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Corrigendum: Dynamics of a susceptible—infected—susceptible epidemic reaction—diffusion model

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This corrigendum corrects a result in [1].

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1. Uniqueness of EE

In Theorem 3.10 of [1], we claimed that the model has a unique EE if $\mathcal{R}_0 > 1$. However, there is an error in the proof of the case $d_S < d_I$. The correct statement and proof of Theorem 3.10 should be as follows. This correction has no impact on the results of the subsequent paper [2], which considered the same model.

THEOREM 1.1. *If $\mathcal{R}_0 > 1$ and $d_S \geqslant d_I$, the model has a unique EE; if $\mathcal{R}_0 > 1$ and $d_S < d_I$, the model has an EE.*

In the proof of Theorem 3.10, the following changes should be made:

- Page 938, Line 2 from the bottom. 'it suffices to show that problem (3.15), (3.16) has a unique positive solution' should be 'it suffices to study the positive solution of problem (3.15), (3.16)'.
- Page 939, Line 9. 'there exists a unique $\tau_0 > 0$' should be 'there exists $\tau_0 > 0$'.

We are able to prove the uniqueness of EE if $d_S < d_I$ with an additional assumption $N/|\Omega| \geqslant \gamma/\beta$. In the proof of the following result, for any $u, v \in C(\overline{\Omega})$, we write $u \leqslant v$ if $u(x) \leqslant v(x)$ for all $x \in \overline{\Omega}$, $u << v$ if $u(x) < v(x)$ for all $x \in \overline{\Omega}$ and $u < v$ if $u \leqslant v$ but $u \neq v$.

THEOREM 1.2. *Suppose $d_S < d_I$ and $N/|\Omega| \geqslant \gamma/\beta$. Then the EE of the model is unique.*

Proof. By Proposition 3.2 and Remark 3.3, the assumption $N/|\Omega| \geqslant \gamma/\beta$ implies $\mathcal{R}_0 > 1$ and hence the model has at least one EE. To prove the uniqueness of the EE, by lemma 3.5, we only need to show that the positive solution of the following problem is unique:

$$d_I \Delta I + I\left(\frac{N}{|\Omega|}\beta - \gamma + d\frac{\beta}{|\Omega|}\int_\Omega I dx - \frac{d_I\beta}{d_S} I\right) = 0, \quad x \in \Omega, \tag{1.1}$$

$$\frac{\partial I}{\partial n} = 0, \quad x \in \partial\Omega, \tag{1.2}$$

where $d = d_I/d_S - 1$.

We first claim that for any positive solution I of (1.1)–(1.2), it satisfies that

$$\frac{d_I}{d_S} I > \frac{d}{|\Omega|}\int_\Omega I dx. \tag{1.3}$$

To see this, we rewrite (1.1) as

$$d_I \Delta I + \beta I\left(\frac{N}{|\Omega|} - \frac{\gamma}{\beta} + \frac{d}{|\Omega|}\int_\Omega I dx - \frac{d_I}{d_S} I\right) = 0. \tag{1.4}$$

Let $I(x_0) = \min\{I(x) : x \in \overline{\Omega}\}$ for some $x_0 \in \overline{\Omega}$. Then similar to the proof of lemma 3.6, by using the maximum principle we can show

$$\frac{d_I}{d_S} I(x_0) \geqslant \frac{N}{|\Omega|} - \frac{\gamma(x_0)}{\beta(x_0)} + \frac{d}{|\Omega|}\int_\Omega I dx.$$

Since $N/|\Omega| \geqslant \gamma/\beta$, we have

$$\frac{d_I}{d_S} I \geqslant \frac{d}{|\Omega|}\int_\Omega I dx. \tag{1.5}$$

If equality holds in (1.5), then I is constant. However, since $d_I/d_S > d$, this is impossible. Thus the claim is valid.

Now suppose I_1 and I_2 are two distinct positive solutions of (1.1)–(1.2). Define

$$k = \max\{\tilde{k} \geqslant 0 : \tilde{k} I_1 \leqslant I_2\}.$$

Interchanging I_1 and I_2 if necessary, we have $k \in (0, 1)$. Moreover, $kI_1 \leqslant I_2$ and $kI_1(x_1) = I_2(x_1)$ for some $x_1 \in \overline{\Omega}$.

Define a function $h : E \subset C_+(\overline{\Omega}) \to C_+(\overline{\Omega})$ by

$$h(I) = (a - d_I\Delta)^{-1} I\left(a + \frac{N}{|\Omega|}\beta - \gamma + d\frac{\beta}{|\Omega|}\int_\Omega I dx - \frac{d_I\beta}{d_S} I\right)$$

for $I \in E$, where

$$E = \left\{ I \in C_+(\overline{\Omega}) : \frac{d_I}{d_S} I \geqslant \frac{d}{|\Omega|}\int_\Omega I dx \text{ and } \|I\| \leqslant C \right\},$$

a is a positive constant, and $C = \max\{\|I_i\|, i = 1, 2\}$. We may choose a large enough so that both I_1 and I_2 are fixed points of h. Moreover, since $d > 0$, if a is large, then h is strictly increasing on E, i.e., $h(u) >> h(v)$ for $u > v$ and $u, v \in E$. For any $I \in E$, using (1.5), we can show $\tilde{k}h(I) \leqslant h(\tilde{k}I)$ for any $\tilde{k} \in (0, 1)$. In addition, if (1.3) holds, then $\tilde{k}h(I) << h(\tilde{k}I)$ for any $\tilde{k} \in (0, 1)$.

Hence, we have

$$kI_1 = kh(I_1) < < h(kI_1) \leqslant h(I_2) = I_2,$$

which contradicts $kI_1(x_1) = I_2(x_1)$. This proves the uniqueness of the positive solution of (1.1)–(1.2). □

References

1 K. Deng and Y. Wu. Dynamics of a susceptible—infected—susceptible epidemic reaction—diffusion model. *Proceedings of the Royal Society of Edinburgh Section A: Mathematics* **146** (2016), 929–946.

2 Y. Wu and X. Zou. Asymptotic profiles of steady states for a diffusive SIS epidemic model with mass action infection mechanism. *Journal of Differential Equations* **261** (2016), 4424–4447.

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