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Influence of ion temperature on plasma sheath transition

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Abstract. Using a two-fluid model, the ion transition from plasma sheath boundary is investigated taking into account the effect of the finite ion temperature. It is shown that by considering the effects of neutral-ion elastic collision on the sheath, there will be an upper as well as a lower limit for the ion transition velocity into the sheath. The dependency of upper and lower limits of the ion transition velocity on the ion temperature is investigated, and it is shown that the finite ion temperature only affects lower limits in non-hot plasmas.

1. Introduction

Plasma is separated from the wall by a sheath. The energy and flux of the ions and electrons that bombard the wall are determined by the properties of the sheath. A sheath forms a potential barrier so that the more mobile species, i.e. the electrons, are confined electrostatically. The subject of plasma sheath is not yet understood, and a variety of models have been used to describe the characteristics of the sheath, such as the energy and number density of the ions and electrons [1].

In the Bohm sheath model, the plasma sheath boundary is not a sharp one, and there is a quasi-neutral region between the plasma and the sheath edge, which has been called the presheath. In this model the plasma potential falls off to zero at the sheath–presheath boundary and decreases from zero to the wall potential inside the sheath.

The electrodynamic properties of plasma sheath boundary are of great importance in a wide range of applications [2]. In these cases, the electrical characteristics and plasma parameter profiles in the sheath are defined by the boundary condition at the plasma sheath interface. Therefore, the ion transition is needed to provide the necessary boundary condition for calculations of the plasma parameters.

The Bohm sheath criterion [3] establishes the necessary condition for the existence of the sheath in a collision-less cold plasma. In the fluid approximation, it requires that the ions enter the sheath region with a velocity that is higher than the ion acoustic velocity. Consequently, the ions must be pre-accelerated by a non-shielded residual field in the quasi-neutral presheath region.

G. C. Das and colleagues [4] have derived the plasma sheath equation in the plasma by considering the finite ion temperature to show the characteristics of the sheath formation in front of the electron-absorbing wall. They derived the Sagdeev potential equation for small-amplitude approximation and described the effect of the finite ion temperature on the necessary condition for the sheath formation in plasmas.

Franklin and colleagues and Kaganovich and colleagues have tried to patch plasma and sheath in their recent works [5–9]. There are a number of options for the boundary condition that determine the boundary electric field in terms of the ion velocity at the sheath edge. Jin Yuan Liu and coworkers [10] have investigated the sheath criterion in a collisional plasma sheath by a two-fluid model. They have shown the existence of upper and lower limits for the sheath criterion when the neutral-ion collisions are taken into account. Effect of the ion temperature on RF plasma sheath was described by Minghao Lei and colleagues [11]. Moreover, G. C. Das and colleagues and B. Alterkop [12] studied the dc plasma sheath formation in thermal plasma; however the transition condition for collisional thermal plasma sheath has not been studied yet.

In this paper we have investigated the effect of the finite ion temperature on the ion velocity in the sheath–presheath boundary by a two-fluid model for a collisional plasma sheath. In the sheath region, it is reasonable to neglect ionization, since the electron energy and density are not enough for remarkable ionization. We will show that there is an upper limit of the velocity for ion transition in addition to the lower limit. The layout of the paper is as follows: The plasma sheath is formulated based on a simple two-fluid model in Sec. 2. In Sec. 3, the numerical results and the corresponding discussions are presented and the role of warm ions in the formation of the sheath is shown. Section 4 gives a brief summary and the conclusion.

2. Basic equations based on the two-fluid model

We consider an unmagnetized collisional plasma in contact with a planar wall. The x axis is selected normal to the wall, and the boundary between plasma and sheath is the origin of the axis; so the plasma and the sheath are placed in the $x < 0$ and $x > 0$, respectively, in a one-dimensional model. The model that is developed here is the same as that of Jin Yuan Liu and colleagues [10] with the addition of the ion temperature in equations.

The electrons are assumed to be in thermal equilibrium; i.e. they are isothermal through the sheath region. Since the thermal velocity of the electrons is much higher than their fluid velocity (because of their high mobility), we evaluate its density as Boltzmann relation:

$$n_{\mathrm{e}} = n_0 \exp\left(\frac{e\phi}{KT_{\mathrm{e}}}\right), \tag{1}$$

where T_{e} is the electron temperature; ϕ is the local potential; and n_0 is the electron and ion density at the sheath edge.

The ions in the plasma sheath are modeled as a warm and collisional fluid. Here we assume that there is only elastic collision in the plasma sheath between ions and neutrals, and these collisions do not originate ionization in the sheath region. In this case, there is no ion source in the sheath region, and so we will have continuity equation in steady state for ions $[\nabla \cdot (n_{\mathrm{i}}\mathbf{v}_{\mathrm{i}}) = 0]$, which can be concluded in one

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dimension as follows:

$$n_0 v_0 = n_{\rm i} v_{\rm i}, \tag{2}$$

where $n_{\rm i}$ and $v_{\rm i}$ are the ion density and the x component of the velocity in the sheath, respectively, and v_0 is the x component of the ion velocity at the sheath edge. The ion equation of motion in steady state is

$$m_{\rm i} v_{\rm i} \frac{dv_{\rm i}}{dx} = -e\frac{d\phi}{dx} - \frac{1}{n_{\rm i}}\frac{dp_{\rm i}}{dx} - m_{\rm i}(n_{\rm n}\sigma v_{\rm i})v_{\rm i}, \tag{3}$$

where $m_{\rm i}$ is the ion mass; $n_{\rm n}$ is the neutral gas density; $v_{\rm i} = n_{\rm n}\sigma v_{\rm i}$ is the ion–neutral collision frequency for momentum transfer; $\sigma = \sigma_{\rm S}(v_{\rm i}/c_{\rm s})^{\gamma}$ is the momentum-transferring cross section [$c_{\rm s} = (KT_{\rm e}/m_{\rm i})^{1/2}$ is the ion acoustic velocity, $\sigma_{\rm s}$ the cross section measured at ion acoustic velocity, γ a dimensionless parameter ranging from zero in the constant mean free path ($\lambda_{\rm i} = 1/n_{\rm n}\,\sigma_{\rm s}$) case to $\gamma = -1$ in the constant collision frequency ($v_{\rm i} = n_{\rm n}\sigma_{\rm s} c_{\rm s}$) case]; and $p_{\rm i} = n_{\rm i} KT_{\rm i}$ is the ion pressure with $T_{\rm i}$ the ion temperature and K the Boltzmann constant. The Poisson's equation relates the self-consistent electrostatic potential ϕ to the electron and ion density as follows:

$$\frac{d^2\phi}{dx^2} = -\frac{e}{\varepsilon_0}(n_{\rm i} - n_{\rm e}). \tag{4}$$

We can simplify these equations with the definitions of some dimensionless variables as follows:

$$\eta = -\frac{e\phi}{KT_{\rm e}}, \quad \xi = \frac{x}{\lambda_{De}}, \quad u = \frac{v_{\rm i}}{c_{\rm s}},$$

$$\alpha = \lambda_{De} n_{\rm n} \sigma_{\rm s}, \quad T = \frac{T_{\rm i}}{T_{\rm e}}.$$

In these definitions, η is the electrostatic potential normalized to $-KT_{\rm e}/e$; ξ introduces the penetration depth in the sheath region normalized to the electron Debye length; u is the x-component velocity of the ion normalized to the ion acoustic velocity; α is the net collision parameter; and T is the ion-to-electron temperature ratio. By writing these four equations by using these new dimensionless variables and eliminating $n_{\rm i}$ and $n_{\rm e}$ we find

$$u\frac{du}{d\xi} = \left(\frac{d\eta}{d\xi} - \alpha u^{2+\gamma}\right)\left(1 - \frac{T}{u^2}\right)^{-1} \tag{5}$$

and

$$\eta'' = \frac{d^2\eta}{d\xi^2} = \frac{u_0}{u} - e^{-\eta}. \tag{6}$$

Solving (5) and (6), η and u are found versus ξ, and so we can obtain $N_{\rm i} = u_0/u$ and $N_{\rm e} = e^{-\eta}$. Multiplying the Poisson's equation (6) by $\eta' d\xi = d\eta$ and integrating from the edge of sheath into the sheath, with the well-known boundary conditions $\eta'_0 \neq 0$ and $\eta_0 = 0$ at the plasma sheath interface, we will have

$$\int_{\eta'_0}^{\eta'} \eta' d\eta' = \int_0^{\eta} \frac{u_0}{u} d\eta - \int_0^{\eta} e^{-\eta} d\eta \tag{7}$$

or

$$\frac{1}{2}\eta'^2 - \frac{1}{2}\eta'^2_0 = -V(\eta, u_0), \tag{8}$$

where η'_0 is dimensionless electric field at the plasma sheath boundary and V is called the Sagdeev potential, which is defined as

$$V(\eta, u_0) = 1 - e^{-\eta} - \int_0^\eta \frac{u_0}{u} d\eta. \tag{9}$$

From (8), it is concluded that the Sagdeev potential values in the sheath region ($\eta' > \eta'_0$) must be negative. According to (9), the Sagdeev potential satisfies the following boundary conditions:

$$V(0, u_0) = 0 \quad \text{and} \quad \frac{\partial V(0, u_0)}{\partial \eta} = 0. \tag{10}$$

These conditions say that the plasma sheath interface is a maximum or minimum point for the Sagdeev potential and possible values for V in the sheath region are either positive or negative. Using (5), (8), and (9), the condition for maximizing V in the sheath edge ($\partial^2 V(0, u_0)/\partial \eta^2 < 0$), and some algebraic operations, in non-hot plasma ($T > (\eta'_0/\alpha)^{2/(2+\gamma)}$), we can find out the new generalized plasma sheath transition condition:

$$\sqrt{\frac{1+T}{1+\alpha/\eta'_0}} \leqslant u_0 \leqslant \sqrt{\frac{\eta'_0}{\alpha}} \quad (\gamma = 0), \tag{11}$$

$$\left[\sqrt{\frac{\alpha^2}{4\eta'^2_0} + (1+T)} - \frac{\alpha}{2\eta'_0}\right] \leqslant u_0 \leqslant \frac{\eta'_0}{\alpha} \quad (\gamma = -1). \tag{12}$$

These inequalities give two upper and lower limits for ion transition velocity in two types of collisional sheath. The upper limit shows the balance between the driving initial electric field and the neutral collision drag. It is sufficient to set $\alpha = 0$ in both (11) and (12) to get the ion transition condition in a non-collisional plasma with the ion temperature effect. In this case, both the relations reduce to

$$\sqrt{1+T} \leqslant u_0. \tag{13}$$

As we can see from inequality (13) in collisionless plasma, the ions do not need the initial electric field for entering to sheath region. As we expect, in cold plasma ($T \ll 1$), this inequality is reduced to the well-known Bohm criterion $1 \leqslant u_0$.

By carrying out similar operations, it is concluded that in hot plasma ($T > (\eta'_0/\alpha)^{2/(2+\gamma)}$) the new generalized plasma sheath transition condition will be

$$\sqrt{\frac{1+T}{1+\alpha/\eta'_0}} \leqslant u_0 \leqslant \sqrt{T} \quad (\gamma = 0), \tag{14}$$

$$\left[\sqrt{\frac{\alpha^2}{4\eta'^2_0} + (1+T)} - \frac{\alpha}{2\eta'_0}\right] \leqslant u_0 \leqslant \sqrt{T} \quad (\gamma = -1). \tag{15}$$

A special case is considered for better explanation of the results. In Fig. 1, the ion velocity condition for sheath formation as a function of the initial electric field has been plotted for collision parameter values $\alpha = 0.1$ [Fig. 1(a)] and $\alpha = 0.15$ [Fig. 1(b)], corresponding to different pressures, $\gamma = 0$ (collision with constant mean free path), and two ion temperature values ($T = 0$ and $T = 0.4$). For the mentioned parameters, this figure shows the allowable ion velocity region for sheath formation, which lies between the upper (U_{u}) and lower (U_l) curves of the ion velocity according to relations (11) and (14). These curves show the dependence of the ion transition

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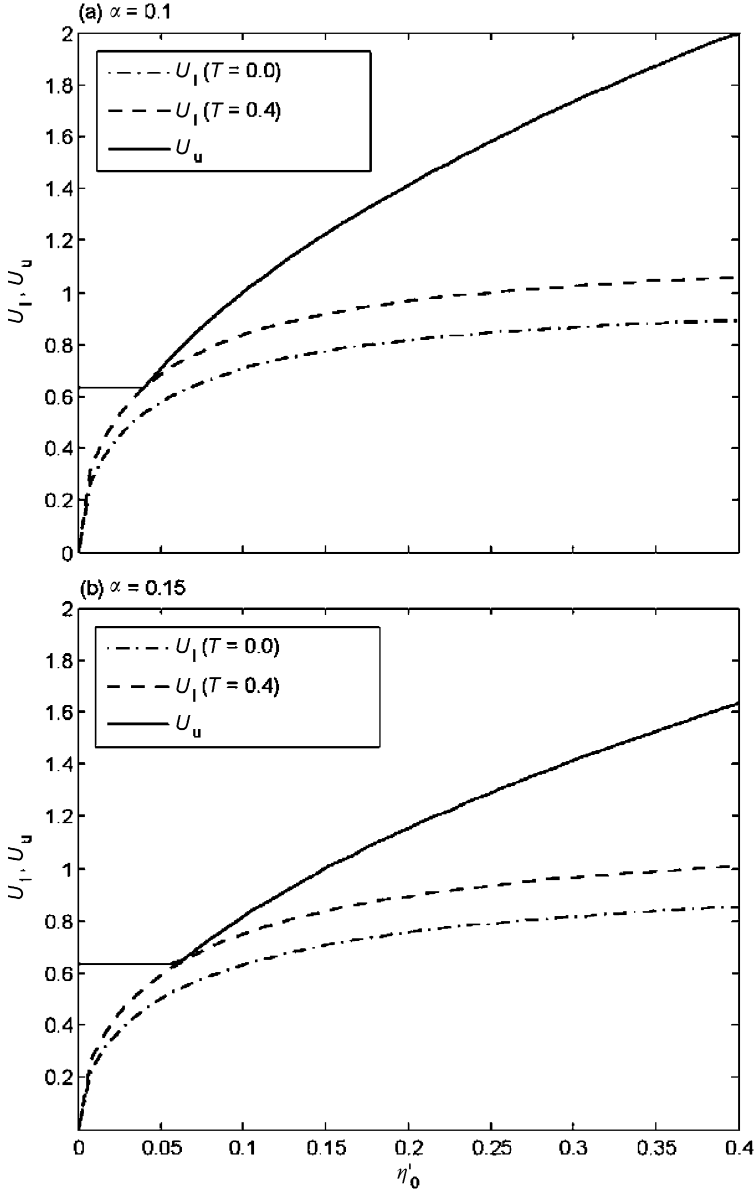

Figure 1. The upper and lower limits of Bohm velocities for two special cases: (a) $\alpha = 0.1$ and (b) $\alpha = 0.15$. The allowable values for Bohm velocities are presented with the region between these two limits.

condition for sheath formation on the collision frequency parameter (α) and the ion temperature (T). We can see that by increasing the values of both α and T, the allowable region for the ion transition condition decreases.

3. An example and discussion

According to relations (11) and (12), it can be deduced that in both collisional and collisionless non-hot plasmas, increasing the ion temperature increases the lower

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limit of ion transition velocity, while its upper limit remains unchanged and is independent of the ion temperature. By solving (5) and (6) numerically, correctness of relations (11) and (12) and their results can directly be examined.

Using (11) and (12), it is obvious that the lower limit of the ion transition velocity is decreased by increasing the frequency of the neutral collision. It means that this velocity can be reduced to being even slower than the ion acoustic velocity c_s.

To review the accuracy of the ion transition condition for sheath formation, the normalized number densities of the electrons and ions, N_e and N_i, as functions of the normalized distance from sheath edge, are shown in Figs 2 and 3. The common parameters in these figures are the normalized electric field at the sheath edge $\eta_0' = 0.2$, collision frequency parameter $\alpha = 0.1$, and $\gamma = 0$. In Fig. 2 $T = 0$, while in Fig. 3 this parameter is $T = 0.4$. According to Fig. 1(a) [and (11)], for $\eta_0' = 0.2$ and $T = 0$, one can find $u_{0\,\min} = 0.82 < u_{0\text{allow}} < 1.41 = u_{0\,\max}$ for allowable values of the ion transition velocity and hence sheath formation. Figure 2 has been plotted for $u_0 = 0.85$, 1.2, and 1, and only $u_0 = 1.8$ is out of the allowable values for u_0. In Figs 2(a) and (b), inequality (11) is satisfied, and the density of the ions is always larger than that of the electrons in the sheath region, but in Fig. 2(c) this inequality is not satisfied because $u_0 = 1.8$ is out of the allowable interval for u_0. Indeed, the neutral collision force on the ions exceeds the electrical force on them. Therefore, the ions are decelerated, which results in the accumulation of ions.

According to Fig. 1(a), it can be found that for $\eta_0' = 0.2$ and $T = 0.4$, the allowable values of the ion velocity are $u_{0\,\min} = 0.96 < u_{0\text{allow}} < 1.41 = u_{0\,\max}$. In Fig. 3, u_0 is the same as in Fig. 2, except that $T = 0.4$ has been used. As can be seen from Fig. 3(a), the ion transition condition is not fulfilled. Indeed the higher the ion temperature, the lower the limit of the condition of the ion transition, and this causes $u_0 = 0.85$ in Fig. 2(a) to satisfy the ion transition condition; however in Fig. 3(a) it does not satisfy this condition.

Since the first two terms on the right-hand side of (3) are always positive ($dn_i/dx < 0$ and $d\phi/dx < 0$ in the sheath region), and the last term of this equation is always negative, we deduce that the first two terms on the right-hand side of (3) accelerate the ion and that the last term of this equation decelerates it into the sheath. Then, at the sheath edge the ion–neutral collision force increases by increasing u_0 for constant ion temperature. So the velocity of the ion at the sheath edge decreases, which leads to increase in the ion density. Since the ion–neutral collision force at the sheath edge does not change for the constant value of u_0, the ion velocity increases by increasing the ion temperature owing to the rise of the accelerating pressure force. This increasing of the ion velocity leads to the decreasing of the ion number density, which may conflict the ion transition condition for the sheath formation [Fig. 3(a), in $\xi < 1.8$].

Finally, as can be seen, there is no difference between Figs 2(c) and 3(c) for the same $u_0 = 1.8 > u_{0\,\max}$. It shows that the increasing of the ion temperature from $T = 0$ to $T = 0.4$ does not affect the upper limit of the ion transition criterion.

4. Conclusion

We have analyzed and investigated a collisional sheath, taking into account the effect of the finite ion temperature, and have obtained the ion velocity criterion for sheath formation. It is found that there is a velocity interval for the entrance of the ion into the sheath. The minimum velocity depends on the ion–neutral collision

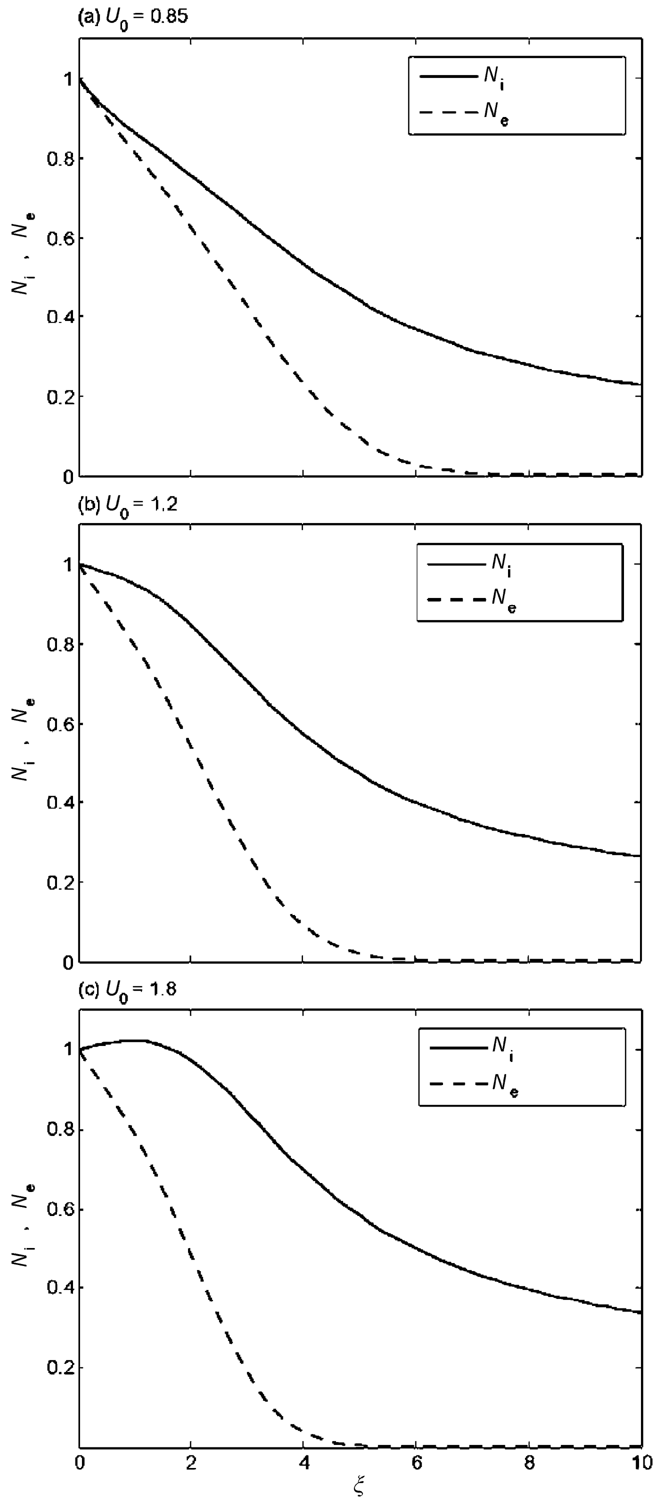

Figure 2. The normalized number densities of the electrons, N_e, and the ions, N_i, as functions of the normalized distance from the sheath edge, ξ, for $\alpha = 0.1$, $\gamma = 0$, $T = 0$, and $\eta_0' = 0.2$ with (a) $u_0 = 0.85$, (b) $u_0 = 1.2$, and (c) $u_0 = 1.8$.

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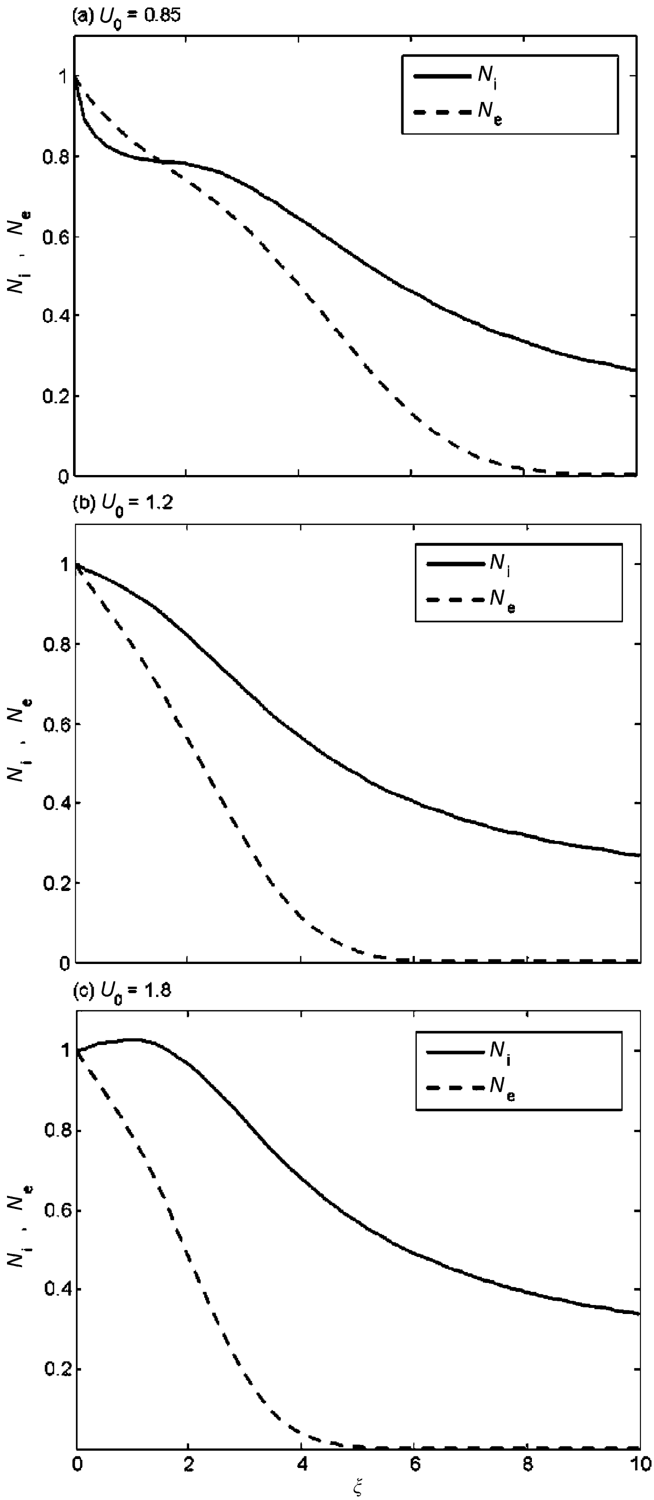

Figure 3. The normalized number densities of the electrons, N_e, and the ions, N_i, as functions of the normalized distance from the sheath edge, ξ, for $\alpha = 0.1$, $\gamma = 0$, $T = 0.4$, and $\eta'_0 = 0.2$ with (a) $u_0 = 0.85$, (b) $u_0 = 1.2$, and (c) $u_0 = 1.8$.

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frequency, initial electric field, and ion temperature, whereas the maximum velocity depends on the collision frequency and initial electric field and is independent of the ion temperature. It may be concluded that the minimum value for the ion velocity is a function of the ion temperature in a non-collisional sheath, and the maximum value for the ion transition velocity trends to the infinite. In other words, there will be only a lower limit for the ion velocity criterion.

For hot plasmas such as non-hot plasmas, the condition of plasma sheath transition has a variety between two minimum and maximum values. In hot plasmas however, in addition to the lower limit, the upper limit of this variety depends on the ion temperature but is independent of the initial electric field.

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