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*Investment strategies in retirement: in the presence of a means-tested government pension**

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Abstract

A simulation approach is used to investigate how various investment strategies affect the ability of retirees to spend at a desired level up until death. Retirees are assumed to maintain all investment and longevity risk, and also have access to a government-sponsored and means-tested Age Pension to provide part of their desired expenditure. It is found that a 100% allocation to growth assets is optimal for large expenditure desires relative to initial balance levels, with allocations outside of this being sensitive to movements in initial balance and desired expenditure level, as well as interactions with the Age Pension.

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1 Introduction

Worldwide, continued increases in life expectancy raise important concerns as to the ability of retirees to provide themselves with a sufficient income throughout their retirement. In many countries, social security programs do not provide what retirees might consider to be a sufficient income; this is unlikely to change due to the strain being placed on government budgets by changing demographics. The broad provision of employer-sponsored defined benefit pensions in many countries removes some of the risks faced by the retiree; however, the provision of defined benefits is rapidly diminishing in most countries.[1] While it is possible for a retiree not covered by an employer-sponsored defined benefit pension to purchase some form of annuity, thus transferring investment and/or longevity risk to the annuity provider, this may

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[1] For example, in the U.K. in 2008, only 42% of the 2.6 million active memberships in defined benefit private-sector schemes were in schemes which were open to new entrants (Table 2.6, Office for National Statistics, 2009). In the U.S. active membership of single-employer defined benefit schemes decreased from 22 million to 17 million from 1985 to 2002, with the proportion of memberships that were inactive increasing from 28% to 50% over this time (Pension Benefit Guaranty Corporation, 2005). In Australia, defined benefit assets decreased from 22% of all retirement benefit assets to 8% from 1995 to 2009 (Australian Prudential Regulation Authority, 2010).

not always be economically and/or practically viable. In this case, investment and longevity risk is maintained in retirement.

Much debate is found in the literature as to the most appropriate asset classes for investment for retirement. Empirical evidence demonstrates the effectiveness of equity investment over long terms, with bonds and cash included in portfolios to reduce short-term volatility, with property also considered to be a relatively risky asset. Shiller (2006), using U.S. data, shows that the probability of a Long-Term (Short-Term) Treasury Bill investment outperforming a diversified equity portfolio over 20 years is only 2% (9%), in addition to showing that the probability of a diversified equity portfolio earning 3% p.a. than Treasury Bills is 65% (75%). Brailsford *et al.* (2008) find the average return on Australian equities over the period 1958–2005 was 6.3% (6.8%) higher than the average return on long-term (short-term) government bonds on a geometric basis. However, the unknown and decreasing time period over which investment is undertaken in retirement leads to a questioning of the appropriateness of equity investment during this period.

Conventional wisdom suggests that investors ought to be less risk averse at younger ages and invest a greater proportion of wealth in growth assets such as equities and property rather than defensive assets such as fixed interest securities and cash, while investment in growth assets should decrease as an individual ages and their investment time horizon decreases. Bodie *et al.* (1992), Bodie (2003) and Samuelson (1989), among others, have agreed with such a proposition. Empirically, Agnew *et al.* (2003) find a downward trend in equity allocation with age when investigating US retirement accounts over the period 1995–98, while other studies find the reduction in equity exposure with age to be insignificant (see e.g., Poterba and Samwick, 2001; Ameriks and Zeldes, 2004; Gomes and Michaelides, 2005). An investment approach that explicitly becomes more defensive with age is usually referred to as a life-cycle strategy.

Optimisation models can be used to make asset allocation decisions in retirement and have been described previously in the literature on numerous occasions. These models optimise by expressing an objective as an explicit function of the inputs of interest. As such, a significant advantage of such models is that they can be analysed and solved for any values of the inputs in question. However, they require simple models for returns, expenditure and mortality to be able to be solved.

Ho *et al.* (1994) provide an initial attempt to analyse the asset allocation decision in retirement only (rather than in the lead up to retirement or over the whole lifecycle). An analysis of the appropriate fixed allocation to equities in terms of the amount required to minimise the probability that the portfolio return will fail to exceed the amount required to fund expenditure until the life expectancy (which is based on Canadian mortality) is performed. Returns on equities and short-term interest rate securities are assumed to be normally distributed and may be correlated. They find an all equity portfolio to be optimal at age 65 for wealth to consumption ratios up to 14.5 (women) and 11.5 (men), with the optimal equity allocation decreasing with age.

Young (2004) assumes that risky asset returns follow standard Brownian motion, and that the force of mortality is constant, in determining analytical solutions using stochastic optimal control techniques for the allocation to risky assets while minimising the probability that the retirement balance reaches zero (i.e., the probability of ruin). The advantage of this optimal control approach is that it defines a dynamic rather than fixed investment strategy automatically; however, the calculations to generate such solutions are complex, require simple investment models and are difficult to solve if boundaries (such as not allowing for borrowing at the risk-free rate) are applied. In Young (2004), since the force of mortality is constant the ruin probability and optimal investment strategy are independent of time/age, making the results illustrative rather than useful. The optimal amount invested in risky assets is negatively correlated with wealth, positively with life expectancy, positively with expenditure and negatively with the volatility of the risky asset. Moore and Young (2006) relax the constant force of mortality assumption and find that the shape of the mortality function has a significant impact on optimal investment strategies. However, the optimal investment strategy can only be approximated and not directly analysed for non-constant mortality functions.

Instead of expressing the objective as a probability calculation, Blake *et al.* (2003) express objectives in terms of utility functions of expenditure and bequest. The variables of interest are the equity allocation and the time at which a life annuity is purchased.[2] The risk-free return is assumed to be fixed, while equity returns are log-normally distributed. This approach requires an assumption of the risk aversion of the retiree in determining solutions. Blake *et al.* (2003) look at a limited number of fixed asset allocations. Later studies, such as Gerrard *et al.* (2004, 2006), use a stochastic optimal control technique similar to that described above to provide dynamic solutions.

Conversely to analytical approaches, a simulation approach allows more complicated and realistic assumptions for investment returns and mortality – but at the expense of only being able to calculate decision metrics for explicit inputs rather than generating decision metric functions that can be analysed. Typically the decision metric is computed for a range of a single input (e.g., by comparing a range of asset allocations from low to high risk; holding all other inputs constant) and compared to determine the optimal approach. While a simulation approach cannot be used to define dynamic optimal strategies, simulation approaches can compare static asset allocation strategies to pre-defined dynamic strategies.

Milevsky *et al.* (1997) follow-up the analytical approach of Ho *et al.* (1994) but update the equity return to log-normal, as a result requiring the use of simulations to investigate the minimum probability.[3] Results are broadly similar to Ho *et al.* (1994). Some information is also presented about the distribution of any bequest at death – although this is not explicitly incorporated into the decision-making process for portfolio allocation. Albrecht and Maurer (2002), in a German context, use a

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[2] This, as well as much of the literature considers investment strategy in conjunction with a choice or requirement to annuitize retirement savings at some point in time. For example, in the U.K. annuitization is compulsory by age 75. In many other countries a significant and competitive life annuity market gives retirees a realistic choice of whether to transfer investment and/or longevity risk to an insurer. This has a significant impact on the objectives of the retiree (who may be wishing to optimise the value of the annuity able to be purchased) and thus the investment decision.

[3] A later paper by Milevsky and Robinson (2000) allowed analytical solutions of the probability of ruin with a log-normal equity return structure by comparing the stochastic present value of expenditure with the initial investable wealth.

simulation approach to look at shortfall probability in the presence of real estate as well as equities and bonds, using log-normal returns. Dus *et al.* (2005) extend the decision metric to take into account the size as well as probability of ruin, by incorporating the amount by which actual expenditure is less than desired expenditure over various constant and variable withdrawal rules. Log-normal returns are used. Stout (2008) looks at the optimal fixed asset allocation for a range of withdrawal rates (fixed and variable) to minimise the probability of ruin; again using log-normal returns.

This study investigates the choice of investment strategies in retirement where a competitive life annuity market is unavailable[4] and the presence of a government-sponsored means-tested Age Pension. Retirees therefore maintain all investment and longevity risk; withdrawals can only be made from the account balance until its exhaustion. The previous literature is extended by investigating the effect of the presence of a means-tested Age Pension.[5]

The presence of a means-tested Age Pension requires a simulation process to project future investment returns for various asset classes and the generation of a random year of death. Explicit risk measures are used, similar to Dus *et al.* (2005), rather than a utility approach. While some basic information on bequests is provided, this is not incorporated directly into the decision making and is left for further investigation in future research. Desired retirement expenditure is assumed to be known and not variable, with portfolio leverage not being allowed for – the maximum allocation to risky assets is assumed to be 100%. Both constant and dynamic investment strategies are tested to investigate their effect on results. Investment returns follow the Wilkie (1995) framework. Mortality rates are explicitly stated for each age and are allowed to improve. The presence of means-tested payments means that a balance to expenditure ratio is insufficient in looking at the question of interest. It is necessary to define explicit balance and expenditure values; outputs are analysed across various retirement balances, expenditure desires, ages, constant and non-constant investment strategies and expenditure patterns.

The structure of this paper is as follows. Section 2 provides some background and basic assumptions underlying modelling retirement balances and retirement expenditure. Section 3 describes the methodology used and various scenarios tested, while Section 4 gives results. Section 5 concludes and provides comments on future research directions.

2 Background and assumptions

2.1 Providing a retirement income

It is assumed that retirement income provision is handled completely through withdrawals from a retirement account balance and the provision of a government-sponsored means-tested Age Pension; details of which are provided in Appendix A. The account balance provides no investment or longevity protection; it may be

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[4] In Australia, where this research originated, there is virtually no life annuity market (see Ganegoda and Bateman, 2008).

[5] This is based on the Age Pension provided in Australia.

withdrawn from until its account balance is exhausted. The retirement account is also impacted by investment earnings dependent on the investment strategy used.

The initial opening balance in the retirement account is set to $500,000[6] and adjusted by 5%, 10% and 20% in both directions in investigations of all scenarios.

2.2 *Expenditure in retirement*

Retirement expenditure desires are assumed to be at a comfortable[7] level, equivalent to a value of $38,611 per annum at December 2009 for a single female. Real expenditure is assumed to be fixed, with indexation occurring at the same rate as the Age Pension. Expenditure desires are adjusted by 5%, 10% and 20% in both directions in investigations of all scenarios. Other approaches to analysing required income in retirement, such as replacement rate,[8] are not considered.

Using a fixed expenditure desire is a simplification of the patterns of expenditure that might be desired in retirement. For example, Chen *et al.* (2007) report that older retirees spend less than younger retirees on all items except medical care and Hatcher (2007) shows increasing age has a negative impact on consumption. These studies are based on US retirees and concentrate on actual consumption. In reality the decreasing expenditure may not be desired, but may be a function of declining wealth and thus ability to consume.

In an ASFA (1999) discussion paper, it is claimed that retirees are likely to pass through three stages of life. The first is a 'Healthy Active' state, where consumption level is high as the retiree desires to engage in expensive leisure and social activities. The second is a 'Plateau' state, where consumption decreases and stabilises due to restricted leisure and social activities as health and mobility declines. The final state is 'Frail/High Cost', where the consumption level is likely to rise substantially due to deteriorating health which requires increasing medication and aged-care costs.

In addition to testing fixed desired expenditure, alternative non-constant scenarios based on the above discussions are investigated.

2.3 *Mortality*

The starting point in generating mortality assumptions is q_x, the probability of a person aged exactly x dying before reaching age $(x+1)$, as calculated by the Australian Government Actuary (AGA, 2009) over 2005–07. Female rates are used, with mortality rate decreases assumed to follow the 25 year improvement factors as published by the AGA. Rates in the first year are assumed to have experienced

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[6] Whilst this figure is very high compared to typical retirement balances, it is necessary to select this high a value in order to display a range of optimal allocations to growth assets, as can be seen from Table 2.

[7] This is as per the Westpac/ Association of Superannuation Funds of Australia (ASFA) "comfortably affluent but sustainable" living standard, which allows "older, healthy and fully active self-funded retired Australian(s) … to engage actively with a broad range of leisure and recreational activities without a substantial disbursement of assets" (Saunders *et al.*, 2004).

[8] The replacement rate measures the required income in retirement as a proportion of pre-retirement income. ASFA (1999) recommended a net income in retirement of 60% of gross pre-retirement income as being adequate.

4 years of mortality improvement already. Mortality is assumed to be independent of retirement balance,[9] expenditure desires and investment returns;[10] and is assumed to be uniformly spread within integer ages in determining year of death between integer ages, while those who survive to age 110 are assumed to die at age 110 exactly.

2.4 *Investment returns*

The Wilkie (1995) model is used as the starting point for modelling investment returns. It is a widely known and applied stochastic economic model in the actuarial field. Equity prices in the Wilkie model are mean reverting,[11] although not to the extent that arbitrage profits can be made (Kemp, 1996). One concern with the model is the stability of parameter estimates for small changes in data period (see Sahin *et al*., 2008).[12] In this research, the most important output of the model is the equity risk premium, of which the geometric average result from the Wilkie model outputs is 5.6%, which is relatively consistent with longer-term historical results in Australia. However, there is much conjecture about the size of the equity risk premium, with many postulating that the historical equity risk premium is higher than what can be expected in future.[13] The effect of changes to the equity risk premium and removal of mean reversion is considered as a sensitivity test in Section 4.4.

Full details of the fitting and parameters of the Wilkie model (including the significance and timing of mean reversion) can be found in Appendix B.

3 Methodology

3.1 *Projection and simulation*

The starting point for projection is a single female who has just retired at age 65 with a retirement balance and no other source of retirement income apart from the Age Pension (see Appendix A). For brevity only a single female is tested, although the principles discussed in the results could be equally extended to other individuals. Balance values are defined at the start of the projection and updated on an annual basis, earning investment returns according to the asset allocation at the start of the

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[9] A specific review of the literature relating to the link between socio-economic factors and mortality can be found in Whitehouse and Zaidi (2008). In general, mortality rates decrease with increases in income. For example, Knox and Nelson (2007) find that, using 2002–05 Australian public-sector pensioner data, mortality is lower for males with higher pension size, with this relationship decreasing as pensioners get older.

[10] There is no consensus on the effect of economic factors (which drive investment returns) on mortality. Ruhm (2004) provides a summary of recent literature and its contradictions.

[11] There is a wide variety of literature debating the existence of mean reversion in equity markets. A seminal paper by Poterba and Summers (1988) finds that U.S. stock returns are positively autocorrelated over short time periods and negatively autocorrelated over periods of greater than 3 years, although this correlation may not be statistically significant. Cutler *et al.* (1991) find significant mean reversion in dividend yields (and hence stock prices) for Australia, Canada, the U.K. and some U.S. time periods but not in other countries modelled.

[12] Other criticisms are collected in Huber and Verrall (1999); the most significant of which is that the model is chosen to best fit empirical data and not linked to financial economic theory. However, as noted in Huber and Verrall (1999), models using financial economic theory often find difficulty in their application to actual events.

[13] Song (2007) provides a detailed literature review of this field.

year (see below) and the investment model described in Appendix B. The desired retirement expenditure (see Section 2.2) is withdrawn from the balance annually[14] (if available), reduced by any receipt from the Age Pension. Cash flows are assumed to occur mid-year, with investment returns applied to these cash flows at half the annual rate of return. It is assumed that investment returns are at the rate of the relevant index with no fees.[15] Other fees are likely to be insignificant compared to the desired expenditure levels and are thus ignored.

The outputs of interest are the length of time over which an income stream is able to be paid to fund desired expenditure before the retirement balance runs out (the 'ruin year')[16] compared to the time of death (the 'death year').[17] Should the death year exceed the ruin year then a ruin event has occurred. The probability of ruin is simply the proportion of simulations in which a ruin event occurs. Extending the risk measure to determining the size of the shortfall, the shortfall years is defined as the death year less the ruin year (counting ruin years greater than the death year as a zero shortfall). This is slightly different to the approach of Dus *et al.* (2005) who used expenditure shortfall rather than shortfall years; however, given the desired expenditure is fixed, shortfall years is an appropriate and simple to interpret measure. The bequest amount is calculated by determining the balance (in real terms) at the exact date of death, projected from the previous year. Expenditure and investment returns for the part of the year the retiree was alive are allowed for.

Investments of the retiree are split between 'growth' (equities) and 'defensive' (all other) classes. Within these classifications the split between the assets classes is as follows.

	Growth assets
Domestic equities	$58\frac{1}{3}\%$
International equities	$41\frac{2}{3}\%$
	Defensive assets
Domestic bonds	30 %
International bonds	20 %
Cash	50 %

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[14] A minimum, age-based withdrawal rate of account balance is applied, as per Australian rules; details are provided in Appendix A. These are unlikely to have any significance on results as they take effect only when the retirement balance is large relative to the desired expenditure; i.e. when the chance of ruin is small. In fact, the effect of the minimum withdrawal is to sometimes require retirees to withdraw a larger amount than necessary for the desired expenditure level, which might be seen as a benefit to the retiree. However, this issue is not measured in this study.

[15] This could be achieved through the use of index funds which charge minimal fees.

[16] The exact value between integer years is determined as the integer start of the year of ruin plus a proportion equal to the actual balance available to be withdrawn during that year divided by the amount needed to be withdrawn to meet the desired retirement expenditure.

[17] The death year is calculated by generating a single uniform random variable and applying it to the mortality model (see Section 2.3 and Appendix C).

Table 1. *Average asset allocation of Australian default options – 30 June 2009*

Australian equities	27.8 %
International equities	22.2 %
Australian bonds	7.8 %
International bonds	5.7 %
Property	10.4 %
Cash	12.0 %
Other	14.2 %

Source: Australian Prudential Regulation Authority (2010).
Note: The 'Other' category contains a range of investments such as derivative, infrastructure, etc.

For a 60/40 split between growth and defensive assets, the asset allocation of the retiree is as follows:

Domestic equities	35 %
International equities	25 %
Domestic bonds	12 %
International bonds	8 %
Cash	20 %

This allocation is broadly consistent with the average asset allocation of the default investment option[18] for Australian schemes at 30 June 2009 as provided in Table 1, but allows for the greater liquidity requirements in retirement by an increased cash allocation.

The split between growth and defensive assets is tested in 5 % increments across the spectrum 0–100 %. This can be considered to be analogous to the two-asset approach used in most previous studies. The asset allocation is assumed to be constant across the projection, with rebalancing occurring at the start of each year. The outputs are investigated over 10,000 simulations. The optimal investment strategy is then determined by minimising the average shortfall years experienced in the simulations. While some distributional information on bequests is provided, this is not incorporated directly into the decision making.

3.2 Scenarios

The assumptions outlined in Section 2 form the base scenario to be investigated. Additional scenarios are investigated as follows.

3.2.1 Retirement expenditure patterns

The following non-constant patterns of expenditure are tested:

- Decreasing (DE) – as per the actual expenditure patterns found in Chen *et al.* (2007) and Hatcher (2007), real expenditure starts 20 % higher than the base rate and decreases by 1.5 % each year.

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[18] This is the investment option provided for those who fail to make an investment choice. Asset allocation data in Australia does not split between pre and post-retirement phases.

- Increasing (IN) – conversely to above, real expenditure starts 20 % lower than the base rate and increases by 1.5 % each year.
- Variable (VA) – as per ASFA (1999), real expenditure starts 20 % higher than the base rate and decreases by 3 % each year for 10 years, before remaining static for 10 years and then increasing by 2.5 % each year thereafter.

These alternative expenditure patterns all have equivalent expected total real expenditure based on the mortality model.

3.2.2 Starting age

The analysis is also run for age 75 and 85. In these scenarios, the initial balance is updated by calculating the median real balance after 10 and 20 years from the simulations in the base scenario, using the optimal growth asset allocation for that scenario. However, the deductible for the income test in the Age Pension is based on the initial opening balance from the base scenario of $500,000.

3.2.3 Non-constant investment strategies

The following non-constant approaches are investigated:

- Life-cycle (LC) – the initial allocation to growth assets is as previous, however, this is reduced by 1 % each year. The growth asset allocation is subject to a minimum of 0 %.
- Dynamic (DY) – the initial allocation to growth assets is as previous, with the actual cumulative real return each year being compared to the expected cumulative real return (not annualised) under the initial allocation to growth assets. The allocation to growth assets is then adjusted down by the same percentage that the actual cumulative real return exceeds the expected cumulative real return,[19] with a corresponding increase where the actual return is lower than the expected return. The growth asset allocation is subject to a minimum of 0 % and a maximum of 100 %.

3.2.4 Sensitivity testing

The following sensitivity tests to the base scenario are investigated:

- No pension (NP) – the Age Pension is removed from the analysis.
- Equity risk premium – the key assumption in the economic model is the equity risk premium; which is 5.6 % p.a. in the fitted Wilkie model. Since the driving force behind equity returns in the Wilkie model is dividend increases, a downward adjustment to μ_d from Table B.2 is made. The current value of 0.0401 implies dividend increases of 4.01 % p.a. greater than price inflation. Alternative values of μ_d of 2 % (EQ2) and 0 % (EQ0) are considered. These effectively reduce the equity risk premium by 2 % p.a. and 4 % p.a., respectively, flowing directly through the investment model to international equity returns as well.

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[19] The expected cumulative real return is calculated by reference to the average per annum compounded real return from the initial growth rate asset allocation in the base scenario.

- Equity volatility (EV) – the volatility of domestic equity investment is increased by 25 % by adjusting the standard error of residuals for domestic dividends and dividend yields by 25 % (see Table B.2). This flows through the investment model to increased international equity volatility.
- Real equity returns random walk (RW) – real autocorrelation in equity returns is removed by assuming a fixed dividend yield (by removing $X_y(t)$ from Table B.2) and removing $\tau_{d,1}$, $\tau_{d,2}$ and θ_d from Table B.2. The standard error of residuals is updated so that the volatility of domestic equity returns is unchanged. This flows directly through the investment model to international equity returns as well. This is tested against a fixed asset allocation (RW) and the dynamic asset allocation[20] (RW + DY).
- Mortality (MO) – the mortality rate is maintained at q_x from AGA (2009) with no allowance for any mortality improvement.

4 Results

An initial investigation is made of the cumulative density function (CDF) of the ruin year across 20 %, 50 % and 80 % growth asset allocations and death year for the simulations in the base scenario and is presented in Figure 1.

The different growth asset allocations have a clear impact on the ruin year, with the larger growth asset allocations leading to later ruin years, except for very low percentiles, shown by the lower CDF values in Figure 1. This is consistent with the time horizon of investment theory (see Shiller, 2006), in that the long time horizon for ruin year leads to a benefit in a large allocation to growth assets, except in the most extreme circumstances. If, for example, a 30-year time horizon is investigated, Figure 1 shows that a 20 % growth asset allocation has only a 16 % chance (CDF = 0.84) of obtaining a ruin year greater than 30 years, while the corresponding percentages for 50 % and 80 % growth asset allocations are 87 % and 92 %, respectively. The steeper 20 % growth asset allocation line represents lower volatility in ruin year; however, the 20 % growth asset allocation is only better than the 80 % growth asset allocation at percentiles lower than 5 %. In other words, an 80 % growth asset allocation has a 95 % chance of obtaining a later ruin year than a 20 % growth asset allocation. The median ruin year for 20 %/50 %/80 % growth asset allocations is 27.7/38.0/45.0 years respectively, compared to a median death year of 25.2 years. In general, the death year is more variable than the ruin year for 20 % and 50 % growth asset allocations, with the ruin year for 80 % growth asset allocations being more volatile at lower CDFs, but being bounded by the maximum death year of 45.

The probability that death year exceeds ruin year for 20 %/50 %/80 % growth asset allocations is 36.7 %/10.8 %/5.9 % respectively and can be seen in the ruin probability plot in Figure 2, which is presented for all growth asset allocations, in addition to a plot of the mean and percentiles for the shortfall years.

If minimising the probability of ruin is the decision metric for optimal asset allocation, a 100 % growth asset allocation would be used, giving a ruin probability

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[20] Since the results from the dynamic asset allocation approach are likely to be extremely sensitive to equity return autocorrelations.

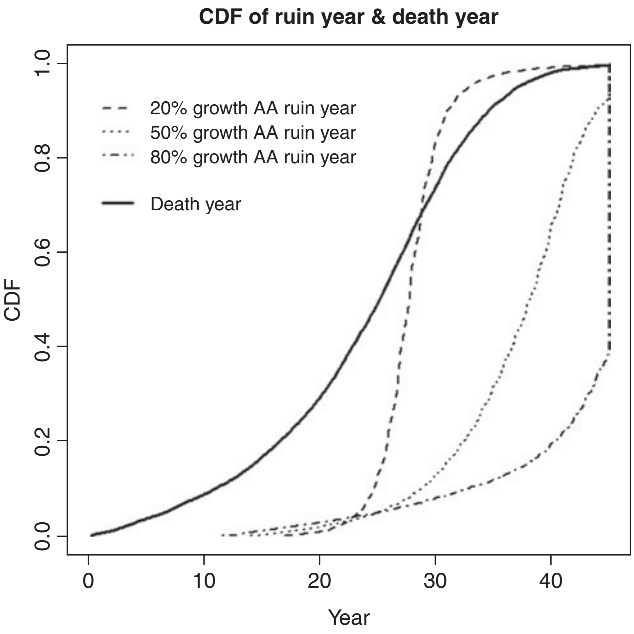

Figure 1. Plot of ruin and death year CDF (base)
Note that ruin year calculations are cut-off after 45 years at age 110; the latest age of death allowed for.

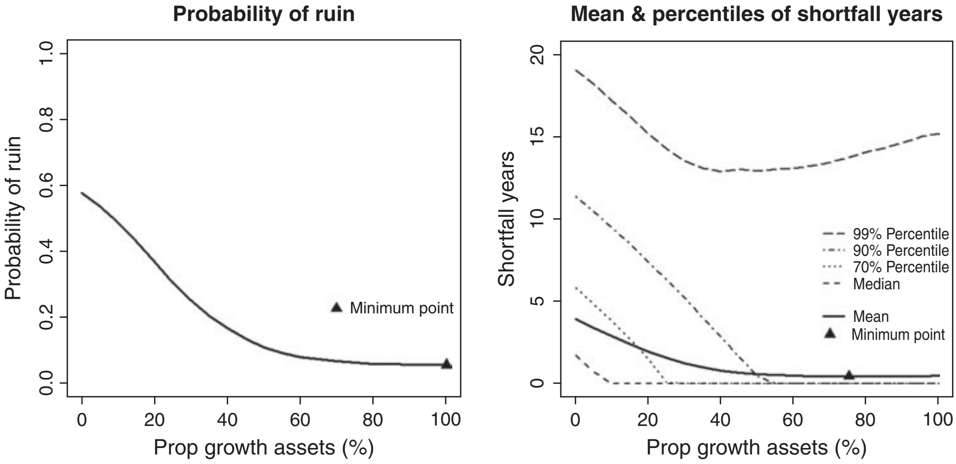

Figure 2. Plots of probability of ruin and shortfall years (base)

of 5.6%. However, the 99% percentile line shows the shortfall years' distribution at high growth asset allocations is longer tailed than for a growth asset allocation of 50%, due to the poor equity returns experienced in these simulations leading to a lower ruin year as the allocation to equities increases. This decreases the optimal growth asset allocation under the average shortfall years metric to 75%, giving an

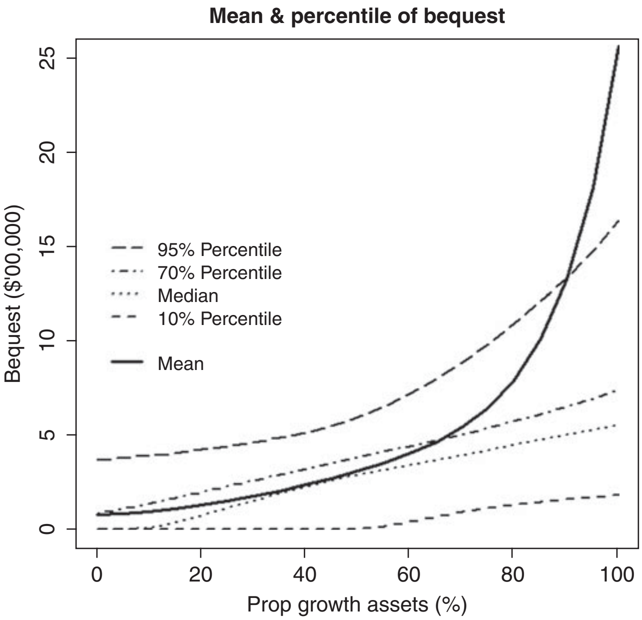

Figure 3. Plot of bequest amount (base)

average shortfall years' of 0.46 years. This is compared to an average shortfall years of 0.50 years for 100 % growth asset allocations. The balance to expenditure ratio of approximately 13 in the base scenario gives a lower optimal asset allocation than previous studies because of the Age Pension component.

In this paper, decision making on optimal growth asset allocations is based on average shortfall years. However, given a retiree may have a desire to bequest any remaining balance at death, the distribution of the bequest amount is presented in Figure 3.

The distribution of bequest amount is extremely skewed at high growth asset allocations, as can be seen by the mean being greater than the 95 % percentile where the growth asset allocation is greater than 90 %. As expected the bequest amount is greater at higher growth asset allocations. In fact the median real bequest amount at 100 % growth asset allocations is $556,000, which is greater than the initial balance of $500,000, indicating that investment returns on the balance are greater than the required withdrawals.[21] There is a potential trade-off here with the average shortfall years – a retiree might be prepared to trade a slightly higher average shortfall years (0.50 at 100 % growth asset allocations compared to 0.46 for 75 %) for a greater median bequest level ($556,000 compared to $451,000). However, this trade-off is not considered any further in this paper, with decision making being based on average

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[21] In this case the minimum withdrawal requirements of the account-based pension would become effective, meaning that withdrawals would actually be greater than desired, creating another "benefit" in addition to the bequest amount for the retiree.

Table 2. *Minimum shortfall years and optimal asset allocation (base)*

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.08	0.39	0.69	1.14	1.88	2.98	5.99	40	70	90	100	100	100	100
−10	0.05	0.23	0.43	0.70	1.09	1.70	3.72	35	55	75	90	100	100	100
−5	0.04	0.18	0.34	0.56	0.87	1.32	2.92	35	50	65	85	100	100	100
0	0.03	0.14	0.27	0.46	0.71	1.06	2.30	30	50	60	75	90	100	100
+5	0.02	0.11	0.22	0.38	0.58	0.86	1.82	30	45	55	70	85	100	100
+10	0.02	0.09	0.18	0.31	0.49	0.72	1.47	30	45	55	65	80	95	100
+20	0.01	0.06	0.12	0.21	0.34	0.51	1.00	30	40	45	55	70	80	100

shortfall years. Any allowance for this trade-off would lead to a higher allocation to growth assets.

The average shortfall years and corresponding optimal growth asset allocation are presented across the range of initial balances and desired expenditures in Table 2.

The average shortfall years of 0.46 years for an optimal growth asset allocation of 75% can be seen in the centre of the tables for the base scenario, with results for alternative initial balances and desired expenditures (by percentage difference) being found vertically and horizontally, respectively, away from the centre.

The top left to bottom right diagonal through the centre of the tables represents a consistent balance to expenditure ratio, as the initial balance and desired expenditure are changed by the same percentage. This does not give the same average shortfall years and optimal growth asset allocation, as the means-testing leads to a greater receipt of Age Pension, while the Age Pension also makes up a higher proportion of desired expenditure at smaller balances and expenditures. Hence, decreasing both initial balance and desired expenditure by 20% reduces the average shortfall years and growth asset allocation to 0.08 years and 40% respectively, while increasing both by 20% increases the results to 1.00 years and 100% respectively. Thus, a rule for the optimal asset allocation for set balance to expenditure ratios cannot be defined directly due to the interaction with the Age Pension.

Other results are as expected, with increases in balance and reductions in expenditure reducing the average shortfall years and vice versa. Clearly the optimal growth asset allocation is related to the average shortfall years, with the optimal allocation reduced as the shortfall years' decrease. However, the change is quite significant for even small revisions in balance or expenditure. For example, a 5% increase in desired expenditure increases the optimal growth asset allocation to 90% from 75%, due to the need to obtain higher returns to fund the additional expenditure. The average shortfall years increases from 0.46 to 0.71 years. Conversely a 5% decrease in desired expenditure reduces the optimal growth asset allocation to 60% and the average shortfall years to 0.27 years. However, some growth asset investment (30%) is required even when the initial balance is increased by 20% and the desired expenditure is

Table 3. *Initial balance/withdrawal and optimal asset allocation (base)*

	Balance/withdrawal							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
Balance (% diff)	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	18.8	15.9	14.8	13.8	12.9	12.2	10.9	40	70	90	100	100	100	100
−10	19.4	16.6	15.5	14.5	13.7	12.9	11.6	35	55	75	90	100	100	100
−5	19.6	16.9	15.8	14.9	14.0	13.3	12.0	35	50	65	85	100	100	100
0	19.8	17.2	16.1	15.2	14.3	13.6	12.3	30	50	60	75	90	100	100
+5	20.1	17.5	16.4	15.5	14.7	13.9	12.6	30	45	55	70	85	100	100
+10	20.3	17.7	16.7	15.8	14.9	14.2	12.9	30	45	55	65	80	95	100
+20	20.6	18.2	17.2	16.3	15.5	14.7	13.5	30	40	45	55	70	80	100

decreased by 20%, as the asset test in this case means virtually no Age Pension is received.

An alternative way of looking at these results is by comparing the optimal asset allocation to the ratio of initial balance to withdrawal (after allowing for any Age Pension receipt), as shown in Table 3.

The maximum initial balance to withdrawal ratio at which a 100% growth asset allocation is recommended is 14.0,[22] which is relatively consistent with the outcome of Ho *et al.* (1994).

4.1 Retirement expenditure patterns

The effects of having a decreasing (DE), increasing (IN) or variable (VA) expenditure pattern are investigated in Tables 4–6, respectively.

The decreasing desired expenditure results from Table 4 show that, for the original initial balance and expenditure compared to the base scenario, the average shortfall years has increased slightly from 0.46 to 0.48 years, with the optimal growth asset allocation decreasing from 75% to 55%. In general, the optimal growth asset allocation is lower for DE, as the lower expenditure in later years does not require as high returns as for the base scenario. The DE structure leads to greater volatility in average shortfall years for changing relative balance and expenditure compared to the base scenario in Table 2.[23] As expenditure increases relative to balance (the top right of the tables) the average shortfall years become much higher due to the higher likelihood of ruin in early years, while the expenditure is higher than the base scenario. Conversely, as expenditure decreases relative to balance (the bottom left of the

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[22] This is achieved by decreasing the initial balance by 5% and increasing initial expenditure by 5%. Note that in this case the initial Age Pension received is $6,666 (i.e. (500,000 × 0.95)/(38,611 × 1.05–6,666) = 14.0); however, this may change depending on future movements in the balance, thus the withdrawal cannot be said to be constant.

[23] However, it should be noted that much of this shortfall occurs in years where the DE expenditure is less than the constant expenditure under the base scenario. Thus a measure which incorporated shortfall in terms of expenditure might show closer results.

Table 4. *Minimum shortfall years and optimal asset allocation (DE)*

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.01	0.41	1.00	1.99	3.56	5.80	10.30	5	45	80	100	100	100	100
−10	0.00	0.14	0.44	0.95	1.75	2.94	6.78	5	25	50	80	100	100	100
−5	0.00	0.08	0.29	0.67	1.27	2.14	5.15	0	20	40	70	90	100	100
0	0.00	0.05	0.19	0.48	0.92	1.59	3.93	0	15	30	55	80	100	100
+5	0.00	0.03	0.12	0.34	0.68	1.19	2.95	0	15	25	45	70	95	100
+10	0.00	0.01	0.07	0.23	0.51	0.91	2.26	0	10	20	35	60	80	100
+20	0.00	0.01	0.03	0.11	0.28	0.54	1.38	0	5	15	25	40	65	100

Table 5. *Minimum shortfall years and optimal asset allocation (IN)*

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.20	0.44	0.63	0.92	1.36	1.98	3.69	80	95	100	100	100	100	100
−10	0.15	0.32	0.46	0.64	0.90	1.27	2.43	70	85	90	100	100	100	100
−5	0.13	0.27	0.39	0.55	0.76	1.04	1.98	65	80	90	95	100	100	100
0	0.11	0.23	0.34	0.47	0.65	0.87	1.62	65	75	85	90	100	100	100
+5	0.10	0.20	0.29	0.41	0.56	0.75	1.34	65	75	80	90	95	100	100
+10	0.08	0.18	0.25	0.36	0.49	0.65	1.12	60	70	75	85	95	100	100
+20	0.07	0.14	0.19	0.27	0.38	0.50	0.83	60	65	70	75	85	95	100

Table 6. *Minimum shortfall years and optimal asset allocation (VA)*

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.14	0.38	0.61	1.04	1.80	3.14	7.31	70	85	90	100	100	100	100
−10	0.10	0.25	0.40	0.62	0.97	1.56	3.98	65	80	85	95	100	100	100
−5	0.08	0.21	0.33	0.50	0.76	1.18	2.88	65	75	80	90	95	100	100
0	0.07	0.18	0.27	0.42	0.62	0.93	2.16	65	75	80	85	95	100	100
+5	0.06	0.16	0.23	0.35	0.52	0.75	1.65	65	70	75	80	90	95	100
+10	0.06	0.13	0.20	0.30	0.43	0.62	1.31	60	70	75	80	85	95	100
+20	0.05	0.10	0.15	0.22	0.32	0.45	0.87	60	65	70	75	80	85	100

tables) the average shortfall years become much lower due to the virtual impossibility of ruin in later years when expenditure is lower than the base scenario.

The increasing desired expenditure results in Table 5 are consistent with the decreasing results. For the original initial balance and expenditure compared to the base scenario, the average shortfall years has again increased slightly from 0.46 to 0.47 years, with the optimal growth asset allocation increasing from 75% to 90%. The optimal growth asset allocation is higher for IN, due to the need to provide for the higher expenditure in future than the base scenario. The IN structure leads to lower volatility in average shortfall years for changing relative balance and expenditure compared to the base scenario in Table 2.[24] As expenditure increases relative to balance (the top right of the tables) the average shortfall years become lower due to the unlikelihood of ruin in early years, while the expenditure is lower than the base scenario. Conversely, as expenditure decreases relative to balance (the bottom left of the tables) the average shortfall years become higher due to the chance of ruin in later years when expenditure is higher than the base scenario.

The variable expenditure results in Table 6 show characteristics of both the DE results in Table 4 and the IN results in Table 5. The higher optimal allocation to growth assets of the IN results is present, due to the spiralling expenditure in the later years of the VA scenario. However, the high variability of average shortfall years with changes to balance and expenditure level seen in DE is also present, due to the initial high expenditure level.[25] For the original initial balance and expenditure compared to the base scenario, the average shortfall years is reduced from 0.46 to 0.42 years due to the increased receipt of the Age Pension in middle years, with the optimal growth asset allocation increasing from 75% to 85%.

4.2 Starting age

The initial balance used at age 75 is $489,424 and at age 85 is $450,506. The results in Tables 7 and 8 show much smaller shortfall years at later ages. There are two effects here compared to the base scenario at age 65. The first is that investment performance has been at the median level with 75% growth assets up until these ages, with the second being that the retiree has not died until this age. Considering the reduction in shortfall years, this indicates that the median investment performance is more significant in reducing shortfall years than survival is in increasing shortfall years, maintaining a balance that is more than sufficient to ensure ongoing retirement income and allowing a reduction in risk at later ages.

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[24] A similar (but opposite) argument applies here as it did to the previous footnote.

[25] Given the previous footnotes, this implies a greater variability in expenditure shortfall for VA than DE.

Table 7. *Minimum shortfall years and optimal asset allocation* (*75*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.00	0.06	0.14	0.27	0.47	0.78	1.82	25	55	70	85	100	100	100
−10	0.00	0.03	0.07	0.15	0.26	0.42	1.01	20	45	55	70	85	95	100
−5	0.00	0.02	0.05	0.11	0.19	0.32	0.76	15	40	50	60	75	90	100
0	0.00	0.01	0.03	0.08	0.15	0.25	0.58	20	35	45	55	70	85	100
+5	0.00	0.01	0.02	0.06	0.12	0.19	0.45	20	30	45	50	65	80	100
+10	0.00	0.01	0.02	0.04	0.09	0.15	0.36	10	30	40	50	60	70	95
+20	0.00	0.00	0.01	0.02	0.05	0.09	0.23	0	20	35	45	50	60	85

Table 8. *Minimum shortfall years and optimal asset allocation* (*85*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.00	0.00	0.01	0.03	0.07	0.12	0.33	0	35	50	60	70	90	100
−10	0.00	0.00	0.00	0.01	0.03	0.06	0.16	0	20	35	50	55	70	95
−5	0.00	0.00	0.00	0.01	0.02	0.04	0.12	0	15	30	40	55	65	90
0	0.00	0.00	0.00	0.00	0.01	0.03	0.09	0	0	25	35	50	55	80
+5	0.00	0.00	0.00	0.00	0.01	0.02	0.06	0	0	15	30	45	55	70
+10	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0	0	15	25	40	50	65
+20	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0	0	0	15	30	40	55

4.3 Non-constant investment strategies

The life-cycle investment strategy is investigated in Table 9.

The results indicate no benefit from a life-cycle strategy, with the shortfall years being greater for the life-cycle strategy than the fixed strategy in Table 3 in every case. The initial optimal growth asset allocation is around 15% higher under the life-cycle strategy to take into account the reduction occurring in later years.

This result is related to a deficiency in the concept of life-cycle investing as it relates to retirees facing investment risk, in that life-cycle investing assumes that wealth must be protected at a target date (usually retirement). However, when a retiree takes on investment risk the concept of a target date is not particularly relevant, as the balance continues to be invested after retirement. Although the age-based results from Tables 7 and 8 indicate a reduction in growth asset allocation is appropriate, this relates specifically to the case where the median investment result has been experienced up until this age.

Table 9. *Minimum shortfall years and optimal asset allocation* (*LC*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.10	0.42	0.72	1.23	2.06	3.24	6.30	55	85	100	100	100	100	100
−10	0.07	0.26	0.46	0.74	1.17	1.86	4.02	55	70	85	100	100	100	100
−5	0.05	0.21	0.37	0.60	0.92	1.44	3.18	50	65	80	95	100	100	100
0	0.04	0.17	0.30	0.49	0.75	1.13	2.52	50	65	75	90	100	100	100
+5	0.04	0.14	0.25	0.41	0.62	0.91	2.00	45	60	70	85	95	100	100
+10	0.03	0.11	0.21	0.34	0.52	0.75	1.60	45	60	65	80	90	100	100
+20	0.02	0.08	0.14	0.24	0.37	0.54	1.07	45	55	60	70	80	95	100

Table 10. *Minimum shortfall years and optimal asset allocation* (*DY*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.16	0.44	0.69	1.11	1.83	2.92	6.23	30	65	75	85	95	100	100
−10	0.11	0.31	0.47	0.70	1.07	1.65	3.69	30	50	70	80	85	95	100
−5	0.10	0.26	0.40	0.58	0.86	1.29	2.86	25	45	60	75	80	90	100
0	0.09	0.23	0.34	0.49	0.71	1.03	2.23	25	45	55	70	80	85	100
+5	0.08	0.20	0.29	0.42	0.60	0.85	1.77	20	40	50	65	75	80	95
+10	0.07	0.17	0.25	0.37	0.51	0.71	1.43	15	35	45	60	70	80	95
+20	0.05	0.13	0.20	0.28	0.39	0.53	0.97	15	30	45	50	60	70	85

This indicates that any decision on changes to investment allocations should not just be a simple reduction in risky assets with age. Noting this, the dynamic investment strategy (DY) may be more appropriate as it makes changes to asset allocations based on performance. It is investigated in Table 10.

The dynamic investment strategy results provide mixed comparisons to the base scenario in Table 2. In general, the initial optimal growth asset allocation has decreased slightly from the base scenario in order to provide room for increases when investment experience is poorer than expected. For the original initial balance and expenditure, the average shortfall years has increased from 0.46 to 0.49 years, with the optimal growth asset allocation decreasing slightly from 75% to 70%.

Some reductions to the average shortfall years have been made where the expenditure is increased relative to the balance (i.e., the upper right of the tables – although it is more concentrated to the right of the table due to Age Pension interactions). This indicates a possible benefit to reducing equity exposure if above expected returns push the balance to a more acceptable level compared to expenditure. An exception to this is the top right cell that has increased from 5.99 to 6.23 shortfall years,

Table 11. *Minimum shortfall years and optimal asset allocation* (*NP*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	2.29	3.86	4.80	5.79	6.81	7.78	9.56	100	100	100	100	100	100	100
−10	1.36	2.29	2.93	3.67	4.48	5.35	7.14	100	100	100	100	100	100	100
−5	1.07	1.80	2.29	2.89	3.59	4.35	6.01	90	100	100	100	100	100	100
0	0.85	1.43	1.82	2.29	2.86	3.52	5.00	85	100	100	100	100	100	100
+5	0.68	1.16	1.47	1.84	2.29	2.83	4.12	75	95	100	100	100	100	100
+10	0.54	0.94	1.20	1.50	1.86	2.29	3.39	65	85	95	100	100	100	100
+20	0.35	0.63	0.81	1.02	1.27	1.56	2.29	55	70	80	90	100	100	100

Table 12. *Initial balance/withdrawal and optimal asset allocation* (*NP*)

Balance (% diff)	Balance/withdrawal							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	12.9	11.5	10.9	10.4	9.9	9.4	8.6	100	100	100	100	100	100	100
−10	14.6	12.9	12.3	11.7	11.1	10.6	9.7	100	100	100	100	100	100	100
−5	15.4	13.7	12.9	12.3	11.7	11.2	10.3	90	100	100	100	100	100	100
0	16.2	14.4	13.6	12.9	12.3	11.8	10.8	85	100	100	100	100	100	100
+5	17.0	15.1	14.3	13.6	12.9	12.4	11.3	75	95	100	100	100	100	100
+10	17.8	15.8	15.0	14.2	13.6	12.9	11.9	65	85	95	100	100	100	100
+20	19.4	17.3	16.4	15.5	14.8	14.1	12.9	55	70	80	90	100	100	100

indicating a need for 100% growth asset allocation at all times due to the low initial ratio of balance to expenditure.

Conversely, increases in the average shortfall years are found where expenditure is decreased relative to the balance. This indicates that the potential additional equity exposure under a dynamic strategy is not required when the balance is already sufficient compared to expenditure.

These results indicate a dynamic strategy that considers returns compared to expectations is not automatically better than a static strategy. It appears that a dynamic strategy must also consider the balance, expenditure and age in more detail in order to develop better outcomes with certainty – this is left to future research.

4.4 Sensitivity testing

The effect of removing the Age Pension from the analysis (NP) is investigated in Tables 11 and 12.

As expected, the removal of the Age Pension has a significant negative impact on the shortfall years, due to the higher withdrawals required to meet desired

Table 13. *Minimum shortfall years and optimal asset allocation* (*EQ2*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.19	1.01	1.90	3.07	4.44	5.92	8.80	35	65	75	90	100	100	100
−10	0.11	0.58	1.12	1.93	2.96	4.14	6.72	30	50	70	80	90	100	100
−5	0.08	0.45	0.87	1.53	2.41	3.45	5.83	30	45	65	75	85	95	100
0	0.07	0.35	0.69	1.22	1.96	2.87	5.03	25	40	55	70	80	90	100
+5	0.05	0.27	0.55	0.97	1.60	2.39	4.33	25	40	50	65	75	85	100
+10	0.04	0.22	0.44	0.78	1.30	1.98	3.72	25	35	45	60	70	80	95
+20	0.03	0.14	0.28	0.52	0.87	1.37	2.74	25	30	40	50	65	75	90

Table 14. *Minimum shortfall years and optimal asset allocation* (*EQ0*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.32	1.99	3.49	5.12	6.72	8.21	10.76	25	55	65	70	80	85	95
−10	0.18	1.15	2.20	3.54	4.97	6.39	8.96	25	45	55	65	70	75	90
−5	0.14	0.87	1.74	2.90	4.24	5.59	8.13	20	40	50	60	70	75	85
0	0.11	0.66	1.37	2.37	3.58	4.86	7.34	20	35	50	60	65	70	80
+5	0.09	0.50	1.08	1.93	3.01	4.21	6.60	20	30	45	55	60	70	80
+10	0.07	0.39	0.85	1.57	2.51	3.62	5.92	20	30	40	50	60	65	75
+20	0.05	0.24	0.53	1.03	1.75	2.63	4.69	20	25	35	45	55	60	70

expenditure. Unlike the base scenario, the results along the top left to bottom right diagonal are identical as the balance to withdrawal ratios[26] without the Age Pension are identical. Allocations to growth assets are significantly increased, particularly at low balance and expenditure amounts where the Age Pension previously provided a significant proportion of expenditure desires.

The maximum initial balance to withdrawal ratio at which a 100% growth asset allocation is recommended is 14.8,[27] again relatively consistent with the outcome of Ho *et al.* (1994). Consistent balance/withdrawal ratios tend to give a slightly lower growth asset allocation for the base scenario (see Table 3) compared to NP, due to the possibility of receiving a higher pension as the balance decreases over time.

Tables 13 and 14 show the effect of reducing the equity risk premium by 2% p.a. and 4% p.a. respectively.

As expected, the shortfall years increase significantly with the reduction in the equity risk premium (from 0.46 years to 1.22 years (EQ2) and 2.37 years (EQ0) for the

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[26] Under NP withdrawals and expenditure are the same.

[27] This is achieved by increasing the initial balance by 20% and increasing initial expenditure by 5% (i.e. (500,000 × 1.2)/(38,611 × 1.05) = 14.8).

Table 15. *Minimum shortfall years and optimal asset allocation* (*EV*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.10	0.51	0.90	1.51	2.36	3.45	6.27	35	60	75	90	100	100	100
−10	0.06	0.30	0.55	0.91	1.44	2.16	4.15	30	50	65	80	90	100	100
−5	0.05	0.24	0.44	0.73	1.15	1.73	3.39	30	45	60	70	85	95	100
0	0.04	0.19	0.35	0.59	0.93	1.40	2.77	25	40	55	65	80	90	100
+5	0.03	0.15	0.28	0.49	0.76	1.14	2.28	25	40	50	60	75	85	100
+10	0.03	0.12	0.23	0.40	0.63	0.94	1.89	25	35	45	55	70	80	100
+20	0.02	0.08	0.15	0.27	0.44	0.66	1.32	25	30	40	50	60	70	90

original initial balance and expenditure). What is surprising is the small magnitude of the reduction in equity allocation, which for the original initial balance and expenditure is only from 75% to 70% for a 2% p.a. reduction in equity risk premium and to 65% for a 4% p.a. reduction in equity risk premium.

Instead of adjusting the equity risk premium, the effect of increasing domestic equity volatility by 25% (EV) is investigated in Table 15.

Again the results are intuitive, with the average shortfall years increasing for the original initial balance and expenditure from 0.46 to 0.59 years, due to the larger growth asset return volatility causing earlier ruin years in simulations with poor returns. The optimal growth asset allocation has also decreased from 75% to 65%, reflective of a transfer to defensive assets as growth assets become more volatile.

The effect of removing real autocorrelation in equity returns (RW) is presented in Table 16, along with a comparison using the dynamic asset allocation (RW+DY) in Table 17.

Removing autocorrelation from equity returns increases shortfall years and reduces the optimal allocation to growth assets, due to the lack of mean reversion causing higher rolling 2–3 year equity return volatility. The average shortfall years increases for the original initial balance and expenditure from 0.46 to 0.56 years, with a reduction in growth assets from 75% to 70%.

However, the effect of using the dynamic investment strategy is unchanged whether using the base investment returns or the non-autocorrelated equity returns. Increases and decreases are found to the average shortfall years in the same cells, with the reduction to growth asset allocation being similar.

The effect of using mortality rates with no improvement (MO) is investigated in Table 18. This has the effect of reducing the additional years of life expectancy (by average) from 23.9 to 21.6 years.

Not surprisingly the average shortfall years has decreased for the original initial balance and expenditure from 0.46 to 0.31 years, due to the earlier death experience. The optimal growth asset allocation has also decreased from 75% to 65%. The maximum initial balance to withdrawal ratio at which a 100% growth asset

Table 16. *Minimum shortfall years and optimal asset allocation* (*RW*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.11	0.48	0.82	1.34	2.12	3.21	6.09	40	65	85	100	100	100	100
−10	0.06	0.30	0.52	0.84	1.29	1.93	3.92	35	55	70	90	100	100	100
−5	0.05	0.23	0.42	0.68	1.04	1.54	3.15	30	50	65	80	95	100	100
0	0.04	0.19	0.34	0.56	0.85	1.25	2.54	30	45	55	70	90	100	100
+5	0.03	0.15	0.28	0.47	0.71	1.03	2.06	30	40	55	65	80	95	100
+10	0.02	0.12	0.23	0.39	0.59	0.86	1.70	25	40	50	60	75	90	100
+20	0.02	0.08	0.15	0.27	0.42	0.62	1.19	25	35	45	55	65	80	100

Table 17. *Minimum shortfall years and optimal asset allocation* (*RW* + *DY*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.22	0.55	0.83	1.31	2.07	3.17	6.27	25	55	70	75	85	95	100
−10	0.15	0.40	0.58	0.84	1.26	1.89	3.90	20	45	60	70	75	85	100
−5	0.13	0.34	0.50	0.70	1.02	1.50	3.11	15	40	55	65	75	80	95
0	0.11	0.30	0.43	0.60	0.85	1.22	2.49	15	35	50	65	70	75	90
+5	0.10	0.26	0.38	0.53	0.72	1.01	2.01	10	30	45	55	70	75	85
+10	0.09	0.23	0.33	0.46	0.63	0.85	1.65	10	30	40	55	65	70	80
+20	0.07	0.18	0.26	0.37	0.49	0.64	1.15	5	20	30	45	55	65	75

Table 18. *Minimum shortfall years and optimal asset allocation* (*MO*)

Balance (% diff)	Shortfall years							Prop growth assets (%)						
	Expenditure (% diff)							Expenditure (% diff)						
	−20	−10	−5	0	+5	+10	+20	−20	−10	−5	0	+5	+10	+20
−20	0.04	0.26	0.48	0.83	1.35	2.18	4.59	30	60	80	100	100	100	100
−10	0.02	0.14	0.29	0.49	0.79	1.22	2.75	30	50	65	80	100	100	100
−5	0.02	0.10	0.22	0.39	0.62	0.95	2.14	25	45	60	75	90	100	100
0	0.01	0.08	0.17	0.31	0.50	0.76	1.66	25	40	50	65	85	100	100
+5	0.01	0.06	0.13	0.25	0.41	0.62	1.31	25	35	45	60	75	90	100
+10	0.01	0.05	0.10	0.19	0.33	0.50	1.06	20	35	45	55	70	85	100
+20	0.00	0.03	0.06	0.12	0.22	0.35	0.72	20	30	35	45	60	70	100

allocation is recommended is 13.8;[28] this is not as big a decrease as seen for a similar drop in life expectancy in Ho *et al*. (1994).

5 Conclusions and future research

This study has used a stochastic investment model and random death year to simulate how different investment strategies affect the ability of a retiree maintaining investment and longevity risk to provide income streams during their retirement, with the objective to minimise the average shortfall years that the income stream will fail to meet desired expenditure levels. The optimal allocation to equity investment is largely dependent on the level of the retirement balance compared to the desired expenditure levels. Where the initial balance is large relative to desired expenditure, the allocation to equities is decreased to reduce the risk of adverse experience, whereas when the initial balance is small, the allocation to equities is increased to attempt to capture the upside of equity investment needed to fund desired expenditure. However, due to the means-tested nature of the Age Pension, these relationships are not constant for given balance to expenditure ratios, with the optimal allocation to equity investment reducing as the balance and expenditure reduces due to the increased proportion of desired expenditure made up by Age Pension receipts.

Assuming that expected lifetime real expenditure is constant, an increase in the weighting to early retirement expenditure reduces the optimal growth asset allocation and increases the effect of changes in the relative balance and desired expenditure on average shortfall years compared to a fixed real expenditure pattern. Conversely, a decrease in the weighting to early retirement expenditure increases the optimal growth asset allocation and decreases the effect of changes in the relative balance and desired expenditure on average shortfall years. Where the weighting is increased in early and later years but reduced in middle years, the optimal growth asset allocation is increased along with an increase in the effect of changes in the relative balance and desired expenditure.

It is found that the optimal allocation to equity investment reduces with age, assuming previous median investment experience. However, a fixed life-cycle strategy reducing equity investment with age is not beneficial in reducing average shortfall years, and a dynamic strategy considering investment performance is only beneficial in some cases. Results indicate that an optimal dynamic strategy must consider balance, expenditure and age, although this is not developed explicitly.

This problem with defining appropriate dynamic investment strategies underlies a larger conceptual problem with the optimisation approach used in simulation studies. The procedure used in Table 2 implicitly assumes a constant investment strategy throughout retirement in obtaining the optimal investment strategy. However, the results also suggest that the optimal investment strategy depends upon circumstances, despite the simulations assuming a constant strategy no matter what the future experience is. However, it may be possible to use the insight from analytical

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[28] This is achieved by decreasing the initial balance by 20% and holding initial expenditure steady (see Table 3).

approaches to generate decision rules to investigate the effect of dynamic strategies in a simulation framework – this is left to future research.

Comparing the base scenario to a scenario where the Age Pension is not available shows that a more consistent relationship with growth asset allocation when comparing balance to withdrawal levels. A 100 % growth asset allocation is recommended for initial balance to withdrawal ratios of up to around 14–15. Further sensitivity testing shows a 2 % drop in equity risk premium reduces the optimal growth asset allocation by approximately 5 %, while a 4 % drop reduces the optimal growth asset allocation by approximately 10 %. Increasing equity volatility by 25 % reduces the optimal growth asset allocation by approximately 10 %, while removing autocorrelation in equity returns reduces the optimal growth asset allocation by approximately 5 % and has no impact on the effectiveness of the dynamic strategy tested. A drop of 2.3 years or 10 % in life expectancy reduces the optimal growth asset allocation by approximately 10 %.

With the increasing transfer of risk from institutions to individuals in retirement, research on how to best manage these risks will become particularly useful for those providing advice to retirees. This paper provides a relatively simple approach to modelling appropriate investment strategies in retirement; in addition to that already described above future work could go into more detail on the following:

- Appropriateness of alternative dynamic investment strategies based on balance, expenditure and age.
- Ability to use other assets, particularly property, to fund retirement expenditure desires.
- Incorporation of joint modelling of couples and scenarios where individuals partially move into retirement while continuing to work part-time.
- A more developed expenditure model to better reflect realistic consumption patterns amongst retirees. This could be done by considering the components driving retirement expenditure in more detail, rather than assuming simple expenditure patterns and indexation. Alternatively, desired expenditure levels could be variable depending on investment performance. In this case revised decision metrics incorporating actual expenditure would be required.
- A more developed mortality model, taking into account uncertainty in future mortality improvements and linkages between mortality and socio-economic factors. A reduction in mortality would increase the optimal growth asset allocation, all other factors remaining equal, due to the need to fund expenditure for a longer period.
- Testing of the effect of other investment models on the results. While the Wilkie model is well known and widely used in actuarial circles, alternative processes such as regime switching and/or vector autoregressions may provide additional insight on the effect of more extreme equity crashes.
- A retiree may be interested in more than the desired level of expenditure as the sole objective, but also the potential for consumption above a desired level. A retiree may also have a bequest motive, necessitating the need to consider a final balance at death. These could be incorporated as additional objectives in an explicit way in developing decision metrics.

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Appendix A – Age pension and other details

The government-sponsored means-tested Age Pension used in this research is based on the Australian system. The details below are provided for a single female homeowner.[29]

The maximum fortnightly payment of the Age Pension from 20 March 2010 is $701.10. The Age Pension payment is indexed according to movements in the Consumer Price Index (CPI). The rate of payment is also subject to wage/salary

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[29] For simplicity the Pension Supplement is considered to be part of the Age Pension for all indexation in this paper. See http://www.centrelink.gov.au/internet/internet.nsf/payments/age_rates.htm for further details.

Table A.1. *Minimum withdrawal rates from account-based pensions*

Age	Minimum annual payment required[32] (% of the account balance)
Under age 65	4%
Age 65–74	5%
Age 75–79	6%
Age 80–84	7%
Age 85–89	9%
Age 90–94	11%
Age 95 and over	14%

indexation, with the maximum single rate maintained at a level which is no less than 27.7% of Male Total Average Weekly Earnings (MTAWE).[30]

An asset test is applied, giving a full pension where assets are less than $178,000, a reduction of $1.50 per fortnight for every $1,000 of assets greater than this, and no pension where assets are greater than $645,500.

An income test is also applied, giving a full pension where income is less than $142 per fortnight, a reduction of $0.50 per fortnight for every $1 of income greater than this, and no pension where income is greater than $1,544.20 per fortnight.

The Age Pension entitlement is calculated under both tests, with the test that results in a lower Age Pension entitlement applying. For the purpose of this paper, assets are considered to be the retirement balance, while income is the amount drawn down from the account-based pension less the appropriate deductible.[31] For simplicity and consistency, the thresholds applied in both tests are assumed to be indexed at the same rate as the Age Pension itself. The tests are assumed to be applied annually based on withdrawals during the year and balance at the start of the year.

Table A.1 shows the minimum annual withdrawal amount from an account-based pension. These rates are assumed to reply throughout the projection period.

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[30] This multi-tiered indexation approach is applied throughout the paper – so that any reference to real rates implies indexation at the same rate as the Age Pension.

[31] This is equal to the initial purchase price of the account-based pension divided by the life expectancy of the relevant individual at the purchase date. In this paper, the initial retirement balance represents the purchase price, while the life expectancy of a female at age 65, which is 21.62 years (AGA, 2009), is used as the relevant life expectancy. This amount does not take into account any mortality improvements, as is convention.

[32] Lower minimum withdrawal rates apply in the years 2008–09 and 2009–10 because of the recent financial crisis. However, these temporary rates are not considered in this paper.

Table B.1. *Investment model – summary of equations used*

Variable	Notation	Equation
Price inflation	$q(t)$	$q(t)=\mu_q(1-\phi_q)+\phi_q q(t-1)+\varepsilon_q(t)$
Salary inflation	$w(t)$	$w(t)=\psi_{w,2}q(t-1)+\mu_w+\varepsilon_w(t)$
Short-term interest rate	$is(t)$	$\ln[is(t)]=\ln[il(t)]-X_{is}(t)$ $X_{is}(t)=\mu_{is}(1-\phi_{is})+\phi_{is}X_{is}(t-1)+\varepsilon_{is}(t)$
Cash	$ac(t)$	$ac(t)=(is(t)+is(t-1))/2$
Long-term interest rate[1]	$il(t)$	$il(t)=\psi_{il}M_{il}(t)+\mu_{il}+X_{il}(t)$ $M_{il}(t)=\rho_{il}q(t)+(1-\rho_{il})M_{il}(t-1)$ $X_{il}(t)=\phi_{il,1}X_{il}(t-1)+\phi_{il,2}X_{il}(t-2)+\phi_{il,3}X_{il}(t-3)+\varepsilon_{il}(t)$
Domestic equity dividend yield	$y(t)$	$\ln[y(t)]=\ln\mu_y+X_y(t)$ $X_y(t)=\phi_y X_y(t-1)+\varepsilon_y(t)$
Domestic equity dividends	$d(t)$	$d(t)=q(t)+\mu_d+\tau_{d,1}\varepsilon_y(t)+\tau_{d,2}\varepsilon_y(t-1)+\varepsilon_d(t)+\theta_d\varepsilon_d(t-1)$
Domestic equities price return[2]	$p(t)$	$p(t)=\ln(D(t)/\ln(1+y(t)))-\ln(P(t-1))$
Domestic equities total return	$de(t)$	$de(t)=p(t)+\ln\begin{pmatrix}1+\ln(1+y(t))\\ \times(\exp(p(t)))^{0.5}\end{pmatrix}$
International equities (total return)	$ie(t)$	$ie(t)=\mu_{ie}+\psi_{ie}de(t)+\varepsilon_{ie}(t)$
Domestic bonds (total return)	$db(t)$	$db(t)=\psi_{db,1}il(t)+\psi_{db,2}il(t-1)$ $+\psi_{db,3}is(t)+\psi_{db,4}is(t-1)+\varepsilon_{db}(t)$
International bonds (total return)	$ib(t)$	$ib(t)=\mu_{ib}+\psi_{ib}db(t)+\tau_{ib}\varepsilon_q(t)+\varepsilon_{ib}(t)$

[1] Constrained to a minimum of 0.001.

[2] Capital forms of small notation (e.g., $D(t)=D(t)\exp(d(t))$ refer to the index value of that variable.

Appendix B – The investment model

The Wilkie (1995) model is used as the starting point for the investment model. A broadly similar approach is used to that described by Butt (2009) in calibrating the Wilkie model; although the model is recalibrated, taking into account data from 30 June 1983 to 30 June 2009 and MTAWE is used instead of Average Weekly Ordinary Time Earnings (all employees) as the salary inflation data source, as this is what Age Pension indexation is based on. In addition, certain parameters in the international models are adjusted such that domestic and international equities have the same expected return. Property variables are not used in this research, due to the relatively low allocation to property of most superannuation schemes (see Table 1), and the difficulty in fitting such models.

The model equations and parameters are found in Tables B.1 and B.2. All interest rates and returns are modelled on a continuously compounding basis. Unless stated, all error terms $\varepsilon(t)$ are independently and identically distributed normally with mean zero and variance σ^2. Summary statistics for the output of the model over 45 years of projections and 10,000 simulations are provided in Table B.3; average returns are geometric. Lagged autocorrelations for domestic equities are provided in Table B.4.

Table B.2. *Investment model – standard error of residuals (SERes), fitted parameter values and standard errors*

Notation	SERes	Parameter	Fitted value	Standard error
$q(t)$	0.0164	μ_q	0.0349	0.0104
		ϕ_q	0.7218	0.1276
$w(t)$	0.0142	$\psi_{w,2}$	0.5407	0.1068
		μ_w	0.0228	0.0051
$is(t)$	0.1939	μ_{is}	0.0937	0.0644
		ϕ_{is}	0.4324	0.1900
$il(t)$	0.0057	ψ_{il}	1.2268	0.0032
		μ_{il}	0.0221	0.0595
		ρ_{il}	0.1720	N/A[33]
		$\phi_{il,1}$	0.1105	0.1859
		$\phi_{il,2}$	−0.2994	0.1641
		$\phi_{il,3}$	−0.3369	0.1620
$y(t)$	0.1654	$\ln\mu_y$	−3.2457	0.0610
		ϕ_y	0.4900	0.1774
$d(t)$	0.0925	μ_d	0.0401	0.0269
		$\tau_{d,1}$	0.3801	0.1195
		$\tau_{d,2}$	−0.2601	0.1236
		θ_d	0.5014	0.1664
$ie(t)$	0.1378	μ_{ie}	0.0074	N/A[34]
		ψ_{ie}	0.9341	0.1860
$db(t)$	0.0075	$\psi_{db,1}$	−3.5495	0.2248
		$\psi_{db,2}$	4.4186	0.2047
		$\psi_{db,3}$	−0.2434	0.1139
		$\psi_{db,4}$	0.3789	0.1152
$ib(t)$	0.0297	μ_{ib}	0.0225	N/A
		ψ_{ib}	0.7788	0.1175
		τ_{ib}	1.2243	0.4134

The results outlined in Table B.3 may give a higher average return for some factors than future expectations, at least in the short-term, might produce.[35] However, since all other investment model factors are driven by price inflation, this is not of particular concern as real returns are unaffected. In addition, the premium on equities over bond returns of 5.6% is relatively consistent with longer-term historical results in Australia. Thus no adjustment is made to the model parameters.

The domestic equity autocorrelation results in Table B.4 exhibit some momentum tendencies over 1-year periods before showing mean-reverting tendencies over 2–3-year periods.

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[33] Due to the nature of the long-term interest rate equation it is not possible to create a linear model to find the value of ρ_{il} which minimises the squared error in the model. As such a range of values was tested to find the minimum standard error of residuals, with $\rho_{il}=0.172$ selected (to three significant figures).

[34] The values of μ_{ie} and μ_{ib} are set to ensure the expected returns under domestic and international investment are consistent.

[35] For example the average price inflation of 3.6% p.a. is outside the Reserve Bank of Australia target of 2–3% p.a.

Table B.3. *Results generated by the investment model*

Factor	Average return % (p.a.)	Standard deviation % (p.a.)	Yearly autocorrelation % (Average)
Price inflation	3.6	2.4	72
Salary inflation	4.3	2.0	32
Domestic equities (total return)	12.3	18.6	13
International equities (total return)	12.4	23.8	7
Australian bonds (total return)	6.7	4.7	7
International bonds (total return)	6.7	4.9	1
Cash	6.3	2.1	86

Table B.4. *Autocorrelations for domestic equities (total return)*

Years of lag	1	2	3	4	5
Autocorrelation %	13	−10	−5	−2	−1

In Australia, there is no tax on investment returns for accounts of individuals in retirement, with taxation offsets for company tax already paid from domestic equity dividends (known as 'franked' dividends) allowed for via a credit of 32%.[36]

Appendix C – Determining the death year

In determining the random death year, it is first necessary to define the probability that an individual currently aged exactly x will die during the year aged y. Using standard actuarial notation, this is defined as ${}_{y-x|}q_x$. The values of q_x from the mortality model in Section 2.3 (incorporating any mortality improvement for all future ages) can be used to calculate ${}_{y-x|}q_x$ recursively as follows:

For $y=x$ $${}_{y-x|}q_x = q_y$$

For $y>x$ $${}_{y-x|}q_x = \left(1 - \sum_{i=x}^{y-1} {}_{i-x|}q_x\right) \times q_y$$

The probability that an individual currently aged exactly x will die before being aged exactly y is defined as ${}_{y-x}q_x = \sum_{i=x}^{y-1} {}_{i-x|}q_x$ where ${}_0q_x = 0$. A uniform random variable z is then obtained; assuming death is uniformly spread within integer ages the death year d is calculated by finding the values ${}_{y-x}q_x$ and ${}_{y-x+1}q_x$ that z lies between and

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[36] Australian Tax Office statistics (2008) reveal that approximately 75% of dividends were franked over the 7 years to 30 June 2008 with the franking credit of 32% calculated as 30% / 70% × 75% assuming a company tax rate of 30% and that 75% of dividends are franked.

performing the following calculation:

$$d = y - x + \frac{z - {}_{y-x}q_x}{{}_{y-x+1}q_x - {}_{y-x}q_x}.$$

Note that if z is greater than ${}_{110-x}q_x$ then d is assumed to be $110 - x$.