manipulate. The set of equivalence classes of integers mod m is denoted, logically but rather messily, by $Z/\equiv_m$. That said, the chapter has the expected ingredients, including public key cryptography, and contains many interesting and challenging exercises (for example showing in several steps that if a is a number of at most 100 decimal digits then gcd (a, b) by the Euclidean algorithm takes at most 479 divisions). Chapter 8 on infinite sets is quite ambitions and covers the inclusion-exclusion formula and the uncountability of the power set of the positive integers, leading to the same for the real numbers, and the Cantor-Schröder-Bernstein theorem.

Altogether this is an ambitious and largely very successful introduction to the writing of good proofs, laced with many good examples and exercises, and with a pleasantly informal style to make the material attractive and less daunting than the length of the book might suggest. I particularly liked the many discussions of fallacious or incomplete proofs, and the associated challenges to readers to untangle the errors in proofs and to decide for themselves whether a result is true.

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Probability and random processes (4th edition) by Geoffrey R. Grimmett and David R. Stirzaker, pp. 669, £40 (paper), ISBN 978-019884759, Oxford University Press (2020)

One thousand exercises in probability (3rd edition) by Geoffrey R. Grimmett and David R. Stirzaker, pp. 580, £29.99 (paper), ISBN 978-0198847618, Oxford University Press (2020)

This pair of books, first published in 1982, has become a classic, well worthy of comparison with those by Feller [1] and Ross [2]. The third edition of *Probability and random processes* (*PRP*) and the then new *One thousand exercises in probability* were reviewed by David Applebaum in March 2002; for those who don't have ready access to the earlier review I shall summarise David's comments here. *PRP* offers a full account of virtually every aspect of the subject that is likely to feature in an undergraduate course, and plenty more. It goes from axioms (including an early discussion of sigma algebras that provides motivation missing in some comparable texts) to generating functions and the key limit theorems, with 'a healthy minimum' of measure theoretic technicalities. There follow chapters on Markov chains (in both discrete and continuous time), convergence and martingales, random processes, queuing theory, martingales and diffusion processes. The authors tell us that for the new edition the section on Markov chains in continuous time has been revised extensively, and sections have been added on coupling from the past, Lévy processes, self-similarity and stability, and time changes. Unlike Feller, *PRP* covers both discrete and continuous distributions in a single volume.

Features of *PRP* include brief but helpful motivational introductions to each subsection, and copious references to historical applications. To aid navigation, definitions, theorems and other key results are highlighted, using three different colours. The tone throughout is rigorous but the touch is human; for instance, one question concerns the lengths of tails of a troupe of chimeras, while the introduction to queuing theory, which '[draws] strongly from our intuitions' adds as a footnote 'and frustrations, including listening to the wrong types of music on a telephone'. There are helpful teaching points, such as, concerning the integral for the characteristic function of the exponential distribution, 'Do not fall into the trap of treating i as if it were a real number, even though this malpractice yields the correct answer in this case.' And later, 'It is not an easy matter to discuss the "stochastic integral" before an audience some of whom have seen little or nothing beyond the Riemann integral. There follows such an attempt.' The number of exercises and problems is now over 1300; *One thousand exercises* (sic!) starts by repeating all the questions in *PRP* and then gives detailed solutions to them all.

I was drawn to this pair of books by a wish to investigate a problem concerning 2-state Markov chains of small finite length. Most texts are content to move rapidly to the long-term properties, but I found all the necessary materials here. I imagine that most users will have similar experiences.

David Applebaum considered that these books were too sophisticated to be a main text for most undergraduates, but for others, and as a work of reference, it is easy to feel that they will meet all their needs for a long time. They should be on the shelves of all mathematicians.

References

1. W. Feller, *An introduction to probability theory and its applications*: volume I, John Wiley & Sons, 1950; volume II, John Wiley & Sons, 1966.
2. Sheldon M. Ross, *Introduction to probability models*, Academic Press, 1972.

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Calculus for cranks by Nets Hawk Katz, pp. 251, £20 (paper), ISBN 978-0-30024-279-9, Yale University Press (2021)

This book, which covers much the same ground as other introductory ones on real analysis such as [1], has a number of unusual features. It originated as class notes for a specific course, Maths 1a, given by the author at Caltech which introduced students from disciplines using calculus to rigorous elements of its foundations. The title refers to them (affectionately) as "cranks" for braving this proof-based "road less travelled by", with their reward being a clear understanding of the fundamental ideas. To sell this approach, the author emphasises ideas of approximation: the real numbers allow the analysis of (real-world) data to arbitrary accuracy, and the characteristic use of inequalities in proofs mirrors the methods needed to track error propagation in floating-point arithmetic.

Standard topics (induction and the real numbers, sequences and series, functions and derivatives, [Riemann] integration) are covered in the first four chapters, but there are several novelties. The real numbers are envisaged as familiar infinite decimal expansions with a truncation operator; using this starting-point leads to some intriguing modifications of standard proofs such as that showing that every non-empty set of reals which is bounded above has a least upper bound (l.u.b.). Indeed, l.u.b.s are used more freely than usual in subsequent proofs such as that showing the convergence of Cauchy sequences and in the definition of x^α (for $x > 1$, $\alpha > 0$ real) as l.u.b.$\left\{x^{\frac{p}{q}} : \frac{p}{q} \in \mathbb{Q}, \frac{p}{q} < \alpha\right\}$, with $x^{\frac{p}{q}}$ defined as l.u.b.$\{y : y^q < x^p\}$. All the standard properties of series (ratio and root tests, radius of convergence for power series), continuous, and differentiable functions are covered with welcome use of o- and O-notation to streamline proofs. There is an unusual proof of Young's version of Taylor's theorem (with error term $o(h^n)$) based, essentially, on numerical integration, and I was also surprised to see a section on e^{-1/x^2}