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EXTERNALITY, CONVEXITY AND INSTITUTIONS

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Economic theory has generally acknowledged the role that institutions have in shaping economic space. The distinction, however, between physical and institutional descriptions of economic activity has not received adequate attention within the mainstream paradigm. In this paper I show how a proper distinction between the physical and institutional space in economic models will help clarify the concept of externality and provide a better interpretation of the relationship between externality and nonconvexity. I argue that within the Arrow-Debreu framework externality should be viewed as incongruence between the physical and institutional descriptions of the economic space. I also argue that, contrary to conventional wisdom, detrimental externality has no special association with nonconvexity. Starrett's (1972) fundamental nonconvexity has to do with the specific institutional structure of Arrow markets rather than the detrimental nature of externality. Indeed, Arrow markets may not eliminate externalities. In a similar vein, it is not detrimental externality, however intense, that causes the production possibility set to become nonconvex, as argued by Baumol and Bradford (1972), but the particular interpretation of intensity that would make even conventional production possibility sets nonconvex. These points become apparent when one distinguishes between the convexity of the physical and institutional production sets.

1. INTRODUCTION: THE PHYSICAL AND THE INSTITUTIONAL

This contradictory meaning, as we have said, turns on failure to distinguish the institution of property from the technology of production. The puzzles are unnecessary if institutional economics is distinguished from engineering economics. Each is an economics of activity. Institutional economics is the

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activity of transactions in the relation of man to man, but engineering economics is the activity of increasing output in the relation of man to nature. But the engineering set-up of society is the march of the physical, biological, and psychological sciences, which give to mankind its command over nature to be used for happiness or destruction according to the collective action of the world's institutions. (Commons 1934, p. 424)

Mainstream models of individual decision-making allow a great degree of flexibility in defining economic space. Commodities can be defined to capture any objects of desire or input requirements for productive activities. A commodity could be an apple to be consumed at 10:00 a.m. in Piccadilly Square, the warm feeling associated with holding hands, or the malicious desire to see a colleague fail to get promotion. Similarly, a productive input may take the form of a quantity of coal, a worker possessing a particular skill and intent on some specific effort level, or the morale of the workforce. This definitional flexibility is undoubtedly a major theoretical strength as it allows for economic models of enormous reach. It also however entails a danger: this flexibility may lead to overlooking the significance of alternative ways of defining the economic space.

A central motivation of economic analysis is to discern the physical possibilities by which human ends can be fulfilled and to investigate the extent to which economies can be organized to exploit these. Objects of desire may be physically unattainable, and it is certainly important to know that, but the more interesting situations are those where ends are physically reachable but the economy seems unable to exploit the possibilities.

It is standard in microeconomic textbooks to state that consumption and production possibility sets, besides being determined by physical or technological constraints, are also determined in part by institutional factors. Familiar explicit reference to institutional factors affecting consumption possibility sets include legal restrictions on work hours, overtime pay beyond the 8 hour work day, limits on overall number of hours worked, child labor, etc. It is less common to see how legal or institutional factors affect production technology.[1]

The ease with which the physical and institutional descriptions are conflated can conceal important insights. Despite recognition that institutions have their role in determining the economic space, mainstream economics does not explicitly address their role and significance. More importantly, while physical constraints can be said to be in a real sense exogenous or beyond control, institutional factors are virtually defined as artifacts of humans' attempt to control physical reality socially. It is

[1] 'The set of feasible production plans is limited first and foremost by technological constraints. However, in any particular model, legal restrictions or prior contractual commitments may also contribute to the determination of the production set' (Mas-Colell et al. 1995, p. 128).

thus particularly disturbing that institutions are not explicitly modeled in economic analysis.

Market institutions, whether formed by evolutionary processes or by conscious design, are artifacts of human interaction seen to assist the coordination of activity. More generally, property rights and entitlements are critical elements of our institutional devices. When describing a production technology we often take for granted that the institutional rules align perfectly with the physical reality, so that institutions almost drop out of our description. It is easy to forget that there are two conceptually separable layers to a production function: a pure description or a blueprint of the physical possibilities of combining inputs in certain ways to generate an output (physical menu) and a description of the institutional rules circumscribing the way that 'institutionally' defined rights can be combined to produce an 'institutionally' defined output.

Consider the input of labor. A physical description might refer to the level of effort of labor of a particular skill required (in combination with other inputs) to produce another good. An institutional description would modify the physical 'possibilities' by the socially determined possibilities: labor input would be measured in time of labor offered and there would be several restrictions on the number of hours worked, conditions of work, etc. A firm may require water from a river to cool down certain production processes. There is a physical description of the possible combinations of water inputs that will lead to the different output levels, and there are institutional rules that circumscribe where a firm can be located along a river, the quantities of water it can extract within specified time periods, etc. What is physically possible and what is institutionally possible are two different things.

The idea of a separate institutional and physical description of the economic space may seem quite obvious, yet the failure to explicitly distinguish the two can obfuscate some important analytical concepts. Indeed, I believe that the concepts and the relationship between externality, jointness and nonconvexity can be substantially clarified with a more explicit understanding of the distinction between the physical and the institutional. The next section describes some of the difficulties in characterizing externality and suggests an unambiguous way of doing so within the context of Arrow-Debreu frictionless models that rely on modelling the physical and institutional economic space separately. Section 3 points to the need to distinguish between the physical and institutional space when we deal with the convexity of production sets; a production function can be convex in one space and nonconvex in the other. Section 4 reinterprets the well-known discussions of Baumol and Bradford (1972) and Starrett (1972) linking detrimental externality to nonconvexity and argues instead that there is no special link between nonconvexity and externality.

2. EXTERNALITY AS INCONGRUENCE BETWEEN PHYSICAL AND INSTITUTIONAL DESCRIPTIONS OF ECONOMIC SPACE

2.1 Many characterizations of externality

There has never been a clear and widely accepted definition of externality.[2] To an extent, the multiple meanings associated with externality depend on the context within which it is used.[3] This paper focuses squarely on conceptual issues that arise within the axiomatic structure of general equilibrium theory and thus abstracts from the many important issues raised by models that attempt to explain endogenously the formation and role of institutions.[4] In this context, a consistent characterization of externality is to view it as synonymous with 'non-market activity'. The Arrow-Debreu framework cannot tell us why markets do not exist. The extent of missing markets is a definitional matter to be determined at the outset by the modeller. The presence of externalities will generally be determined by the way economic units (firms) and markets are defined.[5] In these frictionless models, the failure that arises from externality consists in the shortfall of a hypothetical set of incomplete and costless markets from attaining Pareto optimal outcomes. Comparisons can be made between different configurations of complete and incomplete markets. There are circumstances where models of incomplete markets attain Pareto improvements relative to models with a complete set of markets; for example, incomplete markets may fruitfully obscure certain nonconvexities.

Even within the confines of frictionless Arrow-Debreu general equilibrium models, many economists have viewed externality as being something more than just missing markets. I will argue, however, that the only consistent characterization of externality within the general equilibrium context is to view it as synonymous with 'missing' markets or 'missing' property rights. In this view, all that distinguishes a conventional commodity from an externality is the extent to which a market exists for the 'good' in question. Once property rights have been defined for a good,

[2] 'There is a strong temptation to avoid giving an explicit definition of externality, since even this first step has been a fertile source of controversy, and instead to approach the matter obliquely by putting to work various models in each of which an externality is obviously present' (Cornes and Sandler 1996, p. 39).

[3] In my book, *Externality and Institutions* (1998), I try to clarify the various meanings associated with externality.

[4] This does not mean that I consider these issues unimportant. On the contrary, I believe that a clearer understanding of these concepts in the frictionless framework will go a long way in clarifying conceptual issues in models with transaction costs.

[5] 'The existence, and eventual justification, of these external effects may be understood only after an explanation of the size of economic units and determination of the number of markets is given' (Laffont 1988, p. 7).

and markets are present for the exchange of that good, the externality ceases to be.[6]

The presence of externality is purely a function of how institutions (property rights and markets) are defined. There is nothing intrinsic about the characteristics of 'goods' that bestow on them the property of externality. A loaf of bread will involve externality if no one owns it. One person's decision to eat it will impact another 'directly' without the mediation of exchange through parametric prices. Likewise, the use of air to exhale smoke will not involve externality if the air in question is privately owned and the smoker must purchase the right to emit smoke. Rights to air turn the externality space into conventional commodity space.

2.2 Externality reinterpreted

There is certainly a very rich variety of goods and 'bads' in the real world with characteristics such that it is extremely difficult to imagine that well defined property rights can actually be formed over them – this includes, among others, air space, genetic codes, a sense of security, ideas, and biodiversity. Buchanan and Tullock (1962) vividly pointed this out by contemplating markets for the color of underwear that others wear. The cost of setting up institutions is critical in determining the extent of the market, but in frictionless models institutions are presumed to exist or to be absent.

The literature on externalities in frictionless models nonetheless attests to the many insights that can be gained by comparing (a) models with marketed goods in which the only constraints facing agents are those imposed by budgets and production functions and (b) models that include non-marketed or environmental commodities the quantities of which are exogenous to some agents. Cornes and Sandler (1996, pp. 51–67) provide a good overview of the many ways in which alternative types of externalities or non-marketed commodities can be modeled: general externality, public goods, impure public goods or bads, club goods, etc. The motivation to form models with different types of non-marketed goods usually springs from an interest in capturing features or characteristics of actual goods and their implications for market interaction and market outcomes. But as they point out, one must take care not to impart the fact that they are modeled as non-marketed goods to something intrinsic in their nature. Rather than treating various kinds of non-market activity as 'synonymous with particular goods or services, we prefer to think of

[6] One often sees expressions such as 'externality markets', as if externalities continue to have some special status even when markets have been formed for the activity in question. In the sense discussed here 'externality markets' is an oxymoron. It is *either* markets *or* externality, but not both.

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them as *incentive structures*' (Cornes and Sandler, p. 64, emphasis added).[7] Essentially different ways of modelling market and non-market activity give rise to different incentive structures.

A helpful way to characterize externality in the Arrow-Debreu framework is as a mismatch or lack of isomorphism between a model of the physical economic space and a model of the institutional economic space. The institutional space determines the means of control over the physical environment. It entails the myriad forms of legal and other rules that assign ownership, exchange and other rights. A misalignment between the physical and institutional space implies that some aspects of the physical world may not be adequately controlled through interactions in the institutionally determined economic space.

For simplicity I will present this characterization of externality in production space alone. Envisage underlying netput vectors $(y_1, y_2, \ldots, y_L)^P \in \Re^L$ that describe the net outputs of production at a purely physical level. Superscript P symbolizes that this is a physical description, devoid of any institutional elements. The physical production set is then the set of all feasible physical production plans. If the institutional description of production possibilities aligns perfectly with the physical description, so that we have a complete set of property rights and markets for all relevant physical activities, we can say that there are no externalities. The netput vector in the institutional space would be $(y_1, y_2, \ldots, y_L)^I \in \Re^L$, with superscript I indicating that all inputs and outputs can be expressed in terms of rights over resources or activities.

Externality is said to be present if the physical description is incongruent with the institutional. A simple instance of externality would arise if there were no property rights over valued resources, so that their use might effectively be characterized as open access. In this case the institutional description of production possibilities would be of a lower dimension than the physical description: $(y_1, y_2, \ldots, y_K)^I \in \Re^K$, where $K \leq L$. The actual description of production possibilities should be viewed as a combination of physical and institutional elements so that a netput vector might be $(y_1^I, \ldots, y_k^I; y_{k+1}^P, \ldots, y_L^P) \in \Re^L$ where some inputs and outputs are mediated through the market while others are not. The idea is that the firm has control over k inputs and outputs purchased from and channeled within the market, but also uses, is affected by, or has some form of control

7 The quote continues: 'Which of these structures, if any, characterize the production and consumption of a particular commodity is a matter that may be influenced not only by technological considerations but also by such considerations as the way in which institutions have evolved, the nature of individual preferences, and, indeed, the distribution of those preferences.' This suggests that if we want to understand why we have different kinds of non-market activity, we need to answer the question of how institutions evolved. A very important matter, but as mentioned earlier, it takes us outside the Arrow-Debreu framework and requires modelling of transactions costs and/or information costs.

over, other inputs and outputs outside of the market. This would be one way of viewing 'missing markets'.

The standard way of representing production functions with externalities does not distinguish the physical and institutional descriptions. A typical representation would have $y = f(z_1, \ldots, z_n; e_1, \ldots, e_m)$ where y is the output of a firm, the z_i are marketed inputs used by the firm, and the e_i are non-marketed inputs that are treated as exogenous by the firm though they are decision variables of other firms.[8] In particular, the e_i are generally viewed as being input or output variables of other firms rather than some other form of non-market activity such as open access resources. For instance, apple production may be seen to be directly influenced by the neighbouring production of honey. Honey (a marketed good) is produced jointly with the fertilizing of the orchard (a non-market activity).

The common representation incorporates both physical and institutional aspects, distinguishing marketed from non-marketed, environmental or physical inputs. It would be useful to try to separate these aspects by describing an underlying technology of physical production that is, in a sense, prior to the institutionally determined production function. In this representation there can be no externality since institutions have not been introduced. Such a description will not tell us anything about how these inputs will be controlled; it will only give us the physical relationships or 'blueprint' required for the output to be produced. The typical representation of externalities can then be viewed as arising from the superimposition of an institutional structure that turns some (though not all) of the physical relationships into marketed commodities.

Meade's (1952) orchard-apiary example can be reinterpreted by distinguishing the physical from the institutional descriptions. His original representation took the form:

$$(1) \quad y_a = f(z_{a1}, \ldots, z_{an}; y_h)$$
$$y_h = f(z_{h1}, \ldots, z_{hn}; y_a)$$

where y_a and y_h stand for apple and honey output, z_{ai} and z_{hi} stand for marketed inputs, and y_a and y_h within the production functions stand for non-marketed 'inputs' that are exogenous to the respective firms. The non-marketed inputs represent positive externalities where the apple production provides nectar for the bees and honey production provides fertilization services to the orchard. Subscripts a and h indicate which inputs are 'controlled' by which firm. In the case of marketed inputs, their control derives from market purchases, while non-market 'inputs' are

[8] This notation would not tell us whether there are m firms polluting a single open access resource or several open access resources. It does not provide a separate specification of the physical resources affected by the firms.

controlled by other firms; of necessity, there is ambiguity as to who controls the output of honey and apple. The intuition behind this representation is clear, but it is a valuable exercise to try to consider the possible interpretations of the apple-apiary story in terms of explicitly separate physical and institutional descriptions.

Closer scrutiny of Meade's original representation suggests that it was not necessarily his intent to describe an underlying physical relationship but to find a proxy for some apparent interconnection between the two activities. Indeed a literal reading of the original production functions leaves the impression of a mysterious link between apple output and honey production, i.e., it avoids providing the underlying physical description that might explain precisely how the two processes are connected.

Employing the heuristic description suggested earlier, a purely physical description of production possibilities in the apple-apiary story might take the following netput form:

$$(2) \quad (y_a, y_n, y_p, \ldots, y_L)$$
$$(y_h, y_n, y_p, \ldots, y_K)$$

The first vector shows apple production y_a that is jointly produced with nectar output y_n and requires pollination service y_p as an input. The second vector represents honey production y_h that is jointly produced with pollination service output y_p and requires nectar input y_n. The inputs are not distinguished as marketed or non-marketed since the description is prior to institutions. The physical description sees apple and honey production as a function of a number of inputs and services, where apple production would include a description of fertilization services as an input and the apiary production function would include nectar as a requisite input into its production. This description would include an exhaustive list of inputs so that any potential means of fertilizing apples or providing nectar to bees would be incorporated. As a purely physical description of production possibilities there is no externality.

Externality will arise as soon as we superimpose an institutional structure that does not align with this physical description. The following model might be a likely candidate for externality:

$$(3) \quad \left(y_a^I, y_n^P, y_p^P, \ldots, y_L^I\right)$$
$$\left(y_h^I, y_n^P, y_p^P, \ldots, y_K^I\right)$$

In this case there is externality because property rights and markets have not been formed for nectar and pollination services. If we established conventional markets for these goods then the externality would vanish.

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Bees may turn out to be one of many possible ways of fertilizing apple trees. If the price is right, and it turns out that bee fertilizing services can be jointly produced with honey, then we would expect the apple producer to purchase the service through the market. It may be that alternative fertilizing services come more cheaply (and may or may not be jointly produced with other products). The fact that apples are jointly produced with nectar and that there are potentially two outputs to be sold, while bees are jointly produced with 'fertilizing services', would mean that the more appropriate notational format would involve a netput vector with more than one output. Jointness in outputs does not imply externality.

It is also important to distinguish the two kinds of jointness present in the original Meade representation and in the alternative representation. In the original form there appears to be a fundamental jointness between the outputs of both goods (honey and apples), while in the revised version the jointness appears between two outputs in the separate production functions (honey and pollination, apples and nectar), i.e., not between production functions but within. There are implications also for the nature of property rights envisaged if one wants to set up markets for the relevant activities. In the first case (Meade representation) one would need to have Lindahl markets so that the apple firm could pay for the observation in honey production, while in the second case conventional markets for nectar and pollination services would be required. Clear and separate descriptions of the physical and institutional space helps one distinguish between jointness and externality, and allows a better appraisal of institutional devices in dealing with the underlying physical model.[9]

3. PRODUCTION FUNCTIONS IN PHYSICAL AND INSTITUTIONAL SPACE

By distinguishing between a physical and institutional description of technology, it becomes apparent that the form of a production function (or utility function) will depend on whether we are considering the space of institutions or the underlying physical space. While the physical description must be unique there can be all kinds of institutional structures associated with an underlying physical description. A production function

[9] For the record, apple blossom is not a source of nectar for the production of honey, and fruit pollination often decreases honey production because the bees tend to consume some of their stored honey while pollinating (Cheung 1973, p. 15). Furthermore, there is evidence that nectar and pollination services are controlled effectively through negotiations, i.e., it could almost be said that there are markets for these inputs. See Cheung (1973), Johnson (1973), and Gould (1973) for empirical and theoretical discussions that confound conventional wisdom about this well-known example. The evolution of institutions to deal with complex allocation problems easily surpasses our imagination. Some nice examples can be found in Ostrom (1990) and Eggertsson (1990).

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in physical space may involve increasing returns to scale, while its institutional counterpart may be convex depending on how property rights and markets have been defined over inputs. Take the following example of a beneficial externality: $y = z^a e^b$ where z is a marketed input (raised to the power of a) while e is a beneficial externality (raised to the power of b) which is determined by other firms. Note that in the space of marketed inputs, the production technology is convex as long as $a < 1$. A physical description would involve all inputs, including e^b and the convexity would depend on $a + b$. If $a + b > 1$, then the underlying physical technology would involve increasing returns to scale while the institutionally determined space would be convex in marketed commodities.

While the physical description must be unique, the institutionally determined production function will depend on the particular institutional structure. In the apiary-orchard example the traditional representation may confront both producers with convex technologies in the inputs under their control even when the underlying physical production function involves increasing returns to inputs. In fact, if all inputs were marketed, it could turn out that, due to increasing returns in inputs under the control of firms, it would no longer be profitable to produce privately.

We could also have a situation where the physical space of inputs is convex but the institutionally defined space appears to be nonconvex. Imagine that a firm producing y requires air r to deposit wastes. Its physical production function (abstracting from institutional rules) is defined by the relationship $y = z^a r^b$. With $a + b < 1$, we have a convex technology. The firm does not perceive the dumping of wastes into the atmosphere as the use of an input, i.e., it treats it as a free good. It could be that the firm effectively uses air in such a way that the perceived technology is nonconvex. In particular, envisage a relationship between use of the marketed good and the non-marketed good described by the equation $r = z^c$, so that the perceived relationship in the institutionally defined space is $y = z^{a+bc}$. If $a + bc > 1$, the technology in the space of institutionally defined inputs appears nonconvex. The marketed input is capturing the impacts of another input that is not 'controlled' by institutions.

4. NO SPECIAL LINK BETWEEN NONCONVEXITY AND DETRIMENTAL EXTERNALITY

The distinction between physical and institutional space will be used in this section to show that detrimental externalities are not a cause of nonconvexity. Nonconvexities and externalities have always been seen as enduring foes of the decentralized private-ownership economy. Arrow's thought experiment of setting up artificial commodities for externality activity seemed to provide a way to reconcile the presence of externalities

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with the two Fundamental Welfare Theorems. Commodities could be defined as an agent's observations of every other agent's consumption and production choices. As long as markets could be defined for all activity, failure remained the sole prerogative of nonconvexity.

The work of Starrett (1972) and Baumol and Bradford (1972) came to dispel the notion that the problem of externalities could be overcome by the ingenious redefinition of the institutional space. In different ways, both showed that detrimental externalities had a special relationship to nonconvexities. Starrett argued that with Arrovian markets for detrimental externalities, nonconvexities would be unavoidable; that they are in a sense 'fundamentally' linked to nonconvexities. Baumol and Bradford, on the other hand, argued that increasing intensity of detrimental externality will eventually make the production possibility frontier nonconvex. I will argue, instead, that there is no special link between nonconvexities and detrimental externality and that if the production space is nonconvex in the presence of detrimental externalities, it is largely for the same reasons that the production space would be nonconvex for conventional goods. This insight is made possible by the delineation between the institutional and the physical parameters proposed in this paper.

4.1 Fundamental nonconvexities and bounds

Starrett (1972) has argued that the formation of Arrow markets in detrimental externalities unavoidably leads to nonconvexities in the extended production space. Arrow-type commodities induce a kind of artificial independence of production sets. Consider an economy with M firms and N marketed goods. The firms' ordinary or *institutionally* defined production sets will be subsets of the $\Re^N$ space. Net outputs are now distinguished not only by which firms produce them, but also by which firms are affected by them. Accordingly, y_{ijk} stands for the net output of commodity k by the firm j as observed by firm i. For any amount of commodity k that a firm produces (or uses) it generates M joint commodities which are all other firms' observations of its net output, so that $y_{ijk} = y_k$ for all $i = 1, \ldots, M$. All firms' production sets can be seen as contained in the NM dimensional net output set with commodities distinguished according to the firm that produced them, or they may be contained in the extended NM^2 dimensional net output set distinguished according to which firm is being affected as well. If firm j's production of steel k is associated with smoke that harms firm i's laundry production, firm j will be jointly producing two commodities y_{jjk} and y_{ijk}.

The well-known example discussed by Starrett (1972) is that of a laundry firm suffering from smoke coming from a nearby steel plant. A standard representation has the laundry production depend on the

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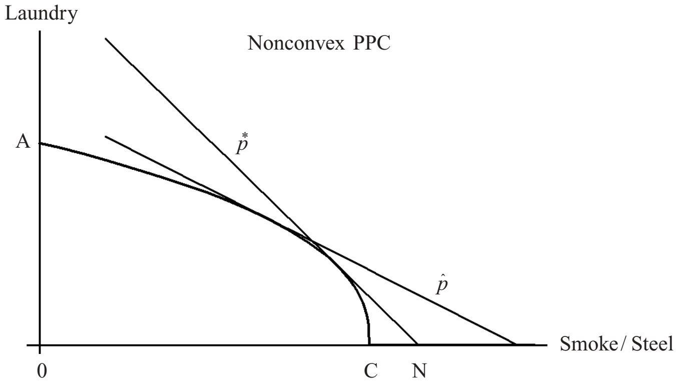

FIGURE 1. Fundamental nonconvexity with Arrow markets.

production of steel standing as a proxy for smoke:

(4) $y_s = h(l_s)$

$$y_L = f(l_L; y_s) \quad \text{or} \quad f(l_L; l_s)$$

The laundry firm's production will be a decreasing function of smoke (or steel output); as smoke increases laundry output for given factor inputs must either fall to zero (see Figure 1) or level off at some point. Either way, the production function appears to be nonconvex with respect to the detrimental activity. If the laundry firm were allowed to sell pollution permits to the steel firm, no equilibrium could arise. At a positive price for the pollution permits, it would always be in the interest of the laundry firm to sell an infinite amount of permits while the steel firm would want a finite quantity. At a zero price the laundry firm would not sell any permits while the steel firm would want to purchase a positive quantity. In Figure 1, levels of smoke beyond point C cannot further reduce laundry production but appear to be part of the production frontier, thus making the production function nonconvex. The production function thus appears to be fundamentally nonconvex in the observation 'smoke emitted by the steel firm'.

A number of authors [10] have pointed out that this problem seems to rest largely on the fact that there are no bounds on the number of permits to be sold. If one were to set bounds on the number of permits, even arbitrarily, it could be that the Arrovian market for detrimental externalities would equilibrate at least for some price ranges. For instance, if the bound were

[10] Baumol and Oates (1988), Boyd and Conley (1997), Cornes and Sandler (1996), Cornwall (1984), Papandreou (1998).

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set at N in Figure 1, the market would come to an equilibrium for permit prices lower than $\overset{*}{p}$ such as $\hat{p}$. It is tempting to set the bound at C, thus convexifying the production space and ensuring an interior solution, but this could arbitrarily prevent an optimal outcome where the laundry firm shuts down.[11]

Boyd and Conley (1997) recently argued that the 'fundamental' nature of nonconvexity stems not from an inherent feature of detrimental externality but from the unboundedness of Arrovian markets *per se*. Unboundedness in any input would unavoidably lead to nonconvexity:

> 'Suppose, for example, that one agent is endowed with an infinite quantity of any input, but that there are no externalities in the economy whatsoever. Suppose also that the first unit of the input has positive value to both the agent with the endowment, and at least one other agent. Then again at any positive price, supply is infinite, and at price zero, supply is zero and there is excess demand. Thus, there do not exist supporting prices. In other words, it is really the unboundedness of the endowment, and not the presence of externalities per se, which drives Starrett's market failure' (Boyd and Conley, p. 394).

Nonetheless, they argue, Starrett's conclusion that Arrow markets for detrimental externality lead to fundamental nonconvexity is strictly correct because the structure of Arrovian markets does not foresee bounds on externality markets. 'It is very hard to see . . . how . . . a bound could be introduced. It is not very appealing to claim that there is a limit on the capacity of firm i to observe the production by firm j of commodity k. In any event, it would be completely counter to the well-established tradition in general equilibrium theory to incorporate such bounds in the production sets themselves. We usually define production sets such that it is possible to contemplate arbitrarily high levels of input. The fact that there exist only a finite amount of possibilities to pollute in the world does not enter into their definition. Such constraints are more appropriately treated through the endowments of agents. The Arrow model does not seem to have sufficient flexibility to allow for this' (p. 396).

Boyd and Conley also dispel the notion that the presence of externalities may imply fundamental unboundedness: 'The land has an inherent limit on the amount of waste that can be stored. It is important to emphasize that *this limit is not imposed by human agency*. A stream can only hold so much effluent. An airport can generate noise pollution at most 24 hours in a given day. Even the atmosphere is finite. The most ambitious factory cannot pollute an infinite quantity of air, because only a finite amount exists to be polluted' (p. 394, emphasis added).

[11] Cornwall (1989) has suggested setting benchmarks so that total profits are maximized. This would require detailed (or global) information on the firms' production functions.

While it is true that Starrett's 'fundamental' nonconvexity has to do with the structure of Arrovian markets, the point is more general. The 'fundamental' nonconvexity is a result of the particular institutional structure used in the model rather than an underlying feature of physical reality or detrimental interaction, however institutionally modeled. Whether bounds are compatible with Arrovian markets or not, it is this aspect of the institutional structure that is causing the 'fundamental' nonconvexity, and lack of bounds would generate nonconvexity for any activity (detrimental or not).

The discussion raises two interrelated issues. First is the question of institutionally determined bounds. We know that problems arise when property rights do not align well with physical reality, as when property rights are not defined for scarce resources. What if property rights have been defined for resources but the total number of rights do not align well with the underlying physical bounds? In the same way that it can be said that property rights have not been adequately or completely defined, one can say that the institutionally determined total quantity of rights have not been adequately defined. Second, is there any compelling reason why one should consider Arrovian markets as a natural candidate for the resolution/removal of externalities? Though bounds may seem to fit uncomfortably with an Arrovian market structure, this problem may not arise under alternative institutional descriptions of the economic space. The next section deals with this issue.

4.2 Resource markets versus Arrow markets

By construction, the Arrovian artificial market setup implicitly assumes that non-market activity (externality) is jointly produced with market activity. For instance, smoke is a spillover that is jointly produced with steel. The Arrovian institutional device would be a consistent means of commodifying externalities if they indeed displayed this intrinsic jointness. However, as argued earlier, externalities and jointness are separate attributes and there is no inherent reason why non-market activity should necessarily be viewed as a by-product of some marketed activity.

If a steel firm is emitting smoke into the air, it is more likely that there are several ways for it to abate the pollution than by reducing steel output. Arrovian markets force a fixed relationship between the steel output and the effluent that may not bear a relationship to the actual *physical* production possibilities. If the steel firm has several means of abating or substituting away from smoke, a market for the observation of steel production will not create the appropriate incentives for selection of the most efficient pollution abatement. Likewise, if the steel firm has several alternative means of disposing of its effluents such as reducing atmospheric deposits by increasing water effluents, the markets for steel

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observations would not capture this. In this respect, if Arrow markets do not align with the underlying physical production possibilities, they will not reconcile the extended market model with either of the Two Welfare Theorems. In fact, Arrow markets will not have eliminated the externality, as the property rights envisaged will not correspond fully with the underlying physical production functions. This problem could be overcome, however, by specifying observation markets for pollution *per se* rather than steel production.

In the case of smoke and laundry, rather than presuming some mysterious link between steel output and laundry, we can envisage a 'conventional' market for the resource 'air' which should have figured in the underlying physical production possibility sets of laundry and steel production. This conventional market captures nicely the substitution possibilities in the output of both goods; indeed it even allows for potential 'defensive' activity on the part of the laundry firm.[12]

4.3 Private bads

While it is plausible to argue that there may not be any intrinsic jointness between steel production and smoke, it is more difficult to maintain that there is no jointness in the 'consumption' of smoke or the use of the input clean air by firms. 'Air' is usually envisaged as a public good. Though Starrett's argument is meant to focus on the detrimental nature of externality, whether the externality is private or public, the specific example he uses inclines one to think in terms of a public bad. I will first treat the example as if the detrimental externality were a private bad (or as if 'air' were a private good) that affects only the laundry firm in order to show that there is no special link between detrimentalness and nonconvexity.[13] Afterwards, I will show how the argument can be extended to account for public bads.

Setting benchmarks or limits on detrimental activity has a more conventional counterpart when treated as missing resource markets. By reinterpreting detrimental externality as the use of some previously missing input, bounds can be envisaged more easily. Instead of the

[12] A more realistic description may involve a full listing of potential residual by-products resulting from steel production and waste deposit inputs reflecting the necessity to deposit residuals somewhere in the environment.

[13] Starrett's example involves smoke from steel affecting the laundry production that could be modelled either as a public or private 'bad'. The fact that the model only involves two agents effectively means that one could just as well assume that smoke is a privately directed externality. It might be helpful to assume that the smoke from the steel firm has strictly local impacts. Alternatively, one might prefer considering another example with a more explicitly private directed externality, e.g., a firm dumping sludge onto open access land that is also being used by a farmer. The argument that detrimental externality is associated with fundamental nonconvexity does not hinge on the publicness of externality.

production of a 'bad' we have the use of a 'good'. Pollution 'output' and the corresponding pollution 'observations' are redefined as air 'inputs' or 'resources' and thus bounds are placed in the endowment set.

Assume that the underlying physical production possibilities for steel incorporate the use of air for waste disposal (this captures the essential difference with Starrett's representation) and that laundry production requires inputs of clean air. The physical production functions are:

(5) $$y_s = h(l, a)$$
$$y_L = f(l, a)$$

where l represents labor and a represents air.

The lack of subscripts is meant to highlight the fact that the means of institutional control over resources have not been established yet. It appears more natural to view the cleaning of laundry as requiring clean atmosphere than viewing laundry as dependent on steel production. In essence, we are recognizing that there are physical attributes of air that affect laundry production and that if these are altered by some means the laundry production will be affected. This representation leaves open all the potential means of maintaining clean air, e.g., locational separation, pollution abatement on the part of the steel firm, defensive action on the part of the laundry firm, etc.

If markets exist for all physically possible activity then we appear to have a totally conventional production model. We can incorporate subscripts to indicate that each firm can control the inputs in question through market exchange. With a full set of markets we have a conventional model for the two institutionally defined production functions:

(6) $$y_s = h(l_s, a_s)$$
$$y_L = f(l_L, a_L)$$

(7) where $l_s + l_L \leq \bar{l}$ and $a_L + a_s \leq \bar{a}$

Externality can be introduced into our model by defining an incomplete set of markets. In this case, air would be the non-marketed activity. This representation treats the problem of externality as a missing input (air, or air of a certain quality) that is not explicitly modeled, rather than viewing one firm's input as a function of another's output. Incomplete markets can be represented with subscripts indicating that control over the use of air is now determined by the steel firm (air is now a non-marketed good):

(8) $$y_s = h(l_s, a_s)$$
$$y_L = f(l_L, a_L) = f(l_L, \bar{a} - a_s)$$

where $l_s + l_L \leq \bar{l}$ and $a_s \leq \bar{a}$

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where $\bar{a}$ reflects the total amount of air available where the firms are located.

The latter representation of the laundry production function is more informative in that it suggests that the steel firm's use of air as a waste deposit is diminishing available air for laundry production.[14] While appearing identical to the Meade form where one firm's input choice is an input into another firm's production function, there is a difference in that now we have acknowledged the presence of a new input (the total number of commodities in our model has increased). In this view, the question of bounds comes up naturally as a question of underlying endowments in 'air'. The introduction of bounds on total quantity of clean 'air' is analogous to setting bounds or benchmarks to pollution permits. Detrimental pollution is simply redefined as reduction in clean air. In the Arrovian extended market model, setting benchmarks seems arbitrary, largely because external activity is modeled as an output being jointly produced with a good. Why should there be limits on how much to produce? However, when viewing the externality as an 'open access resource' that is being used up as an input into a production process, it seems natural that bounds should be present. Once property rights have been formed for the open access resource, the bounds show up exclusively in the endowment space.[15]

In order to appropriately deal with externality it is important to have a clear understanding of the underlying physical production relations. Detrimental externality often involves resources for which property rights have not been formed. Viewing Starrett's model from this perspective makes explicit the restrictions in the Arrovian market setup, i.e., of a fixed relationship between the output of the steel firm and the externality. A more realistic model of externality will recognize that air may provide services that are inputs in steel and laundry production, with numerous substitution possibilities. Detrimental externality is shown not to be a cause of nonconvexity. If there is nonconvexity in the physical production functions, it will be there whether we allow markets for all inputs or leave some activity as non-marketed. The problem of defining benchmarks in

[14] Viewing externality as a missing input provides greater degrees of freedom in terms of the possible forms of production functions but can replicate any functional form of a lower dimension. By appropriate assumptions over the functional form, and by incorporating the constraint in steel production that air and labor must be used in fixed proportions, the representation with air becomes effectively identical with Starrett's representation. Starrett's model is seen as a limiting case of a broader set of missing input models: $y_s = \min\{h(l_s), h(a_s)\}$ which appears as $h(l_s)$ to the firm that treats the air as an open access resource.

[15] When there is externality the bounds show up in the production function only because they are describing the particular form of the physical interaction (depletion of a previously open access resource that is scarce).

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Arrovian markets is overcome by forming conventional markets where bounds show up in the endowment space.

4.4 Public bads

Treating 'air' as a private good simplifies our analysis and suggests how one could deal with cases of private directed externalities, like dumping garbage on a previously unowned plot of land, but it does not clarify how a resource based approach could address the public nature of many bads. I will argue here that, far from being unrealistic, viewing many detrimental externalities as involving the use of some 'unowned' resources that provide public services is likely to lead to a more accurate physical description and will enhance our understanding of the allocational problems involved.

The distinguishing feature of environmental economics is not the detrimental aspect of external interactions, which is in large part a definitional issue, but the nature and complexity of natural resources and environmental media. The pollution that emanates from the steel factory enters into a complex ecosystem that may transform the pollutant (as well as be transformed by the pollutant) before it is eventually observed by the laundry firm. To treat air as simply vacuous space linking a detrimental act with its observation by another agent misses the critical fact that air itself is a physical resource that needs to be managed.

> 'Ecosystems resemble natural multiproduct factories. Powered by the sun, they produce a variety of goods and services of use to humans. The goods and services, or "functions", they provide vary according to the type of management, or lack thereof, they receive. For instance, a forest may produce plant and animal biomass, soil retention, nutrient uptake, groundwater recharge, and many other useful functions. Increasing some functions through management may imply declining or increasing levels of other functions over time. For instance, increased timber harvesting may imply less soil retention and changes in the species mix of animals and their populations' (Gottfried 1996, p. 133).[16]

Similarly, increasing the services of an ecosystem for absorbing, transforming, or retaining wastes is likely to diminish other ecosystem services.

Redefining detrimental externalities so that they involve depletion of some publicly used resource or service is helpful to my argument.[17] Does a bad always involve the reduction of some good? It suggests that a bad

[16] The term multifunctionality has been used to describe the many products or services provided jointly in resource management contexts (see OECD, 2001).

[17] Interestingly, Starrett (1973), in a discussion about whether detrimental externalities like smoking prevent the existence of a core, points to the importance of recognizing that smoking may involve the use of the resource atmosphere. Property rights over this resource will be critical in determining whether a core exists or not.

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must either be the violation of someone's rights over a resource, or the changing of the use of some resource or service that was 'unowned' but utilized. Bromley (1996) asks 'what is the difference between the provision of environmental benefits and preventing damage? ... [T]he burning of crop stubble may allow a farmer to control pests, but may cause damages to those harmed by the smoke. So the same physical condition – or act – can be beneficial or damaging depending upon whom we ask ... If farmers provide a more desirable landscape they are seen as providing a benefit, while if they provide a less desirable landscape they are harming others' (p. 1–3). Bromley emphasizes the Coasean nature of externality that reinterprets 'pollution', 'bads', or 'damage', as simply alternative and conflicting claims to resources. There isn't a good and bad use or a 'damaging' activity but simply conflicting uses. In the case of our laundry example there isn't a steel firm damaging a laundry firm but two conflicting claims to the use of air.[18]

In light of this discussion, what is an appropriate model of the steel versus laundry story? A resource based approach would draw parallels between the joint outputs resulting from land use in agriculture and forestry and the joint outputs and services that derive from alternative uses of air. Instead of seeing the steel firm as generating an impure public bad, we would view it as using air for waste deposit that rivals the public use of air for laundry and breathing. Air can be used for breathing, dumping, viewing, protecting biodiversity, changing climate, humidifying, etc.

A simple model with air treated as a fixed resource that can be allocated to different uses could take the following form:

Production functions:

$$y_s(l_s, a_s)$$
$$y_L(l_L, A)$$
$$y_i(l_i, a_i, A)$$

[18] In the case of services that are consumed jointly it can often be the case that what is deemed by some as a public good is viewed by others as a public bad. Two people may have opposing views of what is a beautiful landscape. Would this mean that a farmer's decision to plant a different crop and thus alter the landscape involve the joint production of a public bad and good? For a landscape alteration to be viewed as a bad it would mean that the affected parties had some claim to the prior state. But if some claim exists in the form of an exchangeable right then the farmer would need to buy the right to alter the landscape. Rather than viewing the changed landscape as a bad, it would be better to view the different kinds of landscape as competing uses of some fixed land resource. In the ideal exchange economy the specific landscape that maximized returns would be selected. The parties that lose out in the market exchange are not damaged by the 'unattractive' landscape because it wasn't theirs in the first place.

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Endowments:

$$\sum l_i = \bar{l}$$

$$\sum a_i + A = \bar{a}$$

Steel production y_s is a function of labor input and availability of air for waste disposal a_s.[19] Laundry production is a function of labor l_L and good quality air A. 'Good quality air' is capitalized to indicate that it is a public good that enters into the production functions of firms requiring clean air (as well as utility functions of individuals). More generally, we have y_i production functions that may require labor and dumping services of air as well as clean air. All resources would have their corresponding endowments owned by individuals.[20] The second endowment constraint is more sophisticated than your standard private resource constraint. It reflects the physical description of a given air space that can be jointly allocated to different uses (in this case different combinations of polluting activity and clean air).[21] For a more conventional example of this type of endowment constraint, consider a fixed area of land being used for landscape amenities (public good) and farm produce (private good).[22]

As long as markets are defined for the resources/goods in this model there is no detrimental externality. What would appear as a public bad in the absence of markets would be the use of air for depositing waste by the steel firm that would infringe on other public uses of the air shed. With standard markets for private goods (air for waste deposit) and Lindahl markets for the public (joint consumption) goods (air for breathing and laundry production), this is a conventional model. Individuals and the laundry firms would bid for the public provision of breathable air while the steel firms would bid for the private use of air for waste dumping. The fixed air shed could be allocated to the two services in different combinations.

[19] I have used lower case for waste disposal services since I am assuming that it is a rivalrous service, in that one firm's ability to dispose diminishes other firms' ability.

[20] Ownership of resources could be private or public. See Milleron's (1972) discussion of public good models that allow for the possibility of public (common) ownership of resources.

[21] Though this simple endowment constraint implies an additive relationship between waste disposal and clean air, this need not be so. One could treat air as a resource that jointly produces several services, e.g., disposal of different kinds of wastes, health, and amenity services. The endowment constraint could be reinterpreted as a netput vector that has one input – air – and several outputs – disposal activity, cleanliness, etc. The number of rights to pollute or enjoy clean atmosphere would be grounded in the physical possibilities. See OECD (2001) for different ways of modeling the multiple services of natural resources.

[22] Burton (1996) models a case of public lands being distributed between the forest industry that has preferences for intensive and nonintensive uses of land and environmentalists that have preferences for wilderness and nonintenisive forestry.

Having redefined the public bad as the private use of a resource rivaling a public use, any remaining nonconvexity is disassociated from the detrimental nature of interaction. Furthermore, by setting endowments in the space of resources rather than outputs (or observations), bounds acquire their usual meaning of scarcity in resources. This does not preclude, however, the possibility that nonconvexity may somehow be associated with publicness *per se*. This would be a rather different thesis but may warrant some consideration.

'[T]he "publicness" of a good refers to its physical nature, and indicates the potential for collective consumption' (Laffont 1988, p. 33) or use. This physical potential does not constitute an externality or a public good, unless we superimpose an institutional setting such that consumption or use in the institutional space is indeed joint (or nonrival) and there is no exclusion. The physical attributes of goods or services with the potential for collective consumption may involve indivisibility or nonconvexity in production. There may be setup costs to build a lighthouse with the capacity to protect vessels moving at a given speed. Ecosystems are inherently indivisible and their protection is likely to involve nonconvexity. For instance, restoring the 'health' of a river may require substantial resources up to a threshold, after which the ecosystem sustains itself. But this does not mean that publicness is inherently associated with nonconvexity. The broadcasting of a radio station may involve publicness in that there is a potential for joint consumption, but it need not involve nonconvexity of the production function. Though there are physical attributes that lend themselves to collective consumption or use, it is ultimately the way institutions are defined that will determine whether a good is public and whether there is externality and nonconvexity.[23]

4.5 Intensity of detrimental interaction and convexity

Baumol and Bradford (1972) have argued that if a detrimental externality is 'intense' enough, eventually the production possibility curve will become

[23] That public goods are a function of how institutions are defined is nicely expressed by Laffont (1988, p. 34): 'Deeper reflection indicates that the distinction between private and public goods according to the notion of exhaustibility by a single individual is superficial. Indeed, when goods are defined in an appropriate manner individual consumption always leads to exhaustion by private use. Instead of talking about national nuclear defense, we could consider the simultaneous protection of millions of units of space. If person i occupies space l, he consumes the good "protection of space l" and prevents person j from benefiting from this good. Then technical and informational difficulties with exclusion will lead to the appropriate definition of such protection as an economic good. Since it is impossible (or at least too costly) to exclude only one particular space l from protection, the good will be called national nuclear defense and we can say that it is not exhausted in private use.'

nonconvex. I will argue that this is not due to some special feature of detrimental externality but has to do in part with an interpretation of 'intensity' of input use that would render any production possibility curve nonconvex. They consider a two-output, one-input economy in which each output is produced by a single industry, and in which each industry has a convex technology in its own inputs. There is also detrimental externality whereby one firm's increases in output lead to increases in the other firm's costs or increases in inputs required to produce given levels of output. The two firms have convex production sets in the inputs under their control.

If two firms' technologies are linear in their own inputs, the slightest presence of detrimental externality will make the production possibility set nonconvex. Without 'interference' of the two production activities, the production possibility set would be linear. The slightest negative interference can only lead to a reduction in total possible output combinations, leaving only the two extreme points unaffected where one firm does not produce, thus pulling inward the production possibility set. For the more general case, an algebraic example is used where each firm has a decreasing returns to scale technology in the inputs it controls. Let y_s and y_L be the output of steel and laundry, l_s and l_L the amounts of labor (negative leisure) used, and w a measure of intensity of the detrimental interaction between the two firms. The labor requirements as well as the implicit equation for the laundry-steel possibility frontier are:

$$(9)\quad l_s = \frac{y_s^2}{2}$$

$$l_L = \frac{y_L^2}{2} + w y_s y_L$$

$$\bar{l} = l_L + l_s = \frac{y_s^2}{2} + w y_s y_L + \frac{y_L^2}{2}$$

Each industry separately is subject to strictly diminishing returns to scale in terms of the inputs it 'controls'. The production possibility frontier is convex for values of w less than or equal to one, whereas for higher values it becomes nonconvex. Figure 2 shows the production possibility curve for different values of w.

If we present the production functions in a more conventional way with output as a function of inputs, rather than input requirements as a function of outputs, we would have:

$$(10)\quad y_L = \sqrt{w^2 y_s^2 + 2 l_L} - w y_s$$

$$y_s = \sqrt{2 l_s}$$

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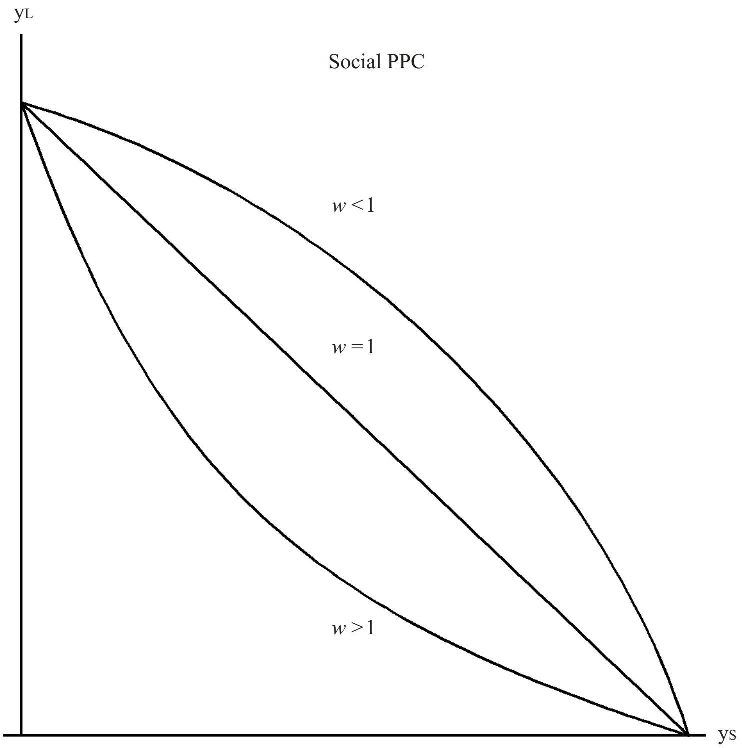

FIGURE 2. Production possibility curve and intense detrimental externality.

This latter representation suggests that the functional form of the detrimental externality is far from intuitive. It is also less clear what the precise meaning of 'intensity' of detrimental interaction is. The easiest way to see what is happening here is to reformulate the problem as a missing input model in the same way we dealt with Starrett's fundamental nonconvexity. We can do so while preserving the identical production structure. Observation of the production functions shows that as output of steel/pollution increases, for a given level of labor input, laundry output will diminish asymptotically to zero. In order to preserve this particular form (whether it is realistic or not), we can suggest the following missing input story. The steel and laundry firm both require a certain 'plot' of air (or space). 'Air' can be treated as a fixed allocable resource that can be allocated between 'residual dumping' (or pollution) and 'cleanliness'. Increases in the percentage of the fixed resource going to dumping reduces the percentage available for cleanliness. One could either view this in quantitative terms as percentage of air filled with particulates (and the corresponding percentage of air free of particulates), or qualitative terms with air of differing qualities having an effect on the production of the two firms. Furthermore, a natural interpretation would be that the production functions of the two firms are essentially short run production functions in that they have already determined their location and accordingly the specific 'plot' of air they are using is fixed, i.e., there is no input in Baumol and Bradford's production function that allows for alternative locations of the firms with the corresponding spatial separation.[24] In this case, the increased pollution on the part of the steel firm corresponds to more of the fixed air resource being allocated to steel production. We can illustrate this interpretation with the following transformations:

$$x_L = 1 - \frac{1}{a_L + 1}$$

$$x_s = 1 - \frac{1}{a_s + 1}$$

$$x_L + x_s = 1$$

$$a_s = \frac{1}{a_L}$$

where x_L, x_s are the percentages of air being allocated to laundry and steel and a_L, a_s are the respective indices of air use by the two activities. Higher levels of steel production require a higher percentage of fixed air allocated to dumping/pollution and higher levels of laundry production require higher levels of fixed air allocated to laundry production (clean air). What we have here is a fixed allocable resource (air) that can jointly produce outputs a_L, a_s, and these are inputs in the production of laundry and steel.[25] The underlying physical production functions can

[24] Though I have described the production functions as short run by assuming that the 'plot' of air is fixed, one could easily construct a model where not only the quality of air (allocation of air) is variable, but the quantity/location of air plots of varying quality are variable. Both firms could determine where to locate according to the varying prices of air of different qualities. If one still wanted to avoid assuming a specific total quantity of air, one could simply assume that larger quantities of steel production involve larger quantities of air of worse quality. As steel production increased, it would require larger spaces of air with higher levels of pollution throughout. The laundry firm would still find the quality of air where it is situated deteriorating.

[25] That percentages of air being used add up to one unit of air is purely arbitrary. By multiplying both sides of the constraint by x, where x represents quantity of air, the model could accommodate any number of units of air being used in different proportions (qualities) by the two activities. More realism could be added to the model by defining air space in different locations as different fixed allocable resources. Firms' locational choice would then incorporate air space quality.

be represented as:

$$(11)\qquad y_L = \sqrt{\left(\frac{w}{a_L}\right)^2 + 2l_L} - \frac{w}{a_L}$$

$$y_s = \min\left\{\sqrt{2l_s}, a_s\right\}$$

$$\text{with } x_L + x_s = 1 \quad \text{and} \quad l_L + l_s = \bar{l}$$

This tells us that laundry production is a function of its use of labor and an index of the availability of clean air, while steel requires labor and an index of air for depositing pollution (where the index is proportional to the production of steel). Note that there is no externality in this representation. This way we imitate the precise production function of Baumol and Bradford without the input air. If we now want to superimpose an institutional description with a missing market in the use of air we could represent the nature of control that replicates Baumol and Bradford's detrimental externality by replacing a_L by $\frac{1}{a_s}$ in the laundry firm's production function. This is one way of showing that the steel firm determines the control of air use/quality. This reinterpretation allows us to see that the nonconvexity is already present in the underlying physical production function, even though the individual institutional production functions are convex in the institutionally determined control variables.

It is not the 'detrimental' nature of externality that leads to nonconvexity in the social production function but rather the underlying nonconvexity inherent in the laundry's production function that is 'hidden' by the institutional structure. The underlying physical production function for laundry is an increasing function in air-use, independent of the intensity of air use w. It is the lack of a market that turns the conventional two-input model into a detrimental externality and masks the nonconvexity present in laundry production. The nonconvexity in the social production possibility set will arise for certain values of w whether we view the model as one of detrimental externality or a standard two-input model with jointness in use of labor and air in steel production. As long as the social production set relates to the underlying physical possibilities, its shape will be determined by the underlying physical production functions. Whether and how we introduce externality will not in any way affect the shape of the social production possibility set. Indeed, in this case, externality is totally irrelevant in determining its shape. Even in the case of 'linear' production functions, it is not detrimental externality that gives rise to nonconvexity but the underlying structure of the production function.[26]

[26] Here is a simple linear system (like the one envisaged by Baumol and Bradford, 1972) where even the slightest interference between steel and laundry production would lead to a nonconvex social production set: $y_s = l_s$, $y_L = l_L/(1 + wy_s)$. Here is an underlying

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As we saw in the model above, the 'intensity of interference', w, did eventually turn the social production function from convex to nonconvex. However it is not the result of 'interference' (which is associated with the institutional setup – externality), but 'intensity' (and the way it is interpreted) that is giving rise to nonconvexity. Indeed, if we look at the same model in its conventional garb and imagine that air is a commodity to be bought and sold by both steel and laundry firm, then it is the 'intensity of air use' that eventually leads to nonconvexity. On this interpretation of intensity, the use of any input in an individual production function will, at a sufficient intensity level, render the social production set nonconvex. The production possibility set associated with a conventional Cobb-Douglas production function model for laundry and steel, $y_L = l_L^a k_L^{wb}$ and $y_s = l_s^c k_s^d$, where w captures the 'intensity of use', will eventually become nonconvex for any fixed positive values of a, b, c, d.[27]

If 'intensity' of input use is strong enough, one firm's production of even a small amount of output may prevent the possibility of production by the other firm. In other words, the specific input is so 'essential' that the firm in question either cannot produce without a significant amount of the input in question or can sustain only small levels of output. This would be the case when production involves significant setup costs in the specific input or, alternatively, there were substantial increasing returns to scale in the specific input. It is not a result of interference as suggested by Baumol and Bradford (1972).

5. CONCLUSION: ON PHYSICAL COMPLEXITY AND THE SOCIAL DEFINITION OF SCARCITY

The distinction between physical and institutional descriptions of economic activity has not received adequate attention within the mainstream paradigm. By modelling the economic space as a combination of physical and institutional descriptions, a number of theoretical insights can be gained and conceptual problems overcome. Within the context of a frictionless Arrow-Debreu setting, externality can be unambiguously defined as incongruence between physical and institutional models of economic space. Properties of production sets are also clarified by a crisper distinction between the physical and institutional. Production functions can be convex in the institutional space and nonconvex in the physical space and vice versa. The presumed special link between detrimental

physical production function with the addition of a missing resource and its externality counterpart that has an identical shape: $y_s = \min\{l_s, a_s\}$ and $y_L = l_L/(1 + w(\bar{a} - a_L))$.

27 That intensity of input use causes nonconvexity depends on the interpretation given to intensity. It could be that the intensity of input use for a Cobb-Douglas production function is captured by w in $y_L = l_L^{(1-w)} k_L^w$, for values of $w \in (0, 1)$, so that the production function remains convex however strong the intensity of input use is.

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externality and nonconvexity dissolves when it becomes apparent that externality cannot bring about nonconvexity in the physical production space. By reinterpreting the detrimental externality as a missing property right over a previously open access resource, conventional property rights can be formed with bounds set in the endowment space.

Different underlying physical structures are likely to require different institutional design. In one of the examples considered in this paper, a proper physical description would help determine whether conventional or Arrow markets are appropriate. Using Arrow markets in cases where conventional markets are required will cause inefficiency by reducing the degree of freedom in input substitution or 'defensive' activity. Clearer descriptions of the underlying physical economic relationships are important in order to determine 'appropriate' or corresponding institutions.

In this light, the question of bounds acquires a new perspective. The possibility that endowment bounds have not been appropriately set can be viewed as another kind of incongruence (externality) between physical and institutional descriptions of economic space. While property rights may be defined for some resource, it may be that the total quantities of rights do not align with the underlying physical availability. In the case of pollution or 'air' use rights we would like the benchmark for total allowable pollution (or air use) to align with physical possibilities. If there are more or fewer pollution permits than physically possible it is unlikely that a competitive equilibrium will be Pareto optimal. Universality of markets, a prerequisite for the first fundamental welfare theorem, must also mean that bounds are adequately determined. This poses a potentially difficult task for public policy: the social definition of scarcity.[28] To the extent that physical bounds are not self-evident, society must find a means discovering them so as to ensure that endowments reflect physical scarcity.[29]

[28] Many of the more intriguing and important questions have to do with how institutions are formed and the normative counterpart, how institutions should be formed. How are bounds to be determined? There are often likely to be physical limits to external activity. But what if we have difficulty discerning these limits? The role of discerning physical limits or the environment's capacity is far from trivial. It is also different than that of trying to discover the optimal levels of pollution which would be subject to a specific distribution of wealth or social welfare objective.

[29] The examples touched on in this paper suggest that deeper reflection on the underlying physical world will uncover limits or bounds that were otherwise missed. This need not always be the case. A better description of the physical world could reveal that limits were falsely assumed. David Reed (2002, 2001) has argued that the allocation of spectrum rights has been based on a false metaphor 'that equates spectrum allocations with rights to physical property, such as land use rights' (2001). In this metaphor, the set of frequencies are limited and setting up a market for these will ensure that they are allocated efficiently. In contrast, Reed alludes to quantum dynamics and says that spectrum is more like color and that there is no more scarcity in spectrum than there is in the color green. If this is the case, then present spectrum policy is seriously inefficient and the right policy is to open up the spectrum for all to use freely.

As long as our models feature a granular world, nicely divisible into parcels that can be individually owned and consumed, conventional markets for private property rights can go a long way in allocating resources efficiently. However, physical jointness and/or indivisibilities call for more sophisticated institutional devices. Lindahl markets are a complex (theoretical) institutional device meant to deal with certain kinds of physical jointness. Physical complexity, on the other hand, can be such that the conventional solutions, envisaging joint consumption and use, are inadequate means of aligning effectively the institutional devices with physical reality. If the task of managing the environment raises so many challenges, it is precisely because the environment departs so starkly from the conventional model of a divisible world. The fundamental continuity of environmental media and ecosystems raises the specter of large-scale indivisibilities and a complex tangle of joint and multiple uses. Many environmental services are better viewed as outputs of natural production processes than as resources to be allocated. The difficult task of designing institutions for managing our environment demands institutional ingenuity rooted in a deep understanding of its physical structure.

By viewing externality as an incongruence between the physical and institutional economic space, we see that the kinds of externality, as well as the institutional means to overcome them, will vary according to how the physical and institutional space is defined. The greater the variety of physical interaction we try to model, the richer the corresponding institutional forms we shall be needing. This paper does not offer an all-encompassing means of eliminating externalities. Instead it proposes an interpretation of externality within the confines of a frictionless Arrow Debreu framework that recognizes the potentially rich variety of physical interdependencies in the world and the institutional challenges that these pose.

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