$$\pi = -\int_{x=-1}^{x=1} 2\sin^2\theta\, d\theta = 2\int_0^{\pi} \sin^2\theta\, d\theta.$$

This means that

$$\int_0^{\pi} \sin^2\theta\, d\theta = \frac{\pi}{2}.$$

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Feedback

On Note 108.28: Erik Vigren writes: At the time of submitting and proof reading my Note [1], I was unaware of [2] and references within.

In [1], we have

$$\gamma(x, y) = \frac{y}{\sqrt{\frac{y}{x} - 1}} \tan^{-1}\sqrt{\frac{y}{x} - 1}, \tag{1}$$

as the limit of a recursively defined sequence in which the initial entries are x and y.

In [2] two sequences are considered generated by $a_{n+1} = M(a_n, b_n)$ and $b_{n+1} = M'(a_{n+1}, b_n)$ where M and M' are means. My construction is in essence the same, which can be realised by viewing the sequence entries with even and odd indices as parts of two separate, yet connected, sequences. Of particular relevance is (1) in [1], equivalent to (but not identical to) (12) in [2]. A proof of (12) in [2] appears in [3] and relations reminiscent of the Corollary (4) in my Note are also to be found in both [2] and [3].

References

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