Null space velocity control with dynamically consistent pseudo-inverse

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SUMMARY
Null space velocity control is essential for achieving good behaviour of a redundant manipulator. Using the dynamically consistent pseudo-inverse, the task and null space motion and forces are decoupled. The paper presents a globally stable null space velocity controller and the gradient projection technique in conjunction with the dynamically consistent pseudo-inverse. The physical meaning and influence of the compensation terms in null the space velocity controller are explained. The performance of the proposed null space controller is tested on 4. d.o.f planar redundant manipulator interacting with the environment.

KEYWORDS: Pseudo-inverse, Velocity control; Null space; Redundant manipulator.

1. INTRODUCTION
Kinematic redundancy is characterized by extra degrees of freedom with respect to the given motion constraints posed by an assigned primary task. It provides a mean for solving sophisticated motion tasks such as avoiding obstacles, avoiding singularity, optimizing manipulability, joint torques, etc., while performing the desired primary task. However, this requires complex solutions on both mechanical and control design.[1,2] In order to solve the kinematic redundancy, many control strategies have been proposed based on two primary approaches – local and global optimization.[1,3,4] Although global optimization is generally more successful, it requires optimization over the entire range of control and is thus not suitable for real-time control. On the other hand, local optimization needs less computation, but can result in a poorer performance from the global viewpoint. Nevertheless, local optimization remains the only choice for sensory guided robots, where the task path is generally not predictable. Regarding the control input the redundancy can be solved at the velocity, the acceleration or the torque level. Most approaches published in the past are based on local optimization techniques with a redundancy resolution at the velocity level using the gradient projection technique. Using the gradient optimization technique a variety of performance criteria, such as the collision avoidance, the torque optimization, the singularity avoidance and the manipulability optimization, can be easily incorporated into the control. The choice of the pseudo-inverse in the redundancy resolution determines

the null space motion of the redundant manipulator. Whitney[5] solved the redundancy using the weighted pseudo-inverse, which minimizes null space velocities. The inertia weighted pseudo-inverse, as proposed by Khatib,[6] minimizes instantaneous kinetic energy of the null space motion. It has been later proved by Khatib and Park[7,8] that this is the only pseudo-inverse, that is consistent with the manipulator dynamics, i.e. the null space motion does not produce the operational space acceleration. Using the null space controller formulation as proposed by Hsu and Sastry,[2] we have shown that the null space controller ensures that no task space acceleration is generated irrespective of the type of the pseudo-inverse used.[9] However, this does not assure a decoupling on the force level and hence the inertia weighted pseudo-inverse remains the only choice. Additionally, the null space motion that optimises a given criteria also depends on the type of pseudo-inverse, i.e. different mapping of the Jacobian to the null space requires different trajectories to optimize the same criteria. Recently, a weighted joint space decomposition has been proposed to find a minimal parametrisation of null-space joint velocities.[10] In this paper we present a new formulation of the null space velocity controller that uses the dynamically consistent pseudo-inverse. It is well known that a null space projector rotates a solution optimized in a task space. Therefore, the desired null space velocity has been obtained using a modified gradient optimisation technique.

2. KINEMATICS OF REDUNDANT MANIPULATOR
Redundant manipulators have more degrees of freedom (n) than the dimension of the work-space (m), required to perform the given task. The joint and task coordinates are related by the following equation

$$x = p(q), \qquad (1)$$

where q is n dimensional vector of joint coordinates, x is m dimensional vector of the task coordinates and p is m dimensional vector representing manipulator forward kinematics. Differentiating Eq. 1 we obtain relation between the joint and the task velocities

$$\dot{x} = J\dot{q}, \qquad (2)$$

where J denotes $m \times n$ manipulator Jacobian matrix. Mapping of joint velocities to task velocities is unique, while mapping of task velocities to joint velocities is not. A general solution of Eq. 2 is given as

$$\dot{q} = \mathbf{J}^{\#}\dot{x} + \mathbf{N}\dot{q}. \tag{3}$$

$\mathbf{J}^{\#}$ is the generalized inverse of $\mathbf{J}$ and $\mathbf{N}$ is $n \times n$ matrix which projects $\dot{q}$ to the null space of $\mathbf{J}$. A general form of null space projection matrix $\mathbf{N}$ is given by

$$\mathbf{N} = (\mathbf{I} - \mathbf{J}^{\#}\mathbf{J}) \tag{4}$$

The first term of the Eq. 3 is the particular solution of the Eq. 2, which enables to calculate joint velocities necessary to perform the given task. The second term is the homogenous solution of the above equation, which contributes to a motion in the null space only, without affecting the task space motion. This motion is the *self motion* of the redundant mechanism. In general, there is an infinite number of configuration of the redundant mechanism which satisfy the desired task coordinates.

3. CONTROL OF REDUNDANT MANIPULATOR

The approach used here is based on splitting the control problem of an redundant manipulator into two separate tasks: end effector motion and null space motion. The general form of the equation of motion of the manipulator interacting with the environment can be described by

$$\boldsymbol{\tau} = \mathbf{H}\ddot{q} + \mathbf{C}\dot{q} + g + \mathbf{J}^{T}\boldsymbol{F} \tag{5}$$

where $\boldsymbol{\tau}$ is n dimensional vector of joint torques, $\mathbf{H}$ is an $n \times n$ symmetric, positive definite inertia matrix , $\mathbf{C}$ is $n \times n$ matrix of nonlinear terms due to the centrifugal, Coriolis and friction forces, g is n dimensional vector of gravitational forces and $\boldsymbol{F}$ is an m dimensional vector of environment contact force acting on the end-effector.

3.1 End effector control
The aim of the end effector control law is to track the desired generalized task coordinates x_d, which includes position and force. Let the control law be given as

$$\boldsymbol{\tau} = \mathbf{J}^{T}(\boldsymbol{\Lambda}(\ddot{x}_c - \dot{\mathbf{J}}\dot{q}) + \boldsymbol{F}) + \tag{6}$$

$$\mathbf{C}\dot{q} + g + \boldsymbol{\tau}_0, \tag{7}$$

where $\mathbf{J}$ denotes $n \times$ m manipulator Jacobian and $\boldsymbol{\Lambda}$ denotes the operational space kinetic energy matrix defined as $\boldsymbol{\Lambda} = (\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^{T})^{-1}$.[6] $\ddot{x}_c$ is the command acceleration vector chosen as

$$\ddot{x}_c = \ddot{x}_d + \mathbf{K}_v(\dot{x} - \dot{x}_d) + \mathbf{K}_p(x - x_d), \tag{8}$$

where $\mathbf{K}_v$ and $\mathbf{K}_p$ are properly chosen positive definite diagonal gain matrices. $\boldsymbol{\tau}_0$ is an arbitrary torque vector. A proper choice of $\boldsymbol{\tau}_0$ enables the manipulator to achieve secondary sub-task without affecting generalised force and trajectory at the manipulator's end-effector.[1] Thus, the appropriate choice of the $\boldsymbol{\tau}_0$ enables to investigate the stability of the primary task independently of the secondary task. The problem of the calculation of the appropriate $\boldsymbol{\tau}_0$ is discussed in the next chapter. First we will investigate the stability regarding the motion in the task space. Pre-multiplying Eq. 5 by $\mathbf{J}\mathbf{H}^{-1}$ and considering $\ddot{x} = \mathbf{J}\ddot{q} + \dot{\mathbf{J}}\dot{q}$ we obtain

$$\ddot{x} - \dot{\mathbf{J}}\dot{q} + \mathbf{J}\mathbf{H}^{-1}(\mathbf{C}\dot{q} + g) + \mathbf{J}\mathbf{H}^{-1}\mathbf{J}^{T}F = \mathbf{J}\mathbf{H}^{-1}\boldsymbol{\tau}. \tag{9}$$

Inserting the control law 7 into Eq. 9 and considering that vector the $\boldsymbol{\tau}_0$ is such that $\mathbf{J}\mathbf{H}^{-1}\boldsymbol{\tau}_0 = 0$, yields the equation describing the closed loop behaviour of the system.

$$\ddot{\mathbf{x}}_d - \ddot{\mathbf{x}} + \mathbf{K}_v(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{K}_p(\mathbf{x}_d - \mathbf{x}) = 0 \tag{10}$$

3.2 Inertia weighted pseudo-inverse
In the previous section we have introduced the vector $\boldsymbol{\tau}_0$, which is required to fulfil the following equation

$$\mathbf{J}\mathbf{H}^{-1}\boldsymbol{\tau}_0 = 0 \tag{11}$$

A proper choice of $\boldsymbol{\tau}_0$ enables the manipulator to achieve secondary sub-task without affecting force and trajectory tracking of the manipulator's end-effector. One possible choice of the $\boldsymbol{\tau}_0$ is projecting control acceleration vector into null space of $\mathbf{J}$ with

$$\boldsymbol{\tau}_0 = \mathbf{H}(\mathbf{I} - \mathbf{J}^{\#}\mathbf{J})\ddot{q}. \tag{12}$$

Short calculation proves that this projection fulfils Eq. 11. The particular solution of the Eq. 7 depends on utilization of the generalized inverse $\mathbf{J}^{\#}$. The inertia weighted pseudo-inverse defined as

$$\bar{\mathbf{J}} = \mathbf{H}^{-1}\mathbf{J}^{T}\boldsymbol{\Lambda} \tag{13}$$

is the solution of the minimization problem of cost function $\dot{q}^{T}\mathbf{H}\dot{q}$,[5] which minimizes the instantaneous kinetic energy of the manipulator. In references 7,8 it was proved that the inertia weighted pseudo-inverse is the only one that does not produce operational space accelerations with arbitrary chosen $\boldsymbol{\tau}_0$, that belongs to the Jacobian transpose null-space. Recently, it was proved by Featherstone and Kathib that an inertial weighted pseudo-inverse is also load independent.[7]

3.3 Null space velocity controller
The proposed control in the case of the redundant robots differs from that of a non-redundant case only in the last term $\boldsymbol{\tau}_0$ in Eq. 7, which defines the self motion of redundant manipulator. The control of the self motion is the major control problem of redundant robots. In this chapter we consider the case when the desired null space velocity $\dot{q}_d^{ns}$ that optimizes a desired criteria is known. The problem of finding desired null space velocity $\dot{q}_d^{ns}$ will be discussed in the next section. The task of the null space velocity controller is to track the desired null space velocity $\dot{q}_d^{ns}$, i.e., the null space velocity error $e^{n} = \dot{q}_d^{ns} - \dot{q}^{ns}$ should converge to zero. Since the null space velocity is the projection of the manipulator velocity to the Jacobian null space, the null space velocity error and it's derivative can be expressed as

$$e^{n} = \dot{q}_d^{ns} - (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\dot{q} \tag{14}$$

$$\dot{e}^{n} = \ddot{q}_d^{ns} - (\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q} - (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\ddot{q}. \tag{15}$$

Inserting the control law 7 into Eq. 5 and denoting $\boldsymbol{\zeta} = \ddot{x}_d + \mathbf{K}_v(\dot{x} - \dot{x}_d) + \mathbf{K}_p(x - x_d) - \dot{\mathbf{J}}\dot{q}$ yields the closed-loop dynamics equation in the form

$$\ddot{q} = \mathbf{H}^{-1}\mathbf{J}^{T}\boldsymbol{\Lambda}\boldsymbol{\zeta} + \mathbf{H}^{-1}\boldsymbol{\tau}_0. \tag{16}$$

Substituting $\ddot{q}$ from Eq. 16 into 15 we get

$$\dot{e}^{n} = \ddot{q}_d^{n} + (\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q} - \tag{17}$$

$$(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\mathbf{J}^{T}\boldsymbol{\Lambda}\boldsymbol{\zeta} -$$

$$(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_0.$$

Considering $\mathbf{H}^{-1}\mathbf{J}^T\mathbf{\Lambda} = \bar{\mathbf{J}}$ and $(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\dot{\bar{\mathbf{J}}} = 0$ the third term on the right side of the above Eq. is zero and hence,

$$\dot{e}^n = \ddot{q}_d^n + (\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q} - (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\boldsymbol{\tau}_0 \quad (18)$$

The derivation of the null-space control law relies on the following relation:

Lemma : If the error vector e^n belongs to the Jacobian null space, where mapping to the null space is defined by $(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})$ and $\bar{\mathbf{J}}$ is the inertia weighted pseudo-inverse, then

$$e^{nT}\mathbf{H}\bar{\mathbf{J}} = 0 \quad (19)$$

Proof: If the e^n belongs to the null space, it can be expressed as $(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})e$. Inserting null space error vector into Eq. 19 yields

$$e^{nT}\mathbf{H}\bar{\mathbf{J}} = e^T(\mathbf{I} - \mathbf{J}^T\bar{\mathbf{J}}^T)\mathbf{H}\bar{\mathbf{J}} \quad (20)$$
$$= e^T(\mathbf{H}\bar{\mathbf{J}} - \mathbf{J}^T(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1}$$
$$(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)(\mathbf{J}\mathbf{H}^{-1}\mathbf{J}^T)^{-1})$$
$$= e^T(\mathbf{H}\mathbf{H}^{-1}\mathbf{J}^T\mathbf{\Lambda} - \mathbf{J}^T\mathbf{\Lambda}) = 0$$

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Let the null space control vector $\boldsymbol{\tau}_0$ be given by

$$\boldsymbol{\tau}_0 = \mathbf{H}(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})(\ddot{q}_d^n + \dot{\bar{\mathbf{J}}}\mathbf{J}\dot{q} + \mathbf{K}_n e^n + \mathbf{H}^{-1}\mathbf{C}e^n). \quad (21)$$

Combining Eqs. 18 and 21 and considering that $\ddot{q}_d^n$ and e^n belong to the Jacobian null-space gives

$$\dot{e}^n = -\mathbf{K}_n e^n + \bar{\mathbf{J}}\mathbf{J}(\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q} - (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\mathbf{C}e^n. \quad (22)$$

Now define a Lyapunov function by

$$\nu = \frac{1}{2}e^{nT}\mathbf{H}e^n. \quad (23)$$

Then

$$\dot{\nu} = e^{nT}\mathbf{H}\dot{e}^n + \frac{1}{2}e^{nT}\dot{\mathbf{H}}e^n \quad (24)$$

$$= -e^{nT}\mathbf{K}_n\mathbf{H}e^n + \frac{1}{2}e^{nT}\dot{\mathbf{H}}e^n$$
$$+ e^{nT}\mathbf{H}\bar{\mathbf{J}}\mathbf{J}\dot{\bar{\mathbf{J}}}\mathbf{J}\dot{q}$$
$$- e^{nT}\mathbf{H}(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\mathbf{C}e^n$$

$$= -e^{nT}(\mathbf{K}_n\mathbf{H})e^n + \frac{1}{2}e^{nT}(\dot{\mathbf{H}} - 2\mathbf{C})e^n$$

$$= -e^{nT}(\mathbf{K}_n\mathbf{H})e^n$$

since $e^{nT}\mathbf{H}\bar{\mathbf{J}}$ is zero for an inertia weighted pseudo-inverse $\bar{\mathbf{J}}$ and $(\dot{\mathbf{H}} - 2\mathbf{C})$ is skew symmetric. Since ν is positive definite and $\dot{\nu}$ is negative definite providing that $\mathbf{K}_N$ is diagonal matrix with positive terms, e^n tends to zero and the proposed controller stabilizes the null-space motion as long as the Jacobian is non-singular. A similar result has been independently derived in reference 11. It is not always straightforward to calculate vectors $\ddot{q}_d^n$ and $\dot{\bar{\mathbf{J}}}$ in analytical form. In our experiments numerical differentiation was sufficient approximation of the above signals. Moreover, the

term $\dot{\bar{\mathbf{J}}}\mathbf{J}\dot{q}$ in Eq. 21 describes the compensation of the task space velocity can be in case of the low task space velocity omitted without serious degradation of the performance.[9] One of the advantages of the explicit null space velocity control is that the desired null space velocity obtained from an appropriate optimization procedure can be scaled to maintain it within the given joint speed limits. As it has been previously mentioned, the inertia weighted pseudo-inverse is the only one that does not produce operational space accelerations with arbitrary chosen $\boldsymbol{\tau}_0$. It can be easily verified by combining Eq. 9 and 21, that with the proposed null-space controller we get no operational accelerations regardless to the choice of the pseudo-inverse. The reason why the use of inertia weighted pseudo-inverse is still preferable is that this is the only one that produces no null-space accelerations applying an external force to the manipulator's end effector. This propriety can be easily verified by expressing null-space acceleration in the form

$$\ddot{q}^n = (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\ddot{q} - (\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q}. \quad (25)$$

Substituting for $\ddot{q}$ from Eq. 5 gives

$$\ddot{q}^n = (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}(\boldsymbol{\tau} - \mathbf{C}\dot{q} - g - \mathbf{J}^T F) - (\bar{\mathbf{J}}\dot{\mathbf{J}} + \dot{\bar{\mathbf{J}}}\mathbf{J})\dot{q} \quad (26)$$

For arbitrary chosen external force, in order not to produce any null space acceleration, it is necessary that

$$(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})\mathbf{H}^{-1}\mathbf{J}^T F = 0. \quad (26)$$

Inserting Eq. 13 into the above equation verifies that the inertia weighted pseudo-inverse fulfills that equation.

3.4 Calculation of the desired null-space velocity
Suppose that the desired performance criterion, which is to be optimized, is represented by a smooth function $p = p(q)$. The joint coordinates are related to task coordinates by

$$\dot{q} = \bar{\mathbf{J}}\dot{x} + (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})K\boldsymbol{\phi}, \quad (28)$$

where $\boldsymbol{\phi}$ is an arbitrary chosen vector. Then

$$\boldsymbol{\phi} = \left(\frac{\partial p}{\partial q_1}, \quad \frac{\partial \mathbf{p}}{\partial q_2}, \quad \ldots, \quad \frac{\partial p}{\partial q_N}\right)^T \quad (29)$$

minimizes p for any scalar $K < 0$ and maximizes p for any $K > 0$. The proof of the above theorem can be found in reference 12 and is based on the positive semi-definition of the form $\boldsymbol{\phi}^T(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})K\boldsymbol{\phi}$. It can be easily shown that for $\bar{\mathbf{J}}$ defined as Moore-Penrose pseudo-inverse the form $\boldsymbol{\phi}^T(\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})K\boldsymbol{\phi}$ is always positive semi-definite. On the contrary, if $\bar{\mathbf{J}}$ is weighted pseudo-inverse as in Eq. 13, this form is no more symmetrical and therefore the semi-definition can not be assured. Therefore, the original gradient optimization can not be utilized. Premultiplying the vector $\boldsymbol{\phi}$ by $\mathbf{H}^{-1}$ gives the optimization solution in the form

$$\dot{q} = \bar{\mathbf{J}}\dot{x} + (\mathbf{I} - \bar{\mathbf{J}}\mathbf{J})K\mathbf{H}^{-1}\boldsymbol{\phi} \quad (30)$$

which assures the best optimization step in the case of inertia weighted pseudo-inverse. The proof of the above relation can be found in reference 13. The problem can be represented also as the problem of finding the best

optimization step of the vector ϕ projected into the null space.[14] Different projection operators cause different rotations of the vector ϕ and thus ϕ has to be previously rotated by $\mathbf{H}^{-1}$ to compensate for the projection operator rotation.

3.5 Optimization Criteria

The force and the position tracking are usually of the highest priority for a force controlled robot. The selection of the sub-tasks with lower priority depends on the specific application.[15] However, the collision avoidance is of great importance, since force controlled robot interacts with the environment. With non-redundant robots we can usually predict the robot configuration necessary to accomplish given task. On the contrary, with redundant robots the robot configuration is not predictable. Another important sub-task for the force controlled robot might be of benefit to the mechanical advantage of the manipulator in order to minimize joint torques when applying a certain force to the end effector. Unfortunately, the local joint torque minimization often brings the robot into the singular configuration. Therefore, the singularity avoidance algorithm has to be implemented as a third criteria. We have accomplished the task by maximizing manipulator manipulability proposed in reference 12. Following the idea of the obstacle avoidance using the potential field[16] we define the cost function $p = \frac{l}{2}\mathbf{V}d_0^2$, where $\mathbf{V}$ is an $l \times l$ rotation matrix describing the direction of an artificial potential field pointing from the obstacle, l is the dimension of the position sub-space and d_0 is the shortest distance between obstacle and the robot body. In our case the desired objective is fulfilled if the imaginary force is applied only on robot joints. In this case we can obtain cost function gradient in simple form as

$$\frac{\partial p}{\partial q} = \mathbf{V}d_0(\mathbf{J}^{0,1} + \mathbf{J}^{0,2} + \ldots + \mathbf{J}^{0,n-1}), \tag{31}$$

where d_0 is the vector of shortest distances between the joint and the obstacle and $\mathbf{J}^{0i}$ denotes Jacobian matrices between base (the first index in the superscript) and ith joint (the second index in the superscript) regarding the robot positions only. The local joint torque minimization as a performance objective was intensively investigated by many authors.[17–19] As the joint torque depends on the system dynamics it is difficult to express the gradient of the cost function related to joint torques. We simplified the problem by minimizing only joint torques related to the force applied to the robot end effector. We define the cost function in the form $p = \tau^T\tau$, where $\tau = \mathbf{J}^T F$ is the joint torque due to the end effector force. Then, the cost function gradient required to minimize the given function can be expressed in the form

$$\frac{\partial p}{\partial q} = 2F^T\mathbf{J}\nabla\tau, \tag{32}$$

$$\nabla\tau = \begin{bmatrix} \frac{\partial \mathbf{J}^{(1)}}{\partial q_1}F & \frac{\partial \mathbf{J}^{(1)}}{\partial q_2}F & \ldots & \frac{\partial \mathbf{J}^{(1)}}{\partial q_n}F \\ \frac{\partial \mathbf{J}^{(2)}}{\partial q_1}F & \frac{\partial \mathbf{J}^{(2)}}{\partial q_2}F & \ldots & \frac{\partial \mathbf{J}^{(2)}}{\partial q_n}F \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \mathbf{J}^{(n)}}{\partial q_1}F & \frac{\partial \mathbf{J}^{(n)}}{\partial q_2}F & \ldots & \frac{\partial \mathbf{J}^{(n)}}{\partial q_n}F \end{bmatrix}, \tag{33}$$

where $\mathbf{J}^{(i)}$ denotes ith column of the Jacobian $\mathbf{J}$. This approach can be justified by the fact that velocities and acceleration during the force tracking are usually low. Another advantage using this approach is that the minimization can be related to the desired end-effector force and therefore the manipulator can be put to the optimal pose before the contact with the environment is established. It is not straightforward how to combine three different optimisation criterion, especially, because optimizing one criteria can prevent the optimisation of the second one. For example, local joint torque optimisation tends to bring the robot to a singular position. We solved the problem using the switching task priority formulation, where priority levels
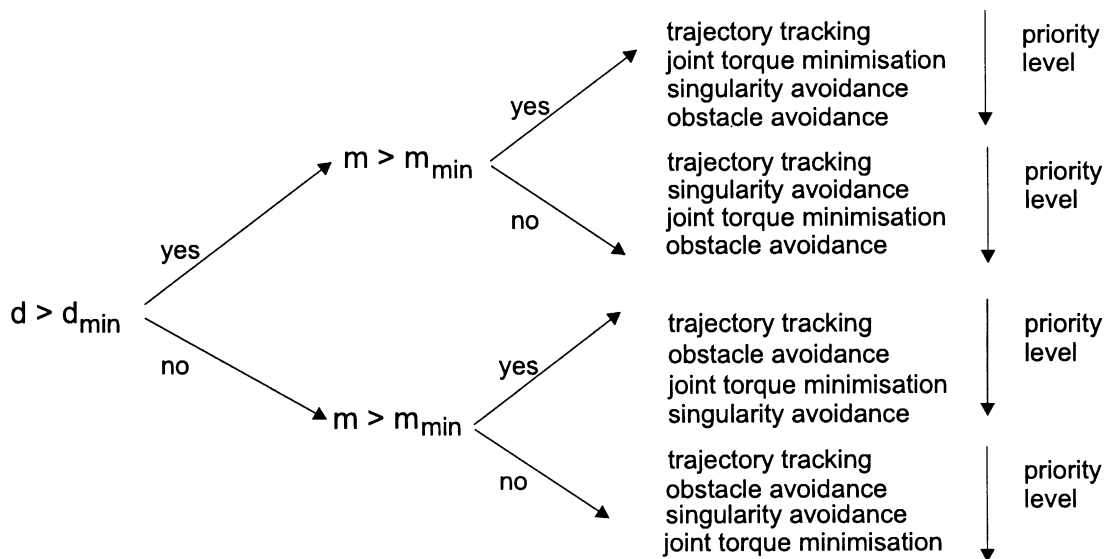

Fig. 1. Decision tree for the task priority selection. d denotes minimal distance between obstacle and a joint and m denotes manipulability index.
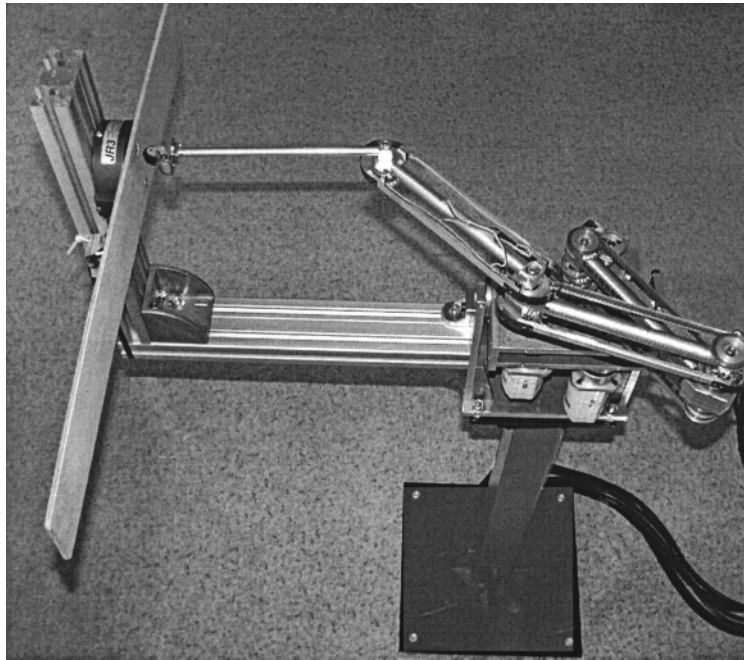
Fig. 2. 4. D.O.F. planar redundant robot.

are dynamically allocated upon the configuration of the robot. The highest priority level task has been always trajectory an force tracking. If the robot joints were near the obstacle, the second priority level was the obstacle avoidance. The joint torque minimization task has had a higher priority level, if the robot manipulability was above the low limit and the joints were not near the obstacle. The priority switching is described by the decision tree in Fig. 1. We have implemented a task priority algorithm for multiple priority levels as proposed in reference 20.

4. EXPERIMENTAL RESULTS FOR A 4. D.O.F. PLANAR REDUNDANT ROBOT

The efficiency of the proposed algorithm has been tested on 4-DOF planar redundant robot with all segments of equal length (see Fig. 2). The robot has no limits in joint angles. All AC brush-less motors are located in the robot base in order to obtain lightweight links. The robot has low gear ratios (6), thus the coupled dynamics of the robot is not negligible. The robot base can be manually rotated from a vertical to a horizontal position, allowing to study the case
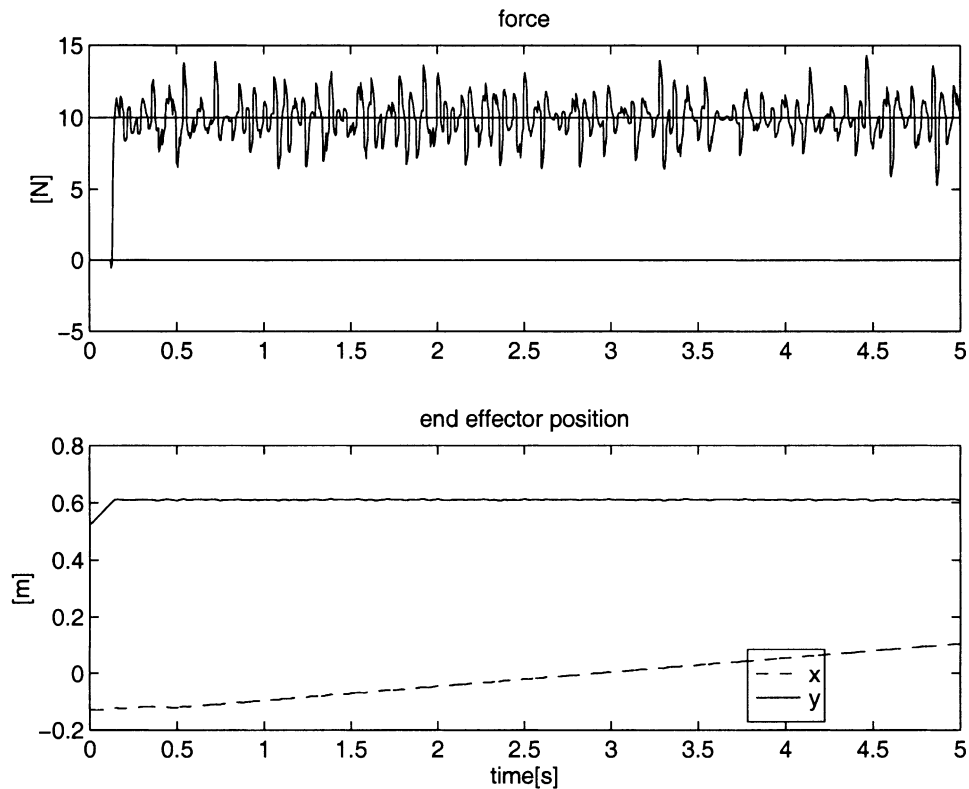

Fig. 3. Force and trajectory tracking.

with the gravity or without gravity's influence on the robot links. We used a JR3 force sensor, which measures forces and torques in all three axes. The sensor was too heavy to be carried by the experimental robot, therefore we mounted the sensor under the plate, which was used as the obstacle. The task of the manipulator was to track the desired force while moving along the obstacle. The robot base has been rotated in a horizontal position. Therefore, gravity has had no influence on the robot dynamics. The secondary subtasks have been the obstacle avoidance and the minimization of the joint torques due to the end-effector force. Obstacle avoidance was carried out by prescribing an imaginary potential field pointing from the obstacle, which forces the robot joints to move away from the obstacle. The proposed control algorithm was realized on SIMULINK and compiled using Simulink Real Time Workshop. We have obtained the control ratio of 500 Hz using a Pentium 200 MHz processor. The calculation of the task velocity compensation term $\dot{\mathbf{J}}\mathbf{J}\dot{\mathbf{q}}$ in Eq. 21, which requires high computational effort, has been omitted without serious performance degradation. Similar, the compensation term $\mathbf{H}^{-1}\mathbf{C}e^n$ in the control law 21 can be also omitted if the null space controller gain $\mathbf{K}_n$ is big enough.[13] The resulting trajectory and force tracking are presented in Fig 3. The first plot shows the force tracking, which is rather noisy and oscillates around the desired force. The y component of the task coordinates changes until the robot hits the wall, while the x coordinate is the ramp function with the constant velocity. Oscillations in y coordinate are due to the force tracking. As it is evident from the figure, we have obtained less satisfactorily results on a real robot compared to a simulated result.[9,13] The cause of the poorer results is in the excessive elasticity of the drive belts between the robot motors and the robot joints and friction in the gears. On the other hand, we obtained better results with an impedance task space control law, as compared to the hybrid force-position control law presented in reference 21, i.e. the position control loop additionally stabilizes the force tracking, but causes the steady state error in the force tracking.

5. CONCLUSIONS
In the paper we presented a new formulation of the null space control law, which utilises the inertia weighted pseudo-inverse. The proposed control law has been used to solve the force control problem of a redundant robot. We have used hierarchical task decomposition, where the primary sub-task was to track the desired force and trajectory, and the secondary sub-tasks were obstacle avoidance, singularity avoidance and joint torque minimization. To accomplish the primary task impedance control has been implemented, but any other force control strategy can also be used with the same null space controller. The secondary task has been carried out by controlling the manipulator's null-space motion. The desired null space motion has been optimised using the gradient projection technique. The optimization procedures and the null-space controller have been adapted in order to use the inertia weighted pseudo-inverse. The proposed controller has been tested on a planar 4. d.o.f redundant robot.

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