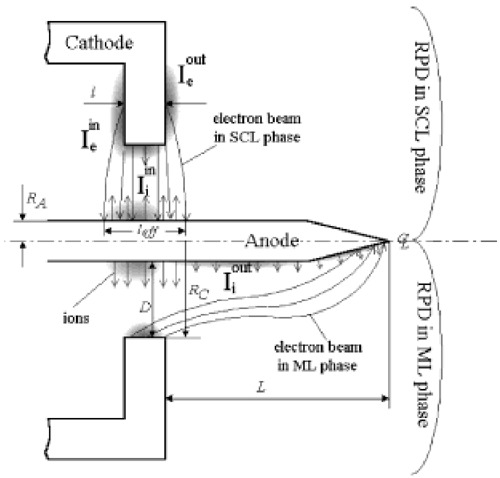

Fig. 1. Current scheme in RPD.

which takes into account the two-dimensional effect (Fig. 1). For a bipolar SCL flow,

$$I_{SCL} = \eta I_{LB} = 14.6\eta \frac{U^{3/2} l_{eff}}{R_A k^2}, \tag{2}$$

where the enhancement factor η is in the range from 1.5 to 3.

An important point of the RPD theory is substantiation of the criteria of the RPD transition to the ML regime. In the NRL model based on the results of calculations and on a model of magnetic insulation in the one-electron approximation, it is supposed that this transition occurs when the total diode current I_D approaches the critical current (Cooperstein *et al.*, 2001)

$$I_D = I_{crit}. \tag{3}$$

The critical current of self-insulation of the electron beam in a rod-pinch diode is therewith determined by the critical current of magnetic insulation in the one-electron approximation, I_C, and by the "scaling factor" α (an empirical factor):

$$I_{crit} = \alpha I_C; \qquad I_C = \frac{I_0}{2} \frac{\sqrt{\gamma^2 - 1}}{\ln(R_C/R_A)}, \tag{4}$$

where $\gamma = 1 + eU/m_e c^2$ is the ratio of the electron energy to the electron rest energy, where $I_0 = mc^3/e$ is the Alfven current, and α is a function of R_C/R_A. For the geometry and voltages used in NRL experiments, $\alpha \approx 1.5$–3.

Criterion (4) is based on analysis of experimental data, and preliminary calculations of the factor α for a given diode geometry are thus required. The physical meaning of this factor has yet to be ascertained. Therefore, the problem of the development of the theory describing the process of magnetic self-insulation in RPDs still remains urgent. Such a theory can be based on the law of conservation of the z-component of the field or particle momentum in the diode (Belomytsev *et al.*, 2001).

2. CONDITION OF MAGNETIC INSULATION OF THE ELECTRON BEAM IN A COAXIAL DIODE

Let there be a coaxial diode with an annular region of electron emission at the cathode (Fig. 2). The ion emission is ignored. In the stationary state, the z-component of the field and particle momentum in the volume marked by the dotted line is constant. Consequently, the flow of the z-component of the momentum through the boundary equals zero:

$$\frac{1}{8\pi} \int_{R_A}^{R_C} \left[\frac{2(I_b + I_{ext})}{cr} \right]^2 2\pi r\, dr - \frac{1}{8\pi} \int_{R_A}^{R_C} \left[\frac{2I_{ext}}{cr} \right]^2 2\pi r\, dr - P_z = 0 \tag{5}$$

where R_A, R_C are the anode and cathode radii, respectively, P_z is the z-component of the momentum transferred by electrons to the anode, c is the velocity of light in vacuum, $2(I_b + I_{ext})/cr$ is the magnetic field strength at the left boundary of the volume, $2I_{ext}/cr$ is that at the right boundary, I_b is the electron beam current, and I_{ext} is the external current. We ignore the contribution of the magnetic field of the ion current, assuming the ion current to be much lower than the electron current, and take no account of the contribution of ions to the z-component of the momentum, considering the ions to be weakly deflected in the magnetic field.

Let us find the maximum values of the currents I_b and I_{ext}, above which the steady-state flow of the current I_b is impossible without passage of the electron beam (or part thereof) through the face boundary of the marked volume. Obviously, this corresponds to the case where all electrons at the anode have only the z-component of velocity, that is,

$$P_z = \frac{I_b}{e} mc\sqrt{\gamma^2 - 1}. \tag{6}$$

By substituting (6) in (5), we obtain

$$I_b + 2I_{ext} = I_0 \frac{\sqrt{\gamma^2 - 1}}{\ln(R_C/R_A)}. \tag{7}$$

If the beam current $I_b \ll I_{ext}$, the external current I_{ext} starts magnetizing the beam current when the latter approaches

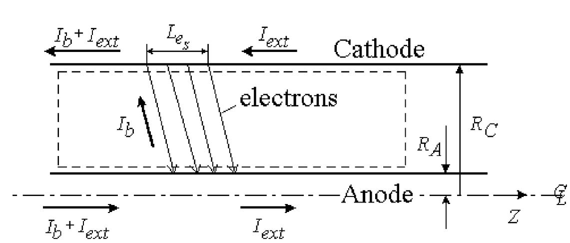

Fig. 2. Current scheme in coaxial diode.

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the known value of the critical current of magnetic insulation I_C in the one-electron approximation:

$$I_{ext} = I_C = \frac{I_0}{2}\frac{\sqrt{\gamma^2 - 1}}{\ln(R_C/R_A)}. \tag{8}$$

In the case of magnetic self-insulation ($I_{ext} = 0$), from (7) it follows that

$$I_b = I_0 \frac{\sqrt{\gamma^2 - 1}}{\ln(R_C/R_A)} = I_s. \tag{9}$$

Consequently, the current required for self-insulation of the beam, I_s, is two times higher than the critical current I_c (Ryzhov *et al.*, 2001):

$$I_S = 2I_C. \tag{10}$$

3. CONDITION FOR MAGNETIC INSULATION OF THE ELECTRON BEAM IN RPDS

To study the range of applicability of the condition of magnetic insulation (7), numerical simulation has been performed for the processes of magnetic insulation of the electron beam in cylindrical diodes of various geometries with the use of the KARAT/MC code. Numerical calculations for the RPDs shows that in such diodes the electron and ion currents $I_{out} = I_e^{out} + I_i^{out}$ to the right of the cathode face should be considered as the external currents responsible for the insulation of the electron beam in the interelectrode gap. Note that the electron current from the right face of the cathode is also external (Fig. 1). The self-insulation of the electron beam is effected by the magnetic field of the currents of electrons and ions, $I_{in} = I_e^{in} + I_i^{in}$, whose emission region is to the left of the right face boundary of cathode edge.

In this case, relation (7) takes the form

$$I_{in} + 2I_{out} = I_e^{in} + I_i^{in} + 2(I_e^{out} + I_i^{out})$$
$$= I_0 \frac{\sqrt{\gamma^2 - 1}}{\ln(R_C/R_A)} = 2I_C = I_S, \tag{11}$$

where R_A is the rod radius and R_C is the internal radius of a thin annular cathode. Relation (11) takes into account that magnetic insulation of the electron beam in a rod-pinch diode is realized both by the magnetic field of self-currents and by that of external currents which ensure a double efficiency of insulation.

The condition of magnetic insulation allows one to find a relation between the diode currents and the empirical "scaling factor" α used in the RPD model proposed at NRL. It is obvious that the total diode current is determined by the expression

$$I_D = I_{in} + I_{out} = I_e^{in} + I_i^{in} + I_e^{out} + I_i^{out}. \tag{12}$$

By substituting (4) in (11) and allowing for (12), we derive

$$\alpha = (2 - (I_e^{out} + I_i^{out})/I_C). \tag{13}$$

It follows from this formula that α depends on the diode geometry not only through the factor $\ln(R_C/R_A)$ involved in the expression for I_C, but also on what part of the diode current falls on the self-currents and external currents at each stage of the RPD operation.

To clarify the effect of the self-currents and external currents on the behavior of the electron beam in the ML regime, simulation has been performed for the modes of the RPD operation studied experimentally on the Gamble II setup. Figures 3 and 4 show examples of the waveforms calculated for the RPD current (Fig. 3) and the electron pulse plot (Fig. 4) obtained in numerical simulation of the RPD operation on the Gamble II setup (Shot #7402, $R_A = 0.32$ cm, $R_C = 0.8$ cm, $L_{rod} = 11$ cm, $U_{max} = 1.2$ MV) by the KARAT/MC code.

Analysis of the results (Ryzhov *et al.*, 2002) of simulation has shown that within the accuracy of calculations, condition (11) is fulfilled for the diode when operating in the mode of magnetic insulation (Fig. 3b, Fig. 4b). In so doing, the process of magnetic insulation can be divided into two stages: the initial stage of magnetic insulation and drift (trans-

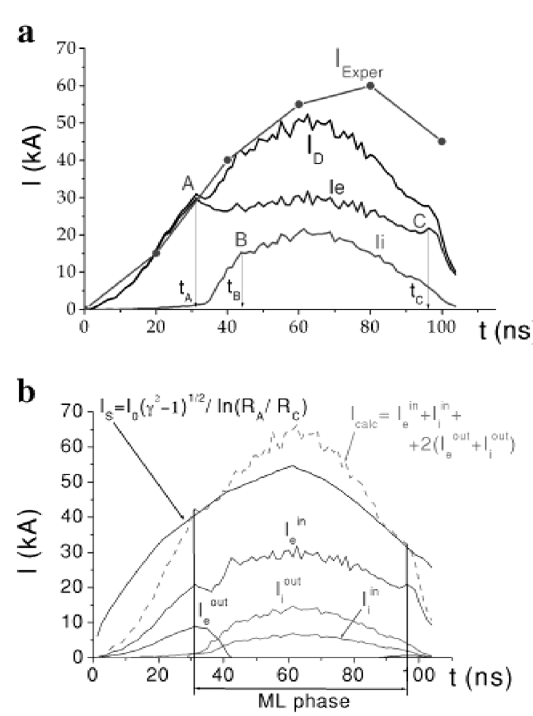

Fig. 3. Currents in rod-pinch diode. Example of the computing modeling (Gamble II Shot #7402, $R_A = 0.32$ cm, $R_C = 0.8$ cm, $L_{rod} = 11$ cm, $U_{max} = 1.2$ MV). a: I_D: diode current, I_i: ion current, I_e: electron current, I_{exper}: diode current measured experimentally. b: Comparison between calculation results and magnetic insulation condition (12); I_S: theoretical self-insulation current, $I_e^{in}, I_i^{in}, I_e^{out}, I_i^{out}$: RPD currents are calculated by PIC code KARAT/MC.

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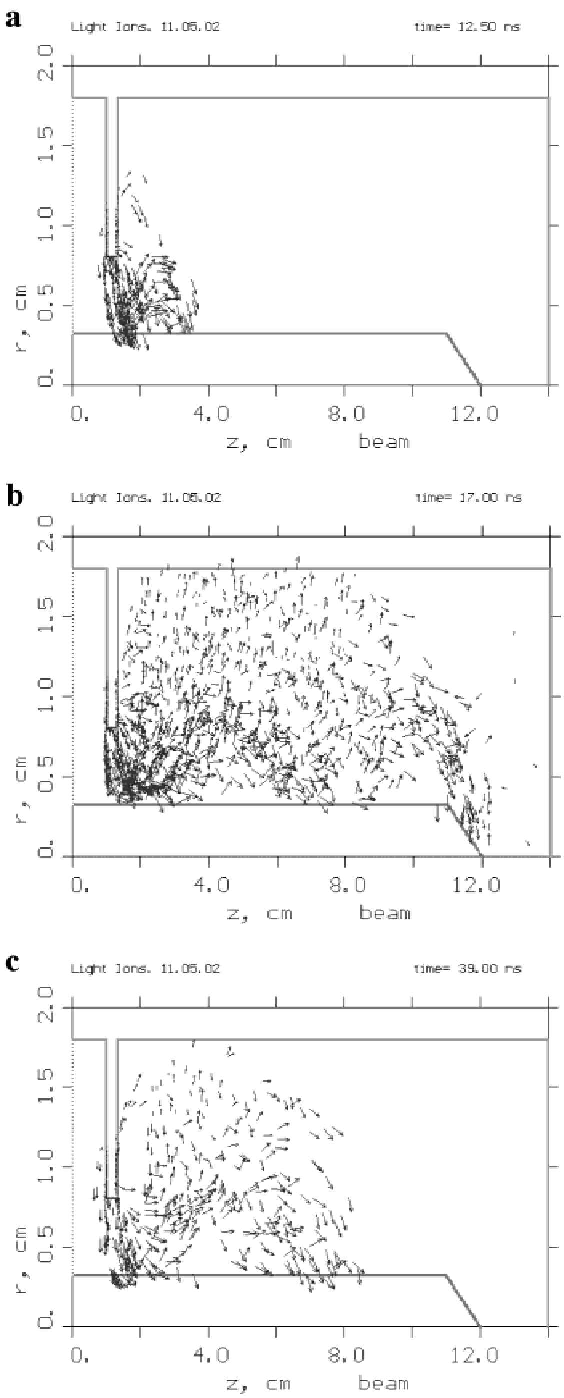

Fig. 4. Electron pulse plot obtained from KARAT simulation for Gambel II rod-pinch diode short #7402. a: $t = t_{Start}$, b: $t = t_{SS}$, and $t = t_{End}$. c: $t > t_{End}$.

portation) of the electron beam (the WP stage) and the steady-state stage of magnetic insulation (the ML stage).

The stage of a weak pinch (the WP regime) or the stage of transportation (from t_A to t_B, Fig. 3a; electron pulse plot, Fig. 4a). At this stage, the beam acted upon by the magnetic self-field starts being forced out of the under-cathode region and then pinches and drifts along the anode rod.

The condition of magnetic insulation for the gap begins to be fulfilled at point A. Essentially the whole of the ion current is still under the "cathode" and is self-current, while part of the electron current mainly from the lateral surface of the emitting part of the cathode is external to the gap. In this regime, the current I_i^{out} can be ignored and therefore the approximate condition of self-insulation will have the form

$$I_e^{in} + I_i^{in} + 2I_e^{out} \approx 2I_C = I_S. \quad (14)$$

To estimate the values of the empirical scaling factor α at this stage, it is possible to use the expression

$$\alpha \approx 2 - I_e^{out}/I_C, \quad \alpha \leq 2. \quad (15)$$

From this point on, the electron beam starts moving rightward along the anode rod, thus broadening the region of ion emission and, consequently, increasing the current I_i^{out}. The transformation of the electron beam into the pinched state and the formation of the space charge to the right of the cathode leads to the electric field redistribution in the gap such that part of the electron current from the lateral surface of the cathode appreciably decreases and nearly the whole of the electron current becomes self-current for the gap.

Once the electron beam has reached the rod tip (t_B), the quasi-steady-state stage of the RPD operation begins (the ML regime). In Figure 3 this regime corresponds to the time interval from t_B to t_C (see also Fig. 4b). The time it takes for this stage to occur determines the pulse duration of the X rays generated on retardation of the focused electron beam at the anode tip. At this stage, a large part of the ion current is found to the right of the cathode, and taking into account the fact that the electron emission from the cathode face is suppressed, the condition of magnetic insulation in the first approximation can be written in the form

$$I_e^{in} + 2I_i^{out} \approx 2I_C. \quad (16)$$

In this approximation, the expression for α takes the form

$$\alpha \approx 2 - I_i^{out}/I_C, \quad \alpha \leq 2. \quad (17)$$

At the moment in time t_C, the condition of magnetic insulation is no longer fulfilled,

$$I_e^{in} + 2I_i^{out} < 2I_C, \quad (18)$$

and the pinch starts decaying (Fig. 4c). The electrons which are out of the gap find themselves at the anode, which results in a slight increase in the electron current of the diode. The electrons emitted by the cathode at $t > t_C$ are not magnetized, and the beam again finds itself in the gap under the cathode. The beam current is determined by the law $I_D = I_{CL} \sim U^{3/2}$.

Up to the point B, there is a good agreement between the experimental and theoretical data for the currents. The fur-

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ther difference in the currents in our model is explained by the effect of expansion of the near-electrode plasma (both the anode and cathode plasmas), which is found in actual setups. This results in an increase in self-insulation current (because of decreasing interelectrode gap width) and, consequently, the current and the space charge of the transported electrons increase, which should increase the ion current I_i, too.

The assumptions postulated in the NRL model of magnetic insulation (4) are in contradiction with the results obtained from the condition of magnetic insulation (11). Actually, it follows from formula (13) that the value of α depends not only on the ratio between the cathode and anode radii, which is involved in the expression for the critical current, but on the external current of the diode as well.

The role of the electron current from the lateral surface of the cathode, I_e^{out}, at the instant the beam gets into the ML regime and the role of the ion current from the anode rode, I_i^{out}, in the steady-state regime of the electron pinch are thus seen to be important. As a last resort, in the case where these currents can be ignored, as opposed to the critical current, the empirical scaling factor α reaches its maximum value $\alpha_{max} = 2$. In actual conditions, $\alpha < 2$. Moreover, calculations have shown that I_{out}^3, I_{out}^i varies within a pulse at different stages of the RPD operation, and, consequently, the assumption that α is constant within a pulse, which is invoked in the NRL model, is invalid.

4. CONCLUSION

The results of numerical calculations for the generation and transportation of the electron beam in rod-pinch diodes by the KARAT/MC code support the theoretical conclusions and agree well with the experimental data gained on the GAMBLE-II (NRL) setups. Conditions for magnetic insulation of the electron beam have been found, showing that the scaling factor α in the NRL model cannot be higher than two and cannot take into account the effect of the currents flowing from the lateral surface of the cathode edge, since it is constant in this model.

Accounting for the influence of the near-electrode plasma will make it possible to allow for variations in the conditions of magnetic insulation within a pulse and to estimate accurately the pinch lifetime.

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